A Study on Fuzzy Relation Equations and Their Applications

Ms.R.Saranya[1], Ms.S.Dhanasri[2], Ms.K.Malathi[3]
[1, 2, 3]Assistant Professor, Dept of Science & Humanities
[1, 2, 3] SRVEC

Abstract- *In this paper we investigate the System of fuzzy relation equations with applications. We base on the knowledge of the system of fuzzy relation equations with a new algorithms. First, the general concepts are explained and then are shown the Properties of Fuzzy relation equation. Finally is investigated the modified fuzzy relation equation has been derived for analyzing passenger preference for a particular hour in a day.*

Keywords- System of fuzzy relation equations, fuzzy comptability relations, super-i-compositions.

I. INTRODUCTION

The notion of fuzzy relational equations based upon the max-min composition was first investigated. He studied conditions and theoretical methods to resolve fuzzy relations on fuzzy sets defined as mappings from sets to [0, 1]. Before we go into the discussion of these Fuzzy Relational Equations (FRE) and its properties it uses and applications we just describe them.

Next we give several of the applications of fuzzy relational equations in studies like chemical engineering, transportation, medicine etc. The fuzzy relational equations happen to be a seemingly simple method but in reality it can be used to solve many complicated problems, problems that even do not have solutions by using linear equations. This chapter is completely devoted to the applications of fuzzy relation equations.

II. PRELIMINARIES

2.1Fuzzy relations:

A classical relation can be considered as a set of tuples, where a tuple is an ordered pair. A binary tuple is denoted by (u, v), an example of a ternary tuple is (u, v, w) and an example of n-ary tuple is $(x_1,\dots ,x_n)$.

2.2 Properties of Fuzzy Relations:

We just recollect the properties of fuzzy relations like, fuzzy equivalence relation, fuzzy comptability relations, fuzzy ordering relations, fuzzy morphisms and super-i-compositions of fuzzy relation.

Now we proceed on to define fuzzy equivalence relation. A crisp binary relation R(X, X) that is reflexive, symmetric and transitive is called an equivalence relation. For each element x in X, we can define a crisp set A_x, which contains all the elements of X that are related to x, by the equivalence relation.

$$A_x = \{y \setminus (x, y) \in R (X, X)\}$$

A_x is clearly a subset of X. The element x is itself contained in A_x due to the reflexivity of R, because R is transitive and symmetric each member of A_x, is related to all the other members of A_x. Further no member of A_x, is related to any element of X not included in A_x. This set A_x is referred to an as equivalence class due to the reflexivity of R, because R is transitive and symmetric each member of A_x, is related to all the other members of A_x. Further no member of A_x, is related to any element of X not included in A_x. This set A_x is referred to an as equivalence class of R(X,X) with respect to x. the members of each equivalence class can be considered equivalent to each other and only to each other under the relation R.

We know every fuzzy relation R can be uniquely represented in terms of its α-cuts by the formula

$$R = \bigcup \alpha_{\alpha\epsilon(0,1)}\, \alpha_R$$

It is easily verified that if R is a similarity relation then each α-cut, $^{\alpha}R$ is a crisp equivalence relation. Thus we may use any similarity relation R and by taking an α - cut $^{\alpha}R$ for any value α ∈ (0, 1], create a crisp equivalence relation that represents the presence of similarity between the elements to the degree α. Each of these equivalence relations form a partition of X.

Fuzzy equivalence is a cut worthy property of binary relation R(X, X) since it is preserved in the classical sense in each α-cut of R. This implies that the properties of fuzzy reflexivity, symmetry and max-min transitivity are also cut worthy. Binary relations are symmetric and transitive but not reflexive are usually referred to as quasi equivalence relations.

