Separation Axioms Via M_I^*-Closed Set

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Abstract- *In this paper we have introduced the concept of separation axioms via M_I^*-closed set and studied some of their characterizations and properties. Also introduced the concept of $M_I^*R_0$-space and $M_I^*R_1$-space and studied their properties.*

Keywords. $M_I^*T_0$-space,$M_I^*T_1$-space,$M_I^*T_2$-space,$M_I^*R_0$-space,$M_I^*R_1$-space.

I. INTRODUCTION

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [8] and Vaidyanathaswamy [9]. Jankovic and Hamlett[7] investigated further properties of an ideal spaces. An Ideal Ion a topological space is a non-empty collection of subsets of X which satisfies the following properties: (i) $A \in I$ and $B \subset A$implies $B \in I$(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. An ideal topological space(or an ideal space) is a topological space (X,τ) with an ideal I on X and is denoted by (X,τ,I). For a subset $A \subset X, A^*(\tau,I) = \{x \in X: A \cap U \notin I \; for\; every\; U \in \tau(x)\}$ is called the local function of A with respect to I and τ [8].We simply writeA^* in case there is no chance for confusion.A Kuratowski closure operator $cl^*(.)$for a topology $\tau^*(I,\tau)$ called the $*$-topology,finer than τ is defined by $cl^*(A) = A \cup A^*$[9].

Caldas and Jafari [3] introduced and studied b_{T_0},b_{T_1} and b_{T_2} via *b*-open sets after that Ekici [5] introduced the notion of b_{R_0} and b_{R_1}spaces.In this article,we introduce new types of separation axioms via M_I^*-closed sets and investigate some of their properties.

II. PRELIMINARIES

Definition.**2.1 [6]**. A subset A of a topological space (X,τ) is called a semi-preopen set(=β-open set) if $A \subset cl(int(cl(A))$ and a semi-preclosed set (=β-closed set) if $int(cl(int(A)) \subset A$.

Definition.**2.2 [2].** A subset A of an ideal topological space (X,τ,I) is called an M_I^*closed if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and$U$ is I_ω-open in (X,τ,I).The class of all M_I^*closed sets in (X,τ,I) is denoted by $M_I^* - C(X)$.That is,
$M_I^* - C(X) = \{A \subset X: A\; is\; M_I^*$closed in $(X,\tau,I)\}$.

Definition.2.3[1]. A subset A of an ideal topological space (X,τ,I) is called an $I_\omega (or\; I_{\hat{g}})$-closed set if $A^* \subseteq U$ whenever $A \subseteq U$and $U$is semi-open in (X,τ).

Definition.2.4.[4] A topological space (X,τ) is said to be

(i) $\alpha - T_0$if for each pair of distinct points in *X*, there is an α-closed set containing one of the points but not the other.
(ii) $\alpha - T_1$ if for each pair of distinct points x and y of X,there exists α-open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.
(iii) $\alpha - T_2$if for each pair of distinct points x and y of$X$,there exists disjoint α-open sets U and V containing x and y respectively.

Definition.2.5.[4] A topological space (X,τ) is said to be
(i) $\alpha - R_0$if for each α-open set *G* in *X* and $x \in G$such that $\alpha cl(\{x\}) \subseteq G$.
(ii) $\alpha - R_1$if for $x,y \in X$ with $\alpha cl(\{x\}) \neq \alpha cl(\{y\})$, there exist disjoint α-open sets *U* and *V* such that $\alpha cl(\{x\}) \subseteq U$ and $\alpha cl(\{y\}) \subseteq V$.

III.$M_I^*T_i$-SPACES,$(i = 0,1,2)$

Definition.3.1. An ideal topological space (X,τ,I) is said to be

(i) $M_I^*T_0$if for each pair of distinct points x,yin$X$,there exist a M_I^*-open set U of X such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

(ii) $M_I^*T_1$if for each pair of distinct points x,yin$X$,there exist two M_I^*-open sets U and V such that $x \in U$ but $y \notin U$ and $x \notin V$ but $y \in V$.

