On Solving Fully Fuzzy Linear Programming Problems With Hexagonal Fuzzy Numbers By Simplex Method

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Abstract- *In this paper, a method for solving fully fuzzy linear programming problems in which the hexagonal fuzzy numbers are involved in both the objective functions and also the constraints are proposed. Simplex procedure is used for solving fully fuzzy linear programming problems by using the ranking function of hexagonal fuzzy numbers. The results are also compared with the solutions obtained from crisp linear programming problems. Numerical examples are also provided for verifying the results.*

Keywords- Fuzzy linear programming, Fully fuzzy linear programming, hexagonal fuzzy numbers, ranking functions.

I. INTRODUCTION

Fuzzy Linear programming was first formulated by Zimmermann [4]. Recently, these problems are considered in several kinds (i) Fuzzy parameters are involved in both the objective function and also the constraints (ii) Fuzzy parameters are involved only in the right hand side of the constraints (iii) Fuzzy variables are involved in both the objective functions and also the constraints (iv) Fuzzy coefficients are involved only in the objective functions (v) Fuzzy parameters are involved in the the coefficients and also the right hand side of the constraints. Nasseri and Ardil [2] deals with the Simplex method for fuzzy variable linear programming problems as in (iii). Nasseri et.al.[3] also deals with the Simplex method for fuzzy number linear programming prolems as in (iv). This paper focused only on the first kind (i). It can be solved by using the ranking functions of hexagonal fuzzy numbers. This paper is organized as follows: In section 2, hexagonal fuzzy numbers , ranking function and arithmetic operations of hexagonal fuzzy numbers are given as in [1]. Section 3 deals with fully fuzzy linear programming problems and also a solution algorithm for soving a fully fuzzy linear programming problem by simplex method is proposed. In section 4, some numerical examples are provided and the results are compared with the results of crisp linear programming problem. The last section draws some concluding remarks.

II. PRELIMINARIES

In this section, we discuss the hexagonal fuzzy numbers and their arithmetic operations as in [1].

2.1 : Hexagonal Fuzzy Number

Let us consider a hexagonal fuzzy number $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$, where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6$ which are real numbers satisfying $a_2 - a_1 \leq a_3 - a_2$ and $a_5 - a_4 \leq a_6 - a_5$ and its membership function is given by,

$$\mu_{\tilde{A}h}(x) = \begin{cases} 0 & if\ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & if\ a_1 \leq x \leq a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & if\ a_2 \leq x \leq a_3 \\ 1 & if\ a_3 \leq x \leq a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & if\ a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & if\ a_5 \leq x \leq a_6 \\ 0 & if\ x \geq a_6 \end{cases}$$

2.2: Ranking function

Let $\mathcal{F}$ (R) be the set of all hexagonal fuzzy numbers.
For $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \in \mathcal{F}$ (R), we define the ranking function, $\check{R}: \mathcal{F}(R) \rightarrow R$ by

$\check{R}(\tilde{A}_h) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} \right)$ as in [1]

2.3: Arithmetic operations on hexagonal fuzzy numbers

For $\tilde{A}_h=(a_1,a_2,a_3,a_4,a_5,a_6)$ and $\tilde{B}_h=(b_1,b_2,b_3,b_4,b_5,b_6)$ in $\mathcal{F}(R)$, we define

1. **Addition** : $\tilde{A}_h + \tilde{B}_h = (a_1+b_1 , a_2+b_2 , a_3+b_3 , a_4+b_4 , a_5+b_5 , a_6+b_6)$

2. **Subtraction:** $\tilde{A}_h - \tilde{B}_h=(a_1-b_6 , a_2-b_5 , a_3-b_4 , a_4-b_3 , a_5-b_2 , a_6-b_1)$

3. **Multiplication:** $\tilde{A}_h * \tilde{B}_h=(a_1\check{R}(\tilde{B}_h), a_2\check{R}(\tilde{B}_h), a_3\check{R}(\tilde{B}_h), a_4\check{R}(\tilde{B}_h), a_5\check{R}(\tilde{B}_h), a_6\check{R}(\tilde{B}_h))$

4. **Division:** $\tilde{A}_h / \tilde{B}_h = (a_1/ \check{R}(\tilde{B}_h), a_2/ \check{R}(\tilde{B}_h), a_3/ \check{R}(\tilde{B}_h), a_4/ \check{R}(\tilde{B}_h), a_5/ \check{R}(\tilde{B}_h), a_6/ \check{R}(\tilde{B}_h))$

Where $\check{R}(\tilde{B}_h) = \frac{b_1 + b_2 + b_3 + b_4 + b_5 + b_6}{6}$

III. FULLY FUZZY LINEAR PROGRAMMING

Fuzzy linear programming problems in which the fuzzy parameters are involved in both the objective functions and also the constraints are referred to as a fully fuzzy linear programming problem and its general form is

Maximize $\tilde{Z} = \sum_{j=1}^{n} \tilde{C}_j \tilde{X}_j$

subject to the constraints

$$\sum_{j=1}^{n} \tilde{A}_{ij} \tilde{X}_j \le \tilde{B}_i, \quad i \in N_m$$

$$\tilde{X}_j \ge 0$$

Here $\tilde{Z}$, $\tilde{A}_{ij}, \tilde{B}_i, \tilde{C}_j$ are considered as hexagonal fuzzy numbers and $\tilde{X}_j$ are decision variables whose states are hexagonal fuzzy numbers.