The notion of fuzzy equations is associated with the concept of compositions of binary relations. The composition of two fuzzy binary relations P (X, Y) and Q (Y, Z) can be defined, in general in terms of an operation on the membership matrices of P and Q that resembles matrix multiplication. This operation involves exactly the same combinations of matrix entries as in the regular matrix multiplication. In the max-min composition for example, the multiplication and addition are replaced with the min and max operations respectively. We shall give the notational conventions. Consider three fuzzy binary relations P (X, Y), Q (Y, Z) and R (X, Z) which are defined on the sets

$$X = \{x_i \mid i \in I\}$$
$$Y = \{y_j \mid j \in J\} \text{ and}$$
$$Z = \{z_k \mid k \in K\}$$

where we assume that $I = N_n$ $J = N_m$ and $K = N_5$. Let the membership matrices of P, Q and R be denoted by P = [p], $Q = [q_{ij}]$, $R = [r_{ij}, y_j)$,$q_{jk} = Q\ (y_j, z_k)$ $r_{ij} = R\ (x_i, z_k)$ for all i∈I $(=N_n)$,j∈J = (N_m) and k∈ˉ K(N_5).this clearly implies that all entries in the matrices P, Q, and R are real numbers from the unit interval [0, 1]. Assume now that the three relations constrain each other in such a way that P°Q = R where ° denotes max-min composition. This means that max $_{j\varepsilon J}$ min $(p_{ij,} q_{jk}) = r_{ik}$ for all i∈I and k∈ˉ K. That is the matrix equation P ° Q = R encompasses n × s simultaneous equations of the form minmax $(p_{ij}, q_{jk}) = r_{ik}$.

2.3 Fuzzy Relation Equation

When two of the components in each of the equations are given and one is unknown these equations are referred to as fuzzy relation equations.

When matrices P and Q are given the matrix R is to determined using P ° Q = R. The problem is trivial. It is solved simply by performing the max-min multiplication like operation on P and Q as defined by Max $x_j {\in}_J$ min $(p_{ij} ,q_{jk}) = r_{ik}$. Clearly the solution in this case exists and is unique. The problem becomes far from trivial when one of the two matrices on the left hand side of P ° Q = R is unknown. In this case the solution is guaranteed neither to exist nor to be unique. Then we can directly utilize this method for solving the second decomposition problem as well. we simply written P ° Q = R in the form Q^{-1} o $P^{-1}=R^{-1}$ employing transposed matrices. We can solve Q^{-1} o P^{-1} = R^{-1} for method and then obtain the solution of P ° Q = R by (Q-1)-1 = Q.

We study the problem of partitioning the equations P ° Q = R. we assume that a pair of matrices R and Q in the equations P ° Q = R is given. Let each particular matrix P that satisfies P ° Q = R is called its solution and let S (Q, R) = {P | P ° Q = R} denote the set of all solutions (the solution set).

It is easy to see this problem can be partitioned, without loss of generality into a set of simpler problems expressed by the matrix equations p_i o Q = r_i for all i∈I where $P_i = [p_{ij} \mid j \in J]$ and $r_i = [r_{ik} \mid k \in K]$.

Indeed each of the equation in max $j \in J$ min $(p_{ij} , q_{jk}) = r_{ik}$ contains simpler expressed by $p_i \circ Q = r_i$.

Thus the matrices P in S (Q, R)= [P | P ° Q = R] problem unknown p_{ij} identified only by one particular value of the index i, that is, the unknown p_{ij} distinguished by different values of i do not appear together in any of the individual equations. Observe that p_i Q, and $r_{i,}$ in p_i o Q = r_i representively, a fuzzy set on Y, a fuzzy relation on Y × Z and a fuzzy set on Z. Let $S_i\ (Q, r_i) = [p_i \mid p_i \text{ o } Q = r_i]$ denote, for each i∈I, the solution set of one of the can be viewed as one column matrix

$$P=\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ . \\ . \\ . \\ p_n \end{bmatrix}$$

where $p_i \in S_i\ (Q, r_i)$ for all i ∈ I = $(=N_n)$. It follows immediately from min max $j \in J$ min $(p_{ij}\ q_{jk}) = r_{ik}$. That if $\max_{j\varepsilon J} q_{jk} < r_{ik}$ for some i ∈ I and some k ∈ K, then no values $p_{ij} \in [0, 1]$ exists (j ∈ J) that satisfy P ° Q = R, therefore no matrix P exists that satisfies the matrix equation.