(iii) $M_I^*T_2$if for each pair of distinct points x,yin$X$,there exist two disjoint M_I^*-open sets U and V of Xcontaining $x$and y respectively.

Example.3.2.(i)

Let$X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.Then M_I^*-open sets of an ideal space are$\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}$.Clearly (X, τ, I) is $M_I^*T_0$,$M_I^*T_1$ and $M_I^*T_2$.

(ii)Let$X = \{a, b, c\}$,$\tau\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $I = \{\emptyset, \{a\}\}$.Then M_I^*-open sets of X are $\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.Clearly (X, τ, I) is not a $M_I^*T_1$-space and also (X, τ, I) is not a $M_I^*T_2$-space.

Proposition.3.3 Every $\alpha - T_i$-space is $M_I^*T_i$-space,$i = 0,1,2$ but not conversely.

Proof. The proof is follows from the fact that every α-open set is M_I^*-open.

Example3.4.(i) Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.Then M_I^*-open sets of X are $\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}$ and α-open sets of X are $\emptyset, X, \{a\}, \{b\}, \{a, b\}$.Clearly (X, τ, I) is $M_I^*T_1$ and $M_I^*T_2$ but not $\alpha - T_1$ and $\alpha - T_2$.

(ii) Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and$I =\{\emptyset, \{a\}\}$.Then M_I^*-open sets of X are power sets of X and α-open sets of $X$are$\emptyset, X, \{a\}, \{b, c\}\}$. Clearly $(X, \tau, I)$is $M_I^*T_0$ but not $\alpha - T_0$.

Theorem.3.5. An ideal topological space (X, τ, I) is $M_I^*T_0$ if and only if for each pair of distinct points $x, y$of$X$,$M_I^*cl(\{x\}) \neq M_I^*cl(\{y\})$.

Proof. Necessity: Let (X, τ, I) be a $M_I^*T_0$-space and x, y be any two distinct points of$X$.There exists a M_I^*-open set U containing x or$y$,say x but not$y$.Then $X\backslash U$ is a M_I^*-closed set which does not contain x but contains y.Since $M_I^*cl(\{y\})$ is the smallest M_I^*-closed set containing$y$,$M_I^*cl(\{y\}) \subseteq X\backslash U$ and therefore $x \notin M_I^*cl(\{y\})$.Consequently $M_I^*cl(\{x\} \neq M_I^*cl(\{y\})$.

Sufficiency: Suppose that$x, y \in X$,$x \neq y$ and $M_I^*cl(\{x\} \neq M_I^*cl(\{y\})$.Let $z \in X$ such that $z \in M_I^*cl(\{x\}$ but $z \notin M_I^*cl(\{y\}$.We claim that $x \notin M_I^*cl(\{y\}$.For,if $x \in M_I^*cl(\{y\}$,then $M_I^*cl(\{x\} \subseteq M_I^*cl(M_I^*cl(\{y\}) = M_I^*cl(\{y\}$.Therefore $z \in M_I^*cl(\{y\}$,a contradiction.Thus $x \notin M_I^*cl(\{y\}$.Then $X\backslash M_I^*cl(\{y\}$ is a M_I^*-open set in X such that $x \in X\backslash M_I^*cl(\{y\}$ and $y \notin X\backslash M_I^*cl(\{y\}$.Thus $(X, \tau, I)$is a $M_I^*T_0$-space.

Theorem.3.6. An ideal topological space (X, τ, I) is a $M_I^*T_1$-space if and only if the singleton sets are M_I^*-closed sets.

Proof. Let (X, τ, I) be $M_I^*T_1$ and x be any point of$X$. Suppose $y \in X\backslash\{x\}$, then $x \neq y$ and so there exists a M_I^*-open set U such that $y \in U$ but $x \notin U$.Consequently $y \in U \subseteq X\backslash\{x\}$,that is $X\backslash\{x\} = \cup\{U : y \in X\backslash\{x\}$ which is M_I^*-open.