3.1: SOLUTION ALGORITHM FOR SOVING A FULLY FUZZY LINEAR PROGRAMMING PROBLEM BY SIMPLEX METHOD

In this section, an algorithm for solving a fully fuzzy linear programming problem is provided.

Step : 1 Transform the problem into standard form.

Step : 2 Set up the initial simplex tableau.

Step : 3 Identify the negative entry which is largest in magnitude among all entries corresponding to non basic fuzzy variables in the objective function row using the ranking function of hexagonal fuzzy number. Ties may be settled arbitrarily. (If all such entries are non- negative, go to step10) Suppose the entry in column i is identified.

Step : 4 Identify all non negative elements in column i.

Step : 5 For each element identified in step 4, form a rank of right hand side hexagonal fuzzy numbers using its ranking function.

Step : 6 Choose the minimum value and identify to which row it belongs, say row j. Ties may be settled arbitrarily.

Step : 7 Identify the basic variable which has a unit entry in row j, say x_k

Step : 8 Replace fuzzy variable x_k by fuzzy variable x_i in the basis using Gauss-Jordan elimination.

Step : 9 Go to step 3.

Step : 10 The optimal solution has been found. Each basic variable is set equal to the entry in the right hand side column corresponding to the row in which the values have the unit entry. All variables are set equal to zero. The optimal solution value is equal to the entry at the intersection of the x_o row and the right hand side column.

IV. NUMERICAL EXAMPLE

Let us consider the following crisp linear programming problem

Maximize $Z = 4 x_1 + 10 x_2$

Subject to the constraints

$2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

$x_1 , x_2 \ge 0$

Using TORA software, the optimal solution is obtained as $x_1 = 0$, $x_2 = 20$ and

Maximize z = 200.

The above linear programming problem is converted in to the following fully fuzzy linear programming problem and the optimal solution is compared

Maximize $\tilde{Z} = \tilde{4}\,\tilde{x}_1 + \tilde{10}\,\tilde{x}_2$

Subject to the constraints

$\tilde{2}\,\tilde{x}_1 + \tilde{x}_2 \leq \tilde{50}$

$\tilde{2}\,\tilde{x}_1 + \tilde{5}\,\tilde{x}_2 \leq \tilde{100}$

$\tilde{2}\,\tilde{x}_1 + \tilde{3}\,\tilde{x}_2 \leq \tilde{90}$

$\tilde{x}_1,\ \tilde{x}_2 \geq 0$

where , $\tilde{4}, \tilde{10}, \tilde{2}, \tilde{3}, \tilde{5}, \tilde{x}_1, \tilde{x}_2$ are fuzzy numbers

Here, we considered the fuzzy numbers as a hexagonal fuzzy numbers then the above fully fuzzy can be transformed in to the following form

Maximize $\tilde{z}$ = (2,3,4,4,5,6) $\tilde{x}_1$ +(8,9,10,10,11,12) $\tilde{x}_2$

Subject to the constraints

(0,1,2,2,3,4) $\tilde{x}_1$ + (-1,0,1,1,2,3) $\tilde{x}_2 \leq$ (48, 49, 50, 50, 51, 52)

(0,1,2,2,3,4) $\tilde{x}_1$ + (3,4 ,5,5,6,7) $\tilde{x}_2 \leq$ (98, 99,100,100,101,102)

(0,1,2,2,3,4) $\tilde{x}_1$ + (1,2,3,3,4,5) $\tilde{x}_2 \leq$ (88, 89, 90, 90, 91, 92)

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5 \geq 0$

Its standard form is

Maximize $\tilde{z}$ = (2,3,4,4,5,6) $\tilde{x}_1$ +(8,9,10,10,11,12) $\tilde{x}_2$ +(0,0,0,0,0,0) $\tilde{x}_3$ +(0,0,0,0,0,0) $\tilde{x}_4$ +(0,0,0,0,0,0) $\tilde{x}_5$

Subject to the constraints

(0,1,2,2,3,4) $\tilde{x}_1$ + (-1,0,1,1,2,3) $\tilde{x}_2$ +(- 1,0,1,1,2,3) $\tilde{x}_3$ + (0,0,0,0,0,0) $\tilde{x}_4$ +(0,0,0,0,0,0) $\tilde{x}_5$ = (48, 49, 50, 50, 51, 52)

(0,1,2,2,3,4) $\tilde{x}_1$ + (3,4 ,5,5,6,7) $\tilde{x}_2$ + (0,0,0,0,0,0) $\tilde{x}_3$ +(-1,0,1,1,2,3) $\tilde{x}_4$ + (0,0,0,0,0,0) $\tilde{x}_5$ = (98, 99,100,100,101,102)