This proposition can be stated more concisely as follows if

$$\max_{j\varepsilon J} q_{jk} \leq \max_{j\varepsilon J}\ r_{ik,}$$

for some k ∈ K then S (Q, R) = φ.

III. NEW ALGORITHMS FOR SOLVING FRE

A new algorithm to solve the fuzzy relation equation

$$P \circ Q = R \quad (1)$$

With max-min composition and max-product composition. This algorithm operates systematically and graphically on a matrix pattern to get all the solutions of P.

Main results

Following are the main algorithms for solving (1) with max-min (or max-product) composition:

Step 1: Check the existence of the solution ***Step 2:*** Rank the elements of r with decreasing order and find the maximum solution $\bar{p}$

Step 3: Build the table M = [m_{jk}], j = 1, 2, …, m; k = 1, 2, …, n, where $m_{jk} \triangleq (\bar{P}, q)$. This matrix M is called "matrix pattern".

Step 4: Mark m_{jk}, which satisfies min $(\bar{P}_j, q_{jk}) = r_k$ (or $p_j . q_{jk} = r_k$), and then let the marked m_{jk} be denoted by $\overline{m}_{jk}$.

Step 5: If k_1 is the smallest k in all marked $\overline{m}_{jk}$, then set $\underline{p}_{j1}$ to be the smaller one of the two elements in $\overline{m}_{jk}$(or set $\underline{p}_{j1}$ to be $\underline{p}_{j1}$)

Step 6: Delete the j[th] row and the kith the column of M, and then delete all the columns that contain marked $\overline{m}_{jk}$., where $k \neq k_1$.

Step 7: In all remained and marked $\overline{m}_{jk}$., find the smallest k and set it to be k_1 , then let $\underline{p}_{j1}$ be the smaller one of the two elements in $\overline{m}_{j2k\,2}$.(or let $\underline{p}_{j2}$ be $\underline{p}_{j1}$).

Step 8: Delete the j_2^{th} row and the k_2^{th} column of M, and then delete all columns that contain marked $\overline{m}_{j\,2k\,2}$, where $k \neq k_2$. Repeat steps 7 and 8 until no marked

Step 9: Repeat steps 7 and 8 until no marked $\overline{m}_{j\,2k2}$ is remained.

Step 10: The other which are not set in steps 5-8, are set to be zero.

IV. APPLICATION OF FUZZY RELATION EQUATION

Modified fuzzy relation equation has been derived for analyzing passenger preference for a particular hour in a day:

Since any transport or any private concern which ply's the buses may not in general have only one peak hour in day, for; the peak are ones where there is the maximum number of passengers traveling in that hour. The passengers can be broadly classified as college student school going children, office going people, vendors etc. Each category will choose a different hour according to their own convenience. For example the vendor group may go for buying good in the early morning hours and the school going children may prefer to travel from 7.00 a.m. to 8 a.m., college students may prefer to travel from 8 a.m. to 9 a.m. and the office going people may prefer to travel from 9.00 a.m. to 10.00 a.m. and the returning hours to correspond to the peak hours as the school going children may return home at about 3.00 p.m. to 4.00 college students may return home at about 2.00 p.m. to 3.30 p.m. and the office going people may return home at about 5.00 p.m. to 6.00 p.m. Thus the peak hours of a day cannot be achieved by solve equation P o Q = R. So we reformulate this fuzzy relation equation in what follows by partition Qi's. This in turn partition the number of preferences depending on the set Q which correspondingly partitions R also. Thus the fuzzy relation equation say P o Q = R reduces to a set of fuzzy relations equations $P_1 \circ Q_1 = R_1$, $P_2 \circ Q_2 = R_2$, … , $P_s \circ Q_s = R_s$ where $Q = Q_1 \cup Q_2 \cup \ldots \cup Q_s$ such that $Q_i \cap Q_j = \varphi$ for $i \neq j$. Hence by our method we get s preferences. This is important for we need at least 4 to 5 peak hours of a day. Here we give a new method by which we adopt the feed forward neural network to the transportation problem.

