Con. Secondary K-Normal Circulant Bimatrices

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Abstract- *We consider con. secondary k-normal circulant bimatrices are introduced and discussed an important results*

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I. INTRODUCTION

Matrices provide a very powerful tool for dealing with linear models. Bimatrices are an advanced tool which can handle over one linear models at a time. Bimatrices will be useful when time bound comparisons are needed in the analysis of the model[3].Unlike bimatrices can be of several types. The concept of s-symmetric matrices, k-symmetric matrices and of s-k symmetric matrices was introduced in [1], [2] and [3] Some properties of symmetric matrices given in [5],[6] .In this paper, our intention is to define s-symmetric circulant matrices, k-symmetric circulant matrices and s-k symmetric circulant matrices also we discussed some results on symmetric circulant matrices.

II. SOME OF DEFINITIONS AND RESULTS

Bimatrix 2.1 [3]

A bimatrix A_B is defined as the union of two square array of numbers A_1 and A_2 arranged into rows and columns. It is written as follows $A_B = A_1 \cup A_2$
where $A_1 \neq A_2$ with

$$A_1 = \begin{bmatrix} a_{11}^1 & a_{12}^1 & \cdots\cdots & a_{1n}^1 \\ a_{21}^1 & a_{22}^1 & \cdots\cdots & a_{2n}^1 \\ \vdots & \vdots & & \vdots \\ a_{m1}^1 & a_{m2}^1 & \cdots & a_{mn}^1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{11}^2 & a_{12}^2 & \cdots\cdots & a_{1n}^2 \\ a_{21}^2 & a_{22}^2 & \cdots\cdots & a_{2n}^2 \\ \vdots & \vdots & & \vdots \\ a_{m1}^2 & a_{m2}^2 & \cdots & a_{mn}^2 \end{bmatrix}$$

`$\cup$` the notational convenience (symbol) only.

DEFINITION:1

For any given $c_0, c_1, c_2, \ldots c_{n-1} \in R^{n \times n}$ the circulant matrix $C=(c_{i,j})_{n \times n}$ is defined by $(c_{i,j}) = c_{j-1 (\mathrm{mod}\ n)}$

$$\begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \cdots & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \cdots & c_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 & \cdots & c_0 \end{bmatrix}$$

Definition 2.2[2]

A bimatrix $A_B = A_1 \cup A_2$ is said to be normal bimatrix , if $A_B A_B^* = A_B^* A_B$

III. ON SECONDARY K-NORMALCIRCULANT BIMATRICES

Definition

A circulant bimatrix $A_B \in C^{n \times n}$ is said to be s-k normal circulant bimatrix if

$$K_B V_B A_B^* A_B V_B K_B = \overline{K_B V_B A_B A_B^* V_B K_B}$$

Theorem

Let $A_B, B_B \in C^{n\times n}$ are s-k normal circulant bimatrix then $A_B \pm B_B$ is also s-k normal circulant bimatrix

Proof

Let $A_B, B_B \in C^{n\times n}$ are s-k normal circulant bimatrix then if

$K_BV_BA_B^*A_BV_BK_B = K_BV_BA_BA_B^*V_BK_B$

$K_BV_BB_B^*B_BV_BK_B = K_BV_BB_BB_B^*V_BK_B$

To prove $A_B \pm B_B$ is s-k normal circulant bimatrix

We will show that ,

$K_BV_B(A_B \pm B_B)(A_B \pm B_B)^* V_BK_B = K_BV_B(A_B \pm B_B)^*(A_B \pm B_B) V_BK_B$

Now

$K_BV_B(A_B \pm B_B)(A_B \pm B_B)^* V_BK_B = K_BV_B(A_B \pm B_B)(A_B^* \pm B_B^*) V_BK_B$

$= K_BV_B(A_B^* V_BK_B \pm B_B^*V_BK_B)(A_B \pm B_B)$

$= K_BV_BA_BA_B^*V_BK_B \pm K_BV_BA_B^*B_BV_BK_B \pm K_BV_BB_B^*A_BV_BK_B \pm K_BV_BB_B^*B_BV_BK_B$

$= K_BV_B(A_B^* \pm B_B^*)A_B \; K_BV_B \pm K_BV_B(A_B^* \pm B_B^*)B_BV_BK_B$

$= K_BV_B(A_B \pm B_B)^*(A_B \pm B_B) V_BK_B$

$K_BV_B(A_B \pm B_B)(A_B \pm B_B)^* V_BK_B = K_BV_B(A_B \pm B_B)^*(A_B \pm B_B) V_BK_B$