(0,1,2,2,3,4) $\tilde{x}_1$ + (1,2,3,3,4,5) $\tilde{x}_2$ + (0,0,0,0,0,0) $\tilde{x}_3$ + (0,0,0,0,0,0) $\tilde{x}_4$ + (-1,0,1,1,2,3) $\tilde{x}_5$ = (88, 89, 90, 90, 91, 92)

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5 \geq 0$

Enter the values into the simplex table

			(2,3,4,4,5,6)	**(8,9,10,10,11,12)**	**(0,0,0,0,0,0)**	**(0,0,0,0,0,0)**	**(0,0,0,0,0,0)**	
C_j	**Y_B**	**X_B**	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	**Ratio**
(0,0,0,0,0,0)	$\tilde{x}_3$	(48,49,50,50,51,52)	(0,1,2,2,3,4)	(-1,0,1,1,2,3)	(-1,0,1,1,2,3)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(48,49,50,50,51,52)
(0,0,0,0,0,0)	$\tilde{x}_4$	(98,99,100,100,101,102)	(0,1,2,2,3,4)	**(3,4,5,5,6,7)**	(0,0,0,0,0,0)	(-1,0,1,1,2,3)	(0,0,0,0,0,0)	**(19.6,19.8,20,20,20.2,20.4)**
(0,0,0,0,0,0)	$\tilde{x}_5$	(88, 89, 90, 90, 91, 92)	(0,1,2,2,3,4)	(1,2,3,3,4,5)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(-1,0,1,1,2,3)	(29.3,29.6,30,30,30.3,30.6)
		$Z_j - C_j$	(-6,-5,-4,-4,-3,-2)	**(-12,-11, -10,-10,-9,-8)**	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	

			(2,3,4,4,5,6)	**(8,9,10,10,11,12)**	**(0,0,0,0,0,0)**	**(0,0,0,0,0,0)**	**(0,0,0,0,0,0)**
C_j	**Y_B**	**X_B**	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$
(0,0,0,0,0,0)	$\tilde{x}_3$	(-12,9,30,30,51,72)	(1.2,0.2,1.6,1.6,3,4.4)	(-4,-2,0,0,2,4)	(-1,0,1,1,2,3)	(-0.6,-0.4,-0.2,-0.2,0,0.2)	(0,0,0,0,0,0)
(8,9,10,10,11,12)	$\tilde{x}_2$	(19.6,19.8,20,20,20.2,20.4)	(0,0.2,0.4,0.40.6,0.8)	(0.6,0.8,1,1,1.2,1.4)	(0,0,0,0,0,0)	(-0.2,0,0.2,0.2,0.4,0.6)	(0,0,0,0,0,0)
(0,0,0,0,0,0)	$\tilde{x}_5$	(-12,9,30,30,51,72)	(-2,-0.6,0.8,0.8,2.2,3.6)	(-4,-2,0,0,2,4)	(0,0,0,0,0,0)	(-1,-0.8,-0.6,-0.6,-0.4,-0.2)	(-1,0,1,1,2,3)
		$Z_j - C_j$	**(-2.8,-1.4,0,0,1.4,2.8)**	**(-4,-2,0,0,0,2,4)**	**(0,0,0,0,0,0)**	**(1.6,1.8,2,2,2.2,2.4)**	**(0,0,0,0,0,0)**

Since all $Z_j - C_j \geq 0$, the optimal solution is obtained.

The values of the decision variables are

$\tilde{x}_1$ =(0,0,0,0,0,0) and $\tilde{x}_2$ = (19.6,19.8,20,20,20.2,20.4)

Therefore,

Maximize $\tilde{z}$ = (160,180,200,200,220,240)

Ř($\tilde{z}$) = 200 , which is same as the crisp linear programming problem.

V. CONCLUSION

We considered the fully fuzzy linear programming problem in which the fuzzy numbers are involved in both the objective functions and also the constraints are successfully dealt with hexagonal fuzzy numbers and find out the optimal solution for the Simplex method using a ranking function which is same as the crisp linear programming problem.

REFERENCES

[1] Stephen Dinagar.D , Hari Narayanan.U,”A note on arithmetic operations of hexagonal fuzzy numbers using the α-cut method” International Journal of Applications of fuzzy sets and Artificial Intelligence,Vol.6 (2016),145-162.

[2] Nasseri.S.H and Ardil.E, “Simplex method for fuzzy variable linear programming problems”,World Academy of Science, Engineering and Technology. 8 (2005) 198-202.

[3] Nasseri.S.H and Ardil.E,Yazdani.A,Zaefarian.R,” “Simplex method for solving linear programming problems with fuzzy numbers”,World Academy of Science, Engineering and Technology. Vol.1,no.10 (2007) 519 – 523

[4] Zimmermann.H.J, “Fuzzy linear programming with several objective functions”fuzzy sets and systems(1978) 45-55.