We briefly describe the modified or the new fuzzy relation equation used here.

We know the fuzzy relation equation can be represented by neural network. We restrict our study to the form

$$PoQ = R \quad (*)$$

where o is the max-product composition; where $P = [p_{ij}]$, $Q = [q_{jk}]$ and $R= [r_{ik}]$, with $i \in N_n$, $j\in N_m$ and $k \in K_s$. We want to determine P. Equation (*) represents the set of equations.

$$\max_{j \in N_m} q_{jk}\, p_{ij} = \tau_{ik} \text{ for all } i \in N_n \text{ and } k \in N_s.$$

To solve equation (*) for $P_{ij}(i\in N_n,\ j \in N_m)$, we use the feed forward neural with m inputs and only one layer with n neurons.

First, the activation function employed by the neurons is not sigmoid function, but the so-called linear activation function *f* defined for all $a \in R$ by

$$f(a)=\begin{cases} o & if\ a < 0 \\ a & if\ a \in [0,1] \\ 1 & if\ a > 1 \end{cases}$$

second, the output y_i of neouron i is defined by $y_i = f\left(\max_{j\in N} W_{ij} \quad X_j\right)$ $(i \in N_n)$

Given equation 1, the training set of the columns q_k of matrix Q as input ($x_j = q_{ik}$ for each $j \in N_m$, $k \in N_s$) and column r_k of matrix R as expected output($y_i = r_{jk}$ for each $i \in N_n$ and $k \in N_s$). Applying this training set to the feed forward neutral network, we obtain a solution to equation 1, when the error function reaches zero. The solution is then expressed by the weight w_{ij} as $p_{ij} = w_{ij}$ for all $i \in N_n$ and $j \in N_m$.

Thus $p = (w_{ij})$ is a $n \times n$ matrix.

It is already known that the fuzzy relation equation is in the dominant stage and there is lot of scope in doing research in this area, further it is to be also tested in real data. Here we are transforming the single equation $P \circ Q = R$ into a collections of equations. When the word preference is said, there should be many preferences. If only one choice is given or the equation results in one solution, it cannot be called as the preference. Further, when we do some experiment in the real data. we may have too many solutions of preferences. For a unique solution sought out cannot or may not be available in reality, so to satisfy all the conditions described above, we are forced to reformulate the equation $P \circ Q = R$. . We partition the set Q into number of partition depending on the number preferences. When Q is partitioned, correspondingly R also gets partitioned, hence the once equation is transformed into the preferred number of equations.

Thus Q and R are given and P is to be determined. we partition Q into s sets, say $Q_1, Q_2, \ldots, Q_s$ such that $Q = Q_1 \cup Q_2 \cup \ldots \cup Qs$, correspondingly R will be partitioned as $R= R_1 \cup R_2 \cup \ldots \cup R_s$. Now the resulting modified fuzzy equations are $P_1 \text{ o } Q_1 = R_2, \ldots, Ps \text{ o } Qs = Rs$ respectively. Hence by our method we obtains preferences.

since in reality it is difficult to make the error function E_p to be exactly zero ,we in our new fuzzy relation equation accept for the error function E_p to be very close to zero. This is a deviation of the formula. Also we do not accept many stages in the arriving of the result. so once a proper guess is made even at the first stage .we can get the desired solution by making E_p very close to zero.

We are to find the passenger preference for a particular hour. The passenger preference problem for a particular hour reduces to finding the peak hours of the day (by peak hours of the day, we mean the number of passengers traveling in that hour is the maximum). since the very term, preference by a passenger for a particular hour is an attribute, we felt it would be interesting if we adopt the modified fuzzy relation equation to this problem.

