Generalized Inverse Matrix For The S-Prime Meet And Reciprocal S-Prime Meet Matrices Of Two Sets

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Abstract- *In this paper,we found the generalized inverse of S-Prime Meet Matrix of two sets with different cardinality and also we find the generalized inverse of S-Prime Join Matrix.Also,we found the generalized inverse of Reciprocal S-Prime Meet Matrix of two sets with different cardinality and also we find the generalized inverse of Reciprocal S-Prime Join Matrix.*

Keywords- Meet Matrix,S-Prime Meet Matrix,generalized inverse matrix

I. INTRODUCTION

Let S = $\{x_1, x_2, ..., x_n\}$ be a set of distinct positive integers and let f be an arithmetical function.Let $(S)_f$ denote the $n \times n$ matrix having $\left(f\left(x_i, x_j\right)\right)$, the image of the greatest common divisor of x_i and x_j as its ij th entry.Also, Let $[s]_f$ denote the matrix having $\left(f\left[x_i, x_j\right]\right)$, the image of the least common multiple of x_i and x_j as its ij th entry.A set S is called factor-closed if it contains every divisor of each of its members.The set S is gcd-closed if $(x_i,x_j) \in S$ for any i and j..In 1875 Smith [14] calculated $\det(S)_f$ when S is factor-closed .Haukkanen [8] generalized the concept of a GCD matrix in to a meet matrix and later Korkee and Haukkanen [10] did the same with the concepts of LCM and join matrices.

Let(P,≼)bealocally finitelattice,letS = $\{x_1, x_2, ..., x_n\}$ be a subset of P and let f be a complex valued function on P.Then × nmatrix$(S)_f$ = $(f(x_i \wedge x_j)$ is called the meet matrix on S associated with f and the n × nmatrix$[S]_f$ = $(f(x_i \vee x_j))$ is called the join matrix on S associated with f.If(P,≼) = (Z+,|),then meet and join matrices become respectively ordinary GCD and LCM Matrices.The properties of meet and join matrices have been studied by many authors(see e.g., [2,6,10,11]).Haukkanen [8] calculated the determinant of $(S)_f$ on an arbitrary set S and obtained the inverse of $(S)_f$ on a lower-closed set S and Korkee and Haukanen obtained the inverse of $(S)_f$ onameet-closed set S.Korkee and Haukkanen[10]present, among others, formulas for the determinant and inverse of $[S]_f$ on meet-closed, join-closed, lower-closed and upper-closed sets S.Most recently,Altinisik,Tuglu and Haukkanen[2]generalized the concepts of meet and join matrices and defined meet and join matrices on two sets.Next we define the GCD matrix on two sets of different cardinality which is the subset of P. RaoandMitrainvestigatedthegeneralizedinversesofmatricesanditsapplicationsin[13].Forsomeauthorshave given some interesting methods for finding their generalized inverse matrices.For example,Rakha[12]defined a method of finding the Moore-Penrose generalized inverse matrix. Furthermore, in 1994 Werner[15] also described the problem for finding a generalized inverse for the product of matrices. Unfortunately there are many type of generalized inverses. Most of the generalized inverse are not unique. Recall that two sided inverse of a matrix A is a matrix A^{-1} for which $AA^{-1} = I = A^{-1}A$.

Herer = n = m; the matrix A has full rank.For a rectangular matrix A,we may have generalized left inverse or left inverse when we multiply theinversefromthelefttogetidentitymatrixA^{-1} left A = I where A^{-} = A^{-1} left = $(AA^T)^{-1}A^T$.Similarly,wemayhave the generalized right inverse or right inverse when we multiple the inverse from the right to get identity matrix AA^{-1} right= I where A^{-} = A^{-1} right= $A^T(AA^T)^{-1}$.

In this paper, we discuss in detail the S-Prime Meet matrix for two sets with different cardinality and also obtained the generalized right inverse of S-Prime Meet Matrix.

II. GENERALIZED INVERSE OF S-PRIME MEET MATRIX

Let X = $\{x_1, x_2, ..., x_m\}$ and Y = $\{y_1, y_2, ..., y_n\}$ be two sets of different cardinality which contains positive integers.

We define the S-Prime Meet Matrix on X and Y with respect to f as

$$(X, Y)_f = f\left(4\left(x_i \wedge y_j\right) + 1\right) = G_A(say)$$

Example:

Let X = {1,2} and Y = {1,2,3} then S-Prime Meet Matrix on X and Y is given by

$$G_A = \begin{bmatrix} 4(1\wedge 1)+1 & 4(1\wedge 2)+1 & 4(1\wedge 3)+1 \\ 4(2\wedge 1)+1 & 4(2\wedge 2)+1 & 4(2\wedge 3)+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1)+1 & 4(1)+1 & 4(1)+1 \\ 4(1)+1 & 4(2)+1 & 4(1)+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 & 5 \\ 5 & 9 & 5 \end{bmatrix}$$

Next, we find the product of its transpose

$$G_A^{\ T} = \begin{bmatrix} 5 & 5 \\ 5 & 9 \\ 5 & 5 \end{bmatrix}$$

$$G_A G_A^{\ T} = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 9 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5 \\ 5 & 9 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 75 & 95 \\ 95 & 131 \end{bmatrix}$$

