A Comparative Study on Existing Methods of An Assignment Problem

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Abstract- *In this paper, we propose some results on comparative study in the existing methods of an assignment problem. We illustrate a numerical example explains the comparative study. Finally we discuss some results.*

Keywords- Assignment problem, Best candidate method, Hungarian method.

I. INTRODUCTION

An assignment problem is a special type of transportation problem where in the number of origins equals number of destinations. The capacity and demand value is exactly one unit. The objective is to determine which origin should supply one unit to which destination so that the total cost is minimum.

Consider a problem of assigning n workers to n jobs so as to minimize the overall cost or time in such a way that each workers can associate with one and only one job.

Let x_{ij} denote the assignment of i^{th} worker j^{th} job, that

$$x_{ij} = \begin{cases} 1, & \text{If worker 'i' is assigned to job 'j'} \\ 0, & \text{otherwise} \end{cases}$$

Then, the mathematical formulation of the assignment problem is as follows.

Minimize Z = $\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\, x_{ij}$

Subject to the constraints. $\sum_{i=1}^{n} x_{ij} = 1$ and

$$\sum_{j=1}^{n} x_{ij} = 1; x_{ij} = 0 \text{ or } 1$$

For all i=1,2,….,n and j =1,2,…,n.

Where C_{ij} is the associated with assigning i^{th} worker to j^{th} job. These concepts are taken from[2].

This type of problem are encountered in many area of human life. In the year (1956), the solution procedure for the problem has been developed by Kuhn and Flood[4]. Also the justification of the steps leading to the solution based on theorems proved by Hungarian mathematicians Konelg (1950) and Egervary(1953) [4]. In the year 2009 Burkard, R.E. and et.al discussed solution to the assignment problem in the first edition of SIAM, Philadelphia[1]. In the year 2009, Rahedi, N.T.A. and J. Atoum proposed an algorithm for solving the travelling salesman problem using new operators in genetic algorithms[5]. Hlayel Abdallah Ahamad proposed the best candidates method for solving Optimization Problems in the year 2012[3].

In this paper consist of four sections fist section explains the introductory concepts. Existing methods Are discussed in section tow. Section three compared the existing methods. Conclustions are given in section four.

II. EXISTING METHODS

In this section we briefly discuss most relevant existing research articles for solving assignment problem.

2.1 Hungarian algorithm:

Step1: Determine the effectiveness matrix. Subtract the minimum element of each row of the given cost matrix from all of the elements of the row. Examine if there is at least one zero in each row and in each column. If it is so, stop here, otherwise subtract the minimum element of each column from all the elements of the column. The resulting matrix is the starting effectiveness matrix

Step 2: Assign the zeroes: (a). Examine the rows of the current effective matrix successively until a row with exactly one unmarked zero is found. Mark this zero, indicating that an assignment will be made there. Mark all other zeroes lying in the column of above encircled zero. The cells marked will not be considered for any future assignment. Continue in this manner until all the rows have taken care of (b). Similarly for columns

Step 3: Check for Optimality. Repeat step 2 successively till one of the following occurs. (a). There is no row and no column without assignment. In such a case, the current assignment is optimal. (b). There may be some row or column without an assignment. In this case the current solution is not optimal. Proceed to next step

Step 4: Draw minimum number of lines crossing all zeroes as follows. If the number of lines is equal to the order of the matrix, then the current solution is optimal, otherwise it is not optimal. Go to the next step.

Step 5: Examine the elements that do not have a line through them. Select the smallest of these elements and subtract the same from all the elements that do not have a line through them and add this element to every element that lies in the intersection of the two lines.