So in our problem we use the fuzzy relation equation P o Q = R , where P denotes the preference of a passenger to particular hour, Q denotes the specific hour under study say h_i, $i = 1, 2, \ldots 17$,where h_i denotes the hour ending at 6.00 am ,h_2 denotes the hour ending at 7.00am ,….h_{17} denotes the hour ending at 10 p.m. and R_i denotes the number of passengers travelling during that particular hour h_i , for i=1,2,….,17.

Here we use the fuzzy relation equation to determine P. we formulate the problem as follows:

If h_i, for $i = 1, 2, \ldots, n$ are the n-hour endings, R_i, for $i=1,2,\ldots n$, denotes the number of passengers traveling during hour hi, for $i = 1, 2, \ldots, n$. We denote by R the set $\{R_1, R_2, \ldots, R_n\}$ and $Q= \{h_1, h_2, \ldots, h_n\}$. To calculate the preference of a passenger to a particular hour we associative with each R_i, a weight w_i. Since R_i Correspond to the number of passenger traveling in that hour hi, is a positive value and hence comparison between any two R_i and R_j's always exist. Therefore ,if $R_i < R_j$, , then we associate a weight w_i to R_i and w_j to R_j such that w_i and w_j take values the interval [0, 1].

Now we solve the matrix relation equation P o Q = R and obtain the preference of the passenger P for a particular time period, which is nothing but the maximum number of passengers traveling in that hour. If we wish to obtain only one peak hour for the day, we take all the n elements and form a matrix equation,

$$P_{max} \text{ o } \begin{bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \cdot \\ \cdot \\ \cdot \\ R_n \end{bmatrix}$$

and find the $n \times n$ matrix $P=(w_{ij})$ using the method described in the beginning.

If we wish to have two peaks hours, we partition Q into w_{ij} Q_1 and Q_2 so that correspondingly R gets partitioned in R_1 and R_2 and obtain the two peak hours using the two equations $P_1 \text{ o } Q_1 = R_1$ and $P_2 \text{ o } Q_2 = R_2$ respectively .

Clearly $P_1 = \begin{pmatrix} 1 \\ w_{ij} \end{pmatrix}$ the weights associated with set R_1 and $P_2 = \begin{pmatrix} 2 \\ w_{ij} \end{pmatrix}$ the weights associated with set R_2

. If we wish to have a peak s, s < n, then we partition for i = 1, 2, …, n into s disjoint sets and find the s peak hours of the day. This method of partitioning the fuzzy relation equation can be real world data problem, though we have described in the context of the transportation problem.

We have tested our hypothesis in the real data got from pallavan transport corporation.

Hour ending Q; 6, 7, 8, 9, 10, 11,12, 13, 14, 15,16,17,18,19, 20,21, 22.

Passenger per hour R: 96,71,222,269,300,220,2441,265,249,114,38,288,356,189,376 ,182,67.

We have partitioned the 17 hours of the day Q.

i) by partitioning Q into three elements each so as to get five preferences,
ii) by partitioning Q into five elements each so as to get three preferences and
iii) by arbitrarily partitioning Q into for classes so as to get four preferences

In all cases from these real data , our predicated value coincides with the real preference value. Since all the concepts are to be realized as fuzzy concepts, we at the first state make the entries of Q and R to lie between 0 and 1. This is done multiplying Q by 10-2 and R by 10-4 respectively.

We partition Q into three elements each by taking only the first 15 elements from the table. That is $Q = Q_1 \cup Q_2 \cup Q_3 \cup Q_4 \cup Q_5$ and $R = R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5$. For

$Q_1 = X_i$	$r_1 = r_{ik}$
0.0096	0.06
0.0096	0.07
0.0071	0

The fuzzy relation equation is

$$P_1 = \begin{bmatrix} 0.06 \\ 0.07 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 0.0096 \\ 0.0071 \\ 0.0222 \end{bmatrix}$$