Theorem

Let $A_B, B_B \in C^{n\times n}$ are s-k normal circulant bimatrices and $A_BB_B=B_BA_B$ then A_BB_B is also s-k normal circulant bimatrix

Proof

Let A_B, B_B are s-k normal circulant bimatrix then

$K_BV_BA_B^*A_BV_BK_B = K_BV_BA_BA_B^*V_BK_B$

$K_BV_BB_B^*B_BV_BK_B = K_BV_BB_BB_B^*V_BK_B$

To prove $A_B B_B$ is s-k normal circulant bimatrix

We will show that ,

$K_BV_B(A_B B_B)(A_B B_B)^* V_BK_B = K_BV_B(A_B B_B)^*(A_B B_B) V_BK_B$

Now

$K_BV_B(A_B \; B_B)(A_B \; B_B)^* \; V_BK_B = K_BV_BA_B B_BA_B^*B_B^*V_BK_B$

$= K_BV_B B_BA_BA_B^*B_B^*V_BK_B$

$= K_BV_B \; B_BA_BA_B^* \; V_BK_B \; B_B^*$

$= K_BV_BB_B \; A_B^* \; B_B^*A_B \; V_BK_B$

$= K_BV_B \; A_B^*B_B \; B_B^* \; A_B \; V_BK_B$

$= K_BV_BA_B^*B_B^*B_B \; A_B \; V_BK_B$

$K_BV_B(A_B B_B)(A_B B_B)^* V_BK_B = K_BV_B(A_BB_B)^*(A_BB_B)V_BK_B$

Theorem

Let $A_B, B_B \in C^{n\times n}$ are s-k normal circulant bimatrices and $A_BB_B=B_BA_B$ then $A_BB_B^*$ is also s-k normal circulant bimatrix

Proof

Let A_B, B_B are s-k normal circulant bimatrix then

$K_BV_BA_B^*A_BV_BK_B = K_BV_BA_BA_B^*V_BK_B$

$K_BV_BB_B^*B_BV_BK_B = K_BV_BB_BB_B^*V_BK_B$ and given $A_BB_B=B_BA_B$

To prove $A_B B_B^*$ is s-k normal circulant bimatrix

We will show that ,

$K_BV_B(A_B B_B^*)(A_B B_B^*)^* V_BK_B = K_BV_B(A_B B_B^*)^*(A_B B_B^*) V_BK_B$

Now

$K_BV_B(A_B \; B_B)(A_B \; B_B)^* \; V_BK_B = K_BV_B(A_BB_B)^*(A_BB_B)V_BK_B$

$K_BV_B \; A_B \; B_BB_B^*A_B^* \; V_BK_B = K_BV_B(B_BA_B)^*(B_BA_B) V_BK_B$ where $A_BB_B=B_BA_B$

$K_BV_B \; A_B B_BB_B^*A_B^*V_BK_B = K_BV_BA_B^*B_B^*B_BA_BV_BK_B$

$K_BV_B \; A_B \; B_B^*B_B \; A_B^* \; V_BK_B = K_BV_BA_B^*B_B B_B^*A_BV_BK_B$ where $B_B^*B_B=B_BB_B^*$

$K_BV_B \; A_B \; B_B^*(B_B^*)^*A_B^*V_BK_B = K_BV_BA_B^*(B_B^*)^*B_B^*A_BV_BK_B$ where $(B_B^*)^*=B$

$K_BV_B(A_B B_B^*)(A_B B_B^*)^* V_BK_B = K_BV_B(A_B B_B^*)^*(A_B B_B^*) V_BK_B$

Theorem

Let $A_B \in C^{m\times n}$ be s-k normal circulant bimatrix then,

1. iA_B is s-k normal circulant bimatrix
2. $-iA_B$ is s-k normal circulant bimatrix

Proof

Let A_B be a s-k normal circulant bimatrix then

$K_BV_BA_B^*A_BV_BK_B = K_BV_BA_BA_B^*V_BK_B$

To prove

1. iA_B is s-k normal circulant bimatrix

we will show that ,

$K_BV_B(iA_B)(iA_B)^*V_BK_B = K_BV_B(iA_B)^*(iA_B)V_BK_B$

$K_BV_BA_BA_B^*V_BK_B = K_BV_BA_B^*A_BV_BK_B$

$K_BV_B\text{-}i^2A_BA_B^*V_BK_B=K_BV_B(-i)A_B^*(iA_B)V_BK_B$

$K_BV_B(iA_B)(-i)A_B^*V_BK_B= K_BV_B(-i)A_B^*(iA_B)V_BK_B$

$K_BV_B(iA_B)(\bar{i})A_B^*V_BK_B= K_BV_B(\bar{i})A_B^*(iA_B)V_BK_B$

where(i)= $\bar{i}$

$K_BV_B(iA_B)(\bar{i})^TA_B^*V_BK_B= K_BV_B(\bar{i})^TA_B^*(iA_B)V_BK_B$ where $(\bar{i})^T=\bar{i}$