By computation of inverse,we have

$$\left(G_A G_A^{\ T}\right)^{-1} = \frac{1}{800}\begin{bmatrix} 131 & -95 \\ -95 & 75 \end{bmatrix}$$

$$\overline{G}_A = G_A^{\ T}\left(G_A G_A^{\ T}\right)^{-1}$$

$$= \begin{bmatrix} 5 & 5 \\ 5 & 9 \\ 5 & 5 \end{bmatrix} \frac{1}{800}\begin{bmatrix} 131 & -95 \\ -95 & 75 \end{bmatrix}$$

$$\therefore\ \overline{G}_A = \begin{bmatrix} 9/45 & -1/8 \\ 1/4 & 1/4 \\ 9/45 & -1/8 \end{bmatrix}$$

Weshallobtaindirectly S-Prime Join Matrixanditisdefinedas

$$G_B = \begin{bmatrix} 5 & 9 & 13 \\ 9 & 9 & 25 \end{bmatrix}$$

We calculate the inverse of S-Prime join matrix

$$G_B G_B^{\ T} = \begin{bmatrix} 5 & 9 & 13 \\ 9 & 9 & 25 \end{bmatrix} \begin{bmatrix} 5 & 9 \\ 9 & 9 \\ 13 & 25 \end{bmatrix} = \begin{bmatrix} 275 & 451 \\ 451 & 787 \end{bmatrix}$$

By computation of inverse,we have

$$\left(G_B G_B^{\ T}\right)^{-1} = \frac{1}{13024}\begin{bmatrix} 787 & -451 \\ -451 & 275 \end{bmatrix}$$

$$\overline{G}_B = G_B^{\ T}\left(G_B G_B^{\ T}\right)^{-1}$$

$$= \begin{bmatrix} 5 & 9 \\ 9 & 9 \\ 13 & 25 \end{bmatrix} \frac{1}{13024}\begin{bmatrix} 787 & -451 \\ -451 & 275 \end{bmatrix}$$

$$\therefore\ \overline{G}_B = \frac{1}{13024}\begin{bmatrix} -124 & 220 \\ 3024 & -1584 \\ -1044 & 1012 \end{bmatrix}$$

III. GENERALIZED INVERSE OF RECIPROCAL S-PRIME MEET MATRIX

Let X = $\{x_1, x_2, ..., x_m\}$ and Y = $\{y_1, y_2, ..., y_n\}$ be two sets of different cardinality which contains positive integers.

We define the Reciprocal S-Prime Meet Matrix on X and Y with respect to f as

$$(X,Y)_{1/f} = f\left(\frac{1}{4(x_i \wedge y_j)+1}\right) = G_{1/A}(say)$$

Example:

Let X = {1,2} and Y = {1,2,3} then Reciprocal S-Prime Meet Matrix on X and Y is given by

$$G_{1/A} = \begin{bmatrix} \frac{1}{4(1\wedge 1)+1} & \frac{1}{4(1\wedge 2)+1} & \frac{1}{4(1\wedge 3)+1} \\ \frac{1}{4(2\wedge 1)+1} & \frac{1}{4(2\wedge 2)+1} & \frac{1}{4(2\wedge 3)+1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4(1)+1} & \frac{1}{4(1)+1} & \frac{1}{4(1)+1} \\ \frac{1}{4(1)+1} & \frac{1}{4(2)+1} & \frac{1}{4(1)+1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{9} & \frac{1}{5} \end{bmatrix}$$

$$G_{1/A}^{\ T} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

Next, we find the product of its transpose

$$G_{1/A}G_{1/A}^{T} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{9} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{3}{25} & \frac{23}{225} \\ \frac{23}{225} & \frac{187}{2025} \end{bmatrix}$$

By computation of inverse,we have

$$\left(G_{1/A}G_{1/A}^{T}\right)^{-1} = \frac{50625}{32}\begin{bmatrix} \frac{187}{2025} & \frac{-23}{225} \\ \frac{-23}{225} & \frac{3}{25} \end{bmatrix}$$

$$\overline{G}_{1/A} = G_{1/A}^{T}\left(G_{1/A}G_{1/A}^{T}\right)^{-1}$$

$$= \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix} \frac{50625}{32}\begin{bmatrix} \frac{187}{2025} & \frac{-23}{225} \\ \frac{-23}{225} & \frac{3}{25} \end{bmatrix}$$

$$\therefore\ \overline{G}_{1/A} = \frac{50625}{32}\begin{bmatrix} -22500/153125 & 4/1125 \\ 145800/20503125 & -8/1125 \\ -22500/153125 & -8/1125 \end{bmatrix}$$

Weshallobtaindirectly S-Prime Join Matrixanditisdefinedas

$$G_{1/B} = \begin{bmatrix} \frac{1}{5} & \frac{1}{9} & \frac{1}{13} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{25} \end{bmatrix}$$

The product of its transpose

$$G_{1/B}G_{1/B}^{T} = \begin{bmatrix} \frac{1}{5} & \frac{1}{9} & \frac{1}{13} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{25} \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} \\ \frac{1}{13} & \frac{1}{25} \end{bmatrix}$$

By computation of inverse,we have

$$\overline{G}_{1/B} = G_{1/B}^{T}\left(G_{1/B}G_{1/B}^{T}\right)^{-1}$$

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