We employ the same method described earlier, where, the linear activation function *f* is defined by

$$f(a) = \begin{cases} 0 \text{ if } a < 0 \\ a \text{ if } a \varepsilon [0,1] \\ 1 \text{ if } a < 1 \end{cases}$$

for all $a \in R$ and output y_i of the neuron i is defined by

$$y_1 = f\left(\max_{j \in N_m} W_{ij} \quad X_j\right) (i \in N_n)$$

calculate $\max_{j \in N_m} W_{ij} \quad X_j$ as follows:

(i) $w_{11}x_1 = 0.03 \times 0.06 = 0.0018$
$w_{12}x_2 = 0.0221875 \times 0.07 = 0.001553125$
$w_{13}x_3 = 0.069375 \times 0.08 = 0.0055$
∴ Max (0.0018, 0.001553125, 0.00555) = 0.00555
$f\left(\max_{j \in N_m} W_{ij} \quad X_j\right) = f(0.00555) = 0.00555$ (Since $0.00555 \in [0, 1]$)
∴ y_1 = 0.00555.

(ii) $w_{21}x_1 = 0.06 \times 0.06 = 0.0036$
$w_{22}x_2 = 0.044375 \times 0.07 = 0.00310625$
$w_{23}x_3 = 0.13875 \times 0.08 = 0.0111$
∴ Max (0.0036, 0.00310625, 0.0111) = 0.0111
$f\left(\max_{j \in N_m} W_{ij} \quad X_j\right) = f(0.0111) = 0.0111$ (Since $0.0111 \in [0, 1]$)
∴ y_2 = 0.0111

(iii) $w_{31}x_1 = 0.12 \times 0.06 = 0.0072$
$W_{32}x_2 = 0.08875 \times 0.07 = 0.0062125$
$W_{33}x_3 = 0.2775 \times 0.08 = 0.0222$
∴ Max (0.0072, 0.0062125, 0.0222) = 0.0222 (Since 0.0222 ∈ [0, 1])
∴ y_3 = 0.0222
$f\left(\max_{j \in N_m} W_{ij} \quad X_j\right) = f(0.0222) = 0.0222$ (Since $0.0222 \in [0, 1]$)
∴ y3 = 0.0222.

Feed Forward Neural Network representing the solution is shown above.

Therefore

$$P_1 = \begin{bmatrix} 0.03 & 0.06 & 0.12 \\ 0.0221875 & 0.044375 & 0.08875 \\ 0.069375 & 0.13875 & 0.2775 \end{bmatrix}$$

Verification:

Consider, P o Q = R

that is $\max_{j \in Nm} \quad q_{jk}\, p_{ij} = r_{ik}$
∴ Max (0.0018, 0.0042, 0.0096) = 0.0096 Max (0.00133125, 0.00310625, 0.0071) = 0.0071

Max (0.0041625, 0.0097125, 0.0222) =0.0222.

Similarly by adopting the above process, we have calculated the passenger preferences P_1, P_2, P_3 and P_5 for the pairs (Q_2, R_2), (Q_3, R_3), (Q_4, R_4) and (Q_5, R_5).

For

Q_2	R_2
0.09	0.0269
0.10	0.0300
0.11	0.0220

we have

$$P_2=\begin{bmatrix} 0.1345 & 0.269 & 0.06725 \\ 0.15 & 0.3 & 0.075 \\ 0.11 & 0.22 & 0.00605 \end{bmatrix}$$

For

Q_3	R_3
0.12	0.0241
0.13	0.0265
0.14	0.0249

We have

$$P_3=\begin{bmatrix} 0.2008 & 0.1004 & 0.0502 \\ 0.2208 & 0.1104 & 0.0552 \\ 0.2075 & 0.10375 & 0.051875 \end{bmatrix}$$

For

Q_4	R_4
0.15	0.0114
0.16	0.0381
0.17	0.0288

we have

$$P_4=\begin{bmatrix} 0.035625 & 0.07125 & 0.0178125 \\ 0.1190625 & 0.23125 & 0.05953125 \\ 0.09 & 0.18 & 0.045 \end{bmatrix}$$