Conversely, suppose $\{p\}$ is M_I^*-closed for every $p \in X$.Let $x, y \in X$ with $x \neq y$.Now $x \neq y$ implies $y \in X\backslash\{x\}$.Hence $X\backslash\{x\}$ is a M_I^*-open set contains y but not$x$.Similarly $X\backslash\{y\}$is a M_I^*-open set contains x but not y.Accordingly(X, τ, I) is a $M_I^*T_1$-space.

Theorem.3.7. The following statements are equivalent for an ideal topological space (X, τ, I):

(i) $(X, \tau, I)$is$M_I^*T_2$.

(ii) Let $x \in X$.For each $y \neq x$, there exist a M_I^*-open set U containing x such that $y \notin M_I^*cl(U)$.

(iii) For each $x \in X$, $\cap\{M_I^*cl(U) : U \in M_I^* - O(X) \text{ and } x \in U = \{x\}$.

Proof.$(i) \Rightarrow (ii)$.Since $(X, \tau, I)$is $M_I^*T_2$,then for each $y \neq x$,there exist two disjoint M_I^*-open sets U and V such that $x \in U$ and $y \in V$ respectively. If $F = X\backslash V$,then F is a M_I^*-closed set such that $U \subseteq F$ which implies $M_I^*cl(U) \subseteq M_I^*cl(F) = F$.Since $y \notin F$,$y \notin M_I^*cl(U)$.

$(ii) \Rightarrow (iii)$.If $y \neq x$,then there exist a M_I^*-open set U such that $x \in U$ and $y \notin M_I^*cl(U)$.Therefore, $y \notin \cup\{M_I^*cl(U) : U \in M_I^* - O(X) \text{ and } x \in U\}$.Therefore $\cap\{M_I^*cl(U) : U \in M_I^* - O(X) \text{ and } x \in U = \{x\}$.This proves (iii).

$(iii) \Rightarrow (i)$.Let $x, y \in X$ and$x \neq y$.Then $y \notin \{x\} = \cap\{M_I^*cl(U) : U \in M_I^* - O(X) \text{ and } x \in U$.This implies that there exist a M_I^*-open set U containing x such that $y \notin M_I^*cl(U)$.Let$V = X\backslash M_I^*cl(U)$,then V is M_I^*-open and $y \in V$.Now $U \cap V = U \cap (X\backslash M_I^*cl(U) \subseteq U \cap X\backslash U = \emptyset$.Therefore, (X, τ, I) is $M_I^*T_2$.

Proposition.3.8. The following statements are true for an ideal topological space (X, τ, I).

(i) Every $M_I^*T_1$-space is $M_I^*T_0$ but not conversely.

(ii) Every $M_I^*T_2$-space is $M_I^*T_1$ but not conversely.

Proof. The proof is straight forward from definitions.

Example.3.9 Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $I = \{\emptyset, \{b\}\}$.Then M_I^*-open sets of X are $\emptyset, X, \{a\}, \{a, b\}, \{a, c\}$.Here (X, τ, I) is $M_I^*T_0$ but not $M_I^*T_1$.

Definition.3.10. A subset A of an ideal topological space (X,τ,I) is called M_I^*-difference set (briefly.M_I^*-D set) if there exist $U,V \in M_I^* - O(X)$ such that $U \neq X$and $A = U\backslash V$.

It is true that every M_I^*-open set U different from X is a M_I^*-D set if $A = U$and $V = \emptyset$, we can observe the following.

Remark.3.11. Every proper M_I^*-open set is a M_I^*-D set but not conversely.

Example.3.12. Let $X = \{a,b,c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$.Then M_I^*-open sets of X are $\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,c\}$.Consider the set $U = \{a,c\} \neq X, V = \{a,b\}$, are M_I^*-open sets in X and $A = U\backslash V = \{c\}$ which is a M_I^*-D set but not a M_I^*-open set

Definition.3.13. An ideal space (X,τ,I) is said to be M_I^*-symmetric, if for $x,y \in X, x \in M_I^*cl(\{y\})$implies $y \in M_I^*cl(\{x\})$.

