Maximum And Minimum of Coloring of Certain Triangular Line Graphs

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Abstract- *The Line graph L(G) of an undirected graph G is another graph L(G) that represents the find adjacencies between the edges of G. In this paper, we find the Maximum and Minimum of total coloring for a certain Line graphs of a snake graph families and further we established the results on maximum number colors required to total coloring to the graph G is denoted by* $\chi_{Max}(G)$ *and similarly we color the vertex of a snake graph and obtained certain results and denoted by* $\chi_{Min}(G)$.

Keywords- Line graphs , Coloring of a graph, Snake graph.

I. INTRODUCTION

In Graph theory, the Line graph *L*(*G*) of undirected graph *G* is another graph *L*(*G*) that represents the adjacencies between the edges of *G*. Other terms used for the Line graph are the covering graph, the edge-to-vertex dual, the conjugate, the representative graph, the edge graph, the interchange graph, the adjoint graph and the derived graph. One of the earliest and most important theorems about Line graphs is due to Hassler Whitney (1932), who proved that with one exceptional case the structure of *G* can be recovered completely from its Line graph.\

The Line graph is defined as follows. The Line graph of *G* denoted by *L*(*G*) is the intersection graph of the edges of G, representing each edge by the set of its two end vertices. Otherwise *L*(*G*) is a graph such that

1. Each vertex of *L*(*G*) represents an edge of *G* .
2. Two vertices of *L*(*G*) are adjacent if their corresponding edges share a common end point in *G*.

A proper coloring of a graph is an assignment of colors (represented by natural numbers) to the vertices of *G* such that no two adjacent vertices are assigned the same color. Equivalently a proper coloring is a partition of the vertex set *V* into independent sets *V*1,*V*2…*Vk*. The sets *Vi* are called *color classes*. The minimum number of colors used to colorize the graph *G* is called the *chromatic number* and is denoted by $\chi_{Min}(G)$.

In this paper we will discuss the coloring for the particular line graph .The maximum coloring of vertex is denoted by $\chi_{Max}(G)$ and minimum coloring of the vertex for the graph is denoted by $\chi_{Min}(G)$.

Line graphs of triangular snakes

The line graph *L*(*G*) of an undirected graph *G* is another graph *L*(*G*) that represents the adjacencies between edges of *G*. In other words, given a graph *G*, its line graph *L*(*G*) is a graph such that (i) each vertex of *L*(*G*) represents an edge of *G* and (ii) two vertices of *L*(*G*) are adjacent if and only if their corresponding edges are adjacent in *G*.

1. Coloring of Triangular Line Snake Graph (L_n) :

Theorem 1 : For the Line graph of G_n and a triangular snake line graph T(n) , for $n \geq 7$, **Then** $\chi[L(T(n)] = n-1.$

Proof : Let T(n) be a triangular line snake graph with n-vertices , Let the vertices set of T(n) be $\{v_1, v_2, v_3 \ldots\ldots\ldots\ldots\ldots\ldots v_n\}$ with v_n as the end vertex, Now let L[T(n)] contains a complete graph of K_{n-1} .with $K_{n-1}=\{x_1,x_2,x_3,\ldots\ldots\ldots\ldots x_n\}$ with $v(c)=\{u_1,u_2,u_3,\ldots\ldots\ldots\ldots u_n\}$. Now assigning the C_i to the vertex v_i for i={1,2,3,.............(n-1)} . Now color the vertices of the cycle, Suppose we wish to introduce the same coloring say as C_n. We should color the vertex of the cycle C. In C each vertex v_i is adjacent with u_{i+1}, for i=2,34,....(n-1). And x_i ,x_{i+1} for i=1,2,3,4.....(n-1). If u_1 i adjacent with u_{i+1} ,u_n ,x_i,x_{i+1}, same way u_n is adjacent with u_{n-1},u_1,

x_{n-1},x_1. Thus the vertex is colored and the max colored used in this graph we denote asand the minimum colored required for this graph is denoted by .

The graph forms a cycle the end color cannot be assigned as the same for the beginning color so that

2. Coloring of Quadrilateral Line Snake graph

Theorem : $\chi[L_n(G)] = (n-1).$

For a Line graph of Gn For $n \geq 3.$

proof : Let Q(n) be a quadrilateral snake graph each vertex will forms cycle, The edge set of Q(n) is given as follows.

a) a{e1,e2,e3 ....en }be a set of edges in Qn .
b) {en+1,en+2,en+3................................e3n} be the edges of cycle Qn.

Let the vertex set of L[Qn] be {v1,v2,v3...........................vn} The vertex set of {v1,v2,v3...........................vn} will forms a complete graph . Now assigning the coloring as follows . Assign the colour *ci* to the vertex *vi* for *i=1, 2, 3….n*.

Next we have to colour the vertices {*vn+1, vn+2, …v3n*}. Since the colouring should be minimum, we cannot introduce new colours to those vertices. So assign only the existing colours to those vertices. The minimum coloring is denoted by . Next we assign the colour *ci* for *i=1, 2,…n* to the vertices *vi* for *i= n+1* to *2n.*

Assign the colours *ci* for *i=1, 2…n-2* to the vertices *i=2n+1* to *3n-2* and the colours C*n* to *v3n-1* and *cn-1* to *v3n*. Clearly the colouring is minimum.

3. Coloring of Star Line graph :

$\chi[L(S_n)] = 2n+1$

Theorem : For a Star Snake Line graph of S_n ,

Proof : Let {v_1,v_2,….*vn*} represent the pendant vertices of S_n, {*vn+1,vn+2,.....v2n*} represent the vertices of the cycle in S*n* , *vij* be the introduced vertex to subdivide (*vi*,*v*j) where *i,j =1* to *2n+1*. In the vertex subset {*v1, v2,…. vn*} and the vertices *vij* for *i = n+1* to *2n, j=2n+1* form *k1,2n* star graph. Now assign the star colouring as follows. Assign the colours *ci* to the pendant vertices *vi* for *i=1* to *n,* the colours *ci* to the vertices of the cycle *vi* for *i=n+1* to *2n*. Also assign the colour *c2n+1* to *v2n+1*. Next assign the colours to *vij* of the cycle as *ci+1* to *vij* *i=2n* and *j=2n+1*. For every *vi* , for *i=n+1* to *2n* has four adjacent vertices that is to be coloured with different colours. Since the colouring should be minimum, Further we denote for this as we try to repeat the colours that are already used. Assign the colours *ci*, for *i=1* to *n-1* to *vij* for *i=n+1* to *2n-1, j=i+1* and the colour *cn* to *vij* for *i=2n, j=n+1*. Also assign the colour *ci+1* to *vij* for *i = 1* to *n-1, j=i+n* and colour *c1* to *vij* for *i=n, j=2n*. This is clearly star colouring, since the four adjacent vertices *vij* to the vertex *vi* get different colours and the colouring is also minimum. $\chi_{Min}[L_n(G)]$ Thus, *X*[L(Sn)] = *2n+1*.

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