For

Q_4	R_4
0.18	0.0114
0.19	0.0381
0.20	0.0288

$$P_5=\begin{bmatrix} 0.0445 & 0.089 & 0.178 \\ 0.023625 & 0.04725 & 0.0945 \\ 0.047 & 0.094 & 0.188 \end{bmatrix}$$

On observing from the table, we see the preference P1, P2, P3, P4, and P5 correspond to the peak hours of the day, h3 that is 8 a.m. with 222 passengers, h_5 that is 10 a.m. with 300 passengers, h_8 that is 1 p.m. with 265 passengers, h_{11} that is 4 p.m. with 381 passengers and h_{15} that is 8 p.m. with 376 passengers. Thus this partition gives us five preferences with coincides with the real data as proved by theworking.Maximumof(0.003108,0.00444,0.00666,0.0222,0.01221)=0.0222.

Maximum of (0.003766, 0.00538, 0.0080694, 0.0269, 0.014795) = 0.0269.Maximum of (0.0042, 0.006, 0.009, 0.03, 0.0165) = 0.03.Maximum of (0.00308, 0.0044, 0.0065997, 0.0222, 0.0121) = 0.0222

Similarly we obtain the passenger preference P for the other entries using the above method. Q_2 , R_2 and Q_3,R_3 and Q_4 and R_4.Now, we partition Q into five elements each by leaving out the first and the last element from the table as Q_1, Q_2 and Q_3 and calculate P_1, P_2 and P_3 as in the earlier case:

For

Q_1	R_1
0.07	0.0071
0.08	0.0222
0.09	0.0269
0.10	0.0300
0.11	0.0220.

The fuzzy relation equation is

$$P_1 \text{ o } \begin{bmatrix} 0.06 \\ 0.08 \\ 0.09 \\ 0.10 \\ 0.11 \end{bmatrix} = \begin{bmatrix} 0.0071 \\ 0..222 \\ 0.0269 \\ 0.0300 \\ 0.0220 \end{bmatrix}$$

Similarly, Calculate $\max_{j\in N_m} W_{ij} \quad X_j$ as follows

$$P_3=\begin{bmatrix} 0.0288 & 0.17933 & 0.1076 & 0.08966 & -.06725 \\ 0.15 & 0.2 & 0.12 & 0.1 & 0.075 \\ 0.11 & 0.14666 & 0.088 & 0.0733 & 0.055 \\ 0.1205 & 0.16066 & 0.0964 & 0.08033 & 0.06025 \\ 0.1325 & 0.17666 & 0.106 & 0.08833 & 0.06625 \end{bmatrix}$$

On observing from the table, we see the preference P1, P2 and P3 correspond to the peak hours of the day, h5 that is 10 a.m. with 300 passengers, h11 that is 4 p.m. with 381 passengers and h15 that is 8 p.m. with 376 number of passengers. Thus this partition gives us three preferences, which coincides with the real data as proved by the working.We now partition Q arbitrarily, that is the number of elements in each partition is not the same and by a adopting the above method we obtain the following results:

For

Q_1	R_1
0.06	0.0096
0.07	0.300
0.08	0.0222

We have

$$P_1=\begin{bmatrix} 0.03 & 0.06 & 0.12 \\ 0.0221875 & 0.044375 & 0.08875 \\ 0.069375 & 0.13875 & 0.2775 \end{bmatrix}$$

For

Q_2	R_2
0.09	0.269
0.10	0.300
0.11	0.220
0.12	0.241
0.13	0.265

We have,

$$P_2=\begin{bmatrix} 0.1345 & 0.17933 & 0.1076 & 0.08966 & -.06725 \\ 0.15 & 0.2 & 0.12 & 0.1 & 0.075 \\ 0.11 & 0.14666 & 0.088 & 0.0733 & 0.055 \\ 0.1205 & 0.16066 & 0.0964 & 0.08033 & 0.06025 \\ 0.1325 & 0.17666 & 0.106 & 0.08833 & 0.06625 \end{bmatrix}$$