$K_BV_B(iA_B)(i)^* A_B^*V_BK_B= K_BV_Bi^*A_B^*(iA_B)V_BK_B$

where $i^*=(\bar{i})^T$

$K_BV_B(iA_B)(iA_B)^* V_BK_B = K_BV_B(iA_B)^*(iA_B)V_BK_B$

iA_B is s-k normal circulant bimatrix

2. $-iA_B$ is s-k normal circulant bimatrix

we will show that ,

$K_BV_B(-iA_B)(-iA_B)^*V_BK_B = K_BV_B(-iA_B)^*(-iA_B)V_BK_B$

$K_BV_BA_BA_B^*V_BK_B = K_BV_BA_B^*A_BV_BK_B$

$K_BV_B\text{-}i^2A_BA_B^*V_BK_B=K_BV_B(-i^2)A_B^*A_BV_BK_B$

$K_BV_B(-i)iA_BA_B^*V_BK_B= K_BV_Bi(-i)A_B^*A_BV_BK_B$

$K_BV_B(-iA_B)(-\bar{i})A_B^*V_BK_B= K_BV_B(-\bar{i})A_B^*(-iA_B)V_BK_B$ where(i)= $-\bar{i}$

$K_BV_B(-iA_B)(-\bar{i})^*A_B^*V_BK_B= K_BV_B(-\bar{i})^*A_B^*(-iA_B)V_BK_B$ where $(-\bar{i})^T=-i^*$

$K_BV_B(-iA_B)(-iA_B)^*V_BK_B= K_BV_B(-iA_B)^*(-iA_B)V_BK_B$

$-iA_B$ is s-k normal circulant bimatrix

Theorem

Let $A_B \in C^{m\times n}$ and A_B^T be the moore penrose inverse of A_B then A_B is s-k normal circulant bimatrix iff A_B^T is s-k normal circulant bimatrix

Proof

Let A_B be a s-k normal circulant bimatrix then

$K_BV_BA_BA_B^*V_BK_B = K_BV_BA_B^*A_BV_BK_B$

To prove A_B^T is s-k normal circulant bimatrix

We will show that,

$K_BV_B(A_B^T)(A_B^T)^*V_BK_B = K_BV_B(A_B^T)^*(A_B^T)V_BK_B$

Now,

$K_BV_BA_BA_B^*V_BK_B = K_BV_BA_B^*A_BV_BK_B$

$(K_BV_BA_BA_B^*V_BK_B)^T = (K_BV_BA_B^*A_BV_BK_B)^T$

$K_BV_B(A_B^{*T})(A_B^T)V_BK_B = K_BV_B(A_B^T)(A_B^*)^T V_BK_B$

$= K_BV_B(A_B^T)^*(A_B^T)V_BK_B$

$= K_BV_B(A_B^T)(A_B^T)^* V_BK_B$

$K_BV_B(A_B^T)(A_B^T)^*V_BK_B = K_BV_B(A_B^T)^*(A_B^T)V_BK_B$

A_B^T is s-k normal circulant bimatrix

Let us assume that A_B^T is s-k normal circulant bimatrix

To prove A_B is s-k normal circulant bimatrix

We will show that,

$K_BV_BA_BA_B^*V_BK_B = K_BV_BA_B^*A_BV_BK_B$

Now,

$K_BV_B(A_B^T)(A_B^T)^*V_BK_B = K_BV_B(A_B^T)^*(A_B^T)V_BK_B$

$(K_BV_B(A_B^T)(A_B^T)^*V_BK_B)^T=(K_BV_B(A_B^T)^*(A_B^T)V_BK_B)^T$

$K_BV_B(A_B^*)(A_B^T)^TV_BK_B = K_BV_B(A_B^T)^T(A_B^*)V_BK_B$

$K_BV_BA_B^*A_BV_BK_B = K_BV_BA_BA_B^*V_BK_B$

$K_BV_BA_BA_B^*V_BK_B = K_BV_BA_B^*A_BV_BK_B$

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