Step 6: Repeat this until an optimal assignment is reached

Example:

Consider the problem of assigning five jobs to five persons. The assignment costs are given below

Job						
		1	2	3	4	5
Person	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

The cost matrix of the given assignment problem

$$\begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix}$$

Row Minima

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Column minima

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 3 & 0 & 5 & (0) \\ (0) & 9 & 5 & 5 & 4 \\ 1 & 6 & 6 & (0) & 4 \\ 4 & 3 & (0) & 0 & 3 \\ 4 & (0) & 2 & 4 & 0 \end{pmatrix}$$

Since each row and each column contains exactly one assignment (i.e,. exactly one encircled zero) the current assignment is optimal

The optimum assignment schedule is $A \to 5, B \to 1, C \to 4, D \to 3, E \to 2$

The optimum assignment cost = $(1+0+2+1+5)$ = 9 units of cost.

2.2 Best Candidates Method (BCM) has the following solution steps:

Step1: Prepare the matrix. If the matrix unbalanced, we balance it and don't use the added row or column candidates in our solution procedure.

Step2: Election the best candidates, that is for minimization problems minimum cost and for maximize profit max cost: Elect the best two candidates in each row, if the candidate repeated more than two times elect it also. Check the columns that not have candidates and elect one candidate for them, if the candidate repeated more than one time elect it also.

Step3: Find the combinations. Determine only one candidate for each row and column starting from the row that have least candidates and delete that row and column If there is situation that have no candidate for some rows or columns select directly the best available candidate. Repeat step3 (1, 2) by determining the next candidate in the row that started from. Compute and compare the total sum of candidates for each combination to determine the best combination that give the optimal solution.

Example: Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows.

Step1: The matrix is balanced, where the number of columns equal to the number of rows as shown in (Table 1).

Table 1:

	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Step 2: Elect the best candidates

$$\begin{pmatrix} 8 & 4 & (2) & 6 & (1) \\ (0) & 9 & 5 & 5 & (4) \\ (3) & 8 & 9 & (2) & 6 \\ 4 & 3 & (1) & (0) & 3 \\ 9 & (5) & 8 & 9 & (5) \end{pmatrix}$$

Step 3: Find the combinations 1

$$\begin{pmatrix} 8 & 4 & (2) & 6 & 1 \\ 0 & 9 & 5 & 5 & (4) \\ (3) & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & (0) & 3 \\ 9 & (5) & 8 & 9 & 5 \end{pmatrix}$$

Step 3: Find the combinations 2

$$\begin{pmatrix} 8 & 4 & 2 & 6 & (1) \\ (0) & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & (2) & 6 \\ 4 & 3 & (1) & 0 & 3 \\ 9 & (5) & 8 & 9 & 5 \end{pmatrix}$$

Here we have just two Combinations.

Combination 1 gives

(A3, B5, C1, D4, E2) = 2 + 4 + 3 + 0 + 5 =14

Combination 2 gives

(A5, B1, C4, D3, E2) = 1 + 0 + 2 + 1 + 5 = 9 which is the optimal one.

III. COMPARISIONS BETWEEN EXISTING METHODS

We take there are only two methods for the comparison to the study. Namely

1. The Best candidates method.
2. The Hungarian method.

1.The Best candidate Method

1. The BCM based on election the best candidates and the alternative in each row and cover all columns with at least one candidate, then we can obtain the combinations that must be have one candidate for each row and column.
2. The BCM comparing to the Hungarian Method as shown from the solution steps can obtained the best combinations with less computation time and without complexity.
3. It is simple to reader comparing with other method.
4. It is clear to understand.

2. Hungarian method

1. To obtain the optimal solution to the assignment problem is maximum computation time comparing with best candidates method.
2. There is complexity to compute the solution of the problem.
3. It is difficult to compare best candidates method.
4. It is not clear to understand.

IV. CONCLUSION

In this study ,I have proposed the BCM for solving assignment problem. The optimization is very important because of their wide applicability in different area, where it refers to choosing the optimal solution from the set of available alternative. The BCM can be used with good deal of success on these problems, because it obtained the optimal solution with minimum computation time, reduce the complexity and it have simple and clearly solution manner which is can be easy to use on different area of optimization problems.From this comparative study the optimal value is same.

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