For

Q_3	R_3
0.14	0.0249
0.15	0.0114

We have $P_3=\begin{bmatrix} 0.14 & 0.0249 \\ 0.15 & 0.0114 \end{bmatrix}$

and for

Q_4	R_4
0.16	0.0381
0.17	0.0288
0.18	0.0356
0.19	0.0189
0.20	0.0376
0.21	0.0182

We have

$$\begin{bmatrix} 0.09525 & 0.0635 & 0.047625 & 0.0381 & 0.1905 & 0.03175 \\ 0.072 & 0.048 & 0.036 & 0.0288 & 0.144 & 0.024 \\ 0.08233 & 0.05322 & 0.04111 & 0.03193 & 0.178 & 0.2411 \\ 0.4725 & 0.0312 & 0.023625 & 0.0189 & 0.0945 & 0.001575 \\ 0.0762 & 0.0508 & 0.0381 & 0.03048 & 0.188 & 0.0254 \\ 0.04275 & 0.0285 & 0.0213756 & 0.0171 & 0.091 & 0.001425 \end{bmatrix}$$

We obtain in the preferences P_1, P_2, P_3 and P_4 by partitioning the given data into a set of three elements, a set of five elements, a set of two elements and a set of six elements. On observing from the table, we see that these preferences correspond to the peak hours of the day, h_3 that is 8 a.m. with 222 passengers, h_5 that is 10 a.m. with 300 passengers, h_9 that is 2 p.m. with 249 number of passengers and h_{11} that is 4 p.m. with 381 number of passengers. Thus this partition gives us four preferences which coincides with the real data as proved by the working. Thus the Government sector can run more buses at the peak hours given and also at the same time restrain the number of buses in the non peak hours we derived the following conclusions:

1. The fuzzy relation equation described given by 1 can give only one preference function P o Q = R but the partition method described by us in this paper can give many number of preferences or desired number of preferences.
2. Since lot of research is needed we feel some other modified techniques can be adopted in the FRE P o Q = R.
3. We have tested our method described in the real data taken from Pallavan Transport Corporation and our results coincides with the given data.
4. We see the number of preference is equal to the number of the partition of Q.
5. Instead of partitioning Q, if we arbitrarily take overlapping subsets of Q certainly we may get the same preference for two or more arbitrary sets.

We see that our method of the fuzzy relation equation can be applied to the peak hour problem in a very successful way. Thus only partitioning of Q can yield non-overlapping unique solution. Finally, in our method we do not force the error function Ep to become zero, by using many stages or intermittent steps. We accept a value very close to zero for Ep as a preference solution.

V. CONCLUSION

Fuzzy relation equation used in almost every branch of Mathematics. This has made fuzzy relation equation is one of the great unifying ideas of mathematics. Finally, I conclude that the fuzzy relation equation may be helpful in future estimation.

REFERENCE

[1] Li, G. and Fang, S.G., On the Resolution of Finite Fuzzy Relation Equations,

[2] Pedrycz, W., Fuzzy Fuzzy Relational Equations with Generalized connectives and their Applications, Fuzzy Sets and systems.

[3] De Jong, K.A., An Analysis of the Behavior of a Class of Genetic Adaptive Systems, Dissertation Abstracts Internet.

[4] Pedrycz, W., Inverse Problem in fuzzy Relational Equations, Fuzzy Sets and Systems.

[5] Czogala, E., Drewniak, J., and Pedrycz, W., Fuzzy relation Applications on Finite Set, Fuzzy Sets and Systems.
[6] Zimmermann, H.J., Fuzzy Set Theory and its Applications, Kluwer, Boston.
[7] Di Nola, A., Pedrycz, W., Sessa, S., and Sanchez, E., Fuzzy Relation Equations Theory as a Basis of Fuzzy Modeling: An Overview, Fuzzy Sets and Systems.
[8] Higashi, M., and Klir, G.J., Resolution of Finite Fuzzy Relation Equations, Fuzzy Sets and Systems.