Proposition.3.14. The following statements are equivalent for an ideal space (X,τ,I):

(i) $(X,\tau,I)$is a M_I^*-symmetric space.
(ii) $\{x\}$isM_I^*-closed for each $x \in X$.

Proof.$(i) \Rightarrow (ii)$.Assume that $\{x\} \subseteq U \in M_I^* - O(X)$,but$M_I^*cl(\{x\}) \nsubseteq U$.Then $M_I^*cl(\{x\}) \cap (X\backslash U) \neq \emptyset$.Now,we take $y \in M_I^*cl(\{x\}) \cap (X\backslash U)$,then by hypothesis $x \in M_I^*cl(\{y\}) \subseteq (X\backslash U)$ and $x \notin U$,which is a contradiction.Therefore,$\{x\}$ is M_I^*-closed for each $x \in X$.
$(ii) \Rightarrow (i)$Assume that $x \in M_I^*cl(\{y\})$,but$y \notin M_I^*cl(\{x\})$.Then $\{y\} \subseteq X\backslash M_I^*cl(\{x\})$ and hence $M_I^*cl(\{y\}) \subseteq X\backslash M_I^*cl(\{x\})$.
Therefore,$x \in X\backslash M_I^*cl(\{x\})$, which is a contradiction and hence $y \in M_I^*cl(\{x\})$.

Corollary.3.15. If an ideal space (X,τ,I) is a $M_I^*T_1$-space, then it is M_I^*-symmetric.

Proof. In a $M_I^*T_1$-space, by Theorem 3.6, every singleton set is M_I^*-closed. Also by Proposition 3.14, (X,τ,I) is M_I^*-symmetric.

Corollary.3.16. In an ideal space (X,τ,I) is M_I^*-symmetric and $M_I^*T_0$,then (X,τ,I) is $M_I^*T_1$.

Proof. Let $x \neq y$ and as $(X,\tau,I)$is $M_I^*T_0$,we may assume that $x \in U \subseteq X\backslash\{y\}$ for some $U \in M_I^* - O(X)$.Then $x \notin M_I^*cl(\{y\})$ and hence $y \notin M_I^*cl(\{x\})$.Then there exist a M_I^*-open set V such that $y \in V \subseteq X\backslash\{x\}$ and thus (X,τ,I) is $M_I^*T_1$.

Definition.3.17. Let (X,τ,I) be an ideal space and $A \subseteq X$.Then the M_I^*-kernal of$A$ denoted by $M_I^* - \ker(A)$ and is defined to be the set $M_I^* - \ker(A) = \cap\{U \in M_I^* - O(X) : A \subseteq U\}$.

Example.3.18. Let $X = \{a,b,c\}, \tau = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$ and $I = \{\emptyset, \{b\}\}$.Then M_I^*-open sets of $X$are $\emptyset, X, \{a\}, \{a,b\}, \{a,c\}$.Here
$M_I^* - \ker(\{a\}) = \{a\}$,$M_I^* - \ker(\{b\}) = \{a,b\}$,
$M_I^* - \ker(\{c\}) = \{a,c\}$,$M_I^* - \ker(\{a,b\}) = \{a,b\}$,
$M_I^* - \ker(\{a,c\}) = \{a,c\}$,$M_I^* - \ker(\{b,c\}) = X$.

Theorem.3.19. Let (X,τ,I) be an ideal space and $x \in X$.Then $y \in M_I^* - \ker\{x\}$ if and only if $x \in M_I^*cl(\{y\})$.

Proof.Neceesity: Suppose that $y \notin M_I^* - \ker\{x\}$.Then there exist a M_I^*-open set V containing x such that $y \notin V$.Therefore,we have $x \notin M_I^*cl(\{y\})$.

Sufficiency: Assume that $x \notin M_I^*cl(\{y\})$.Then there exist a M_I^*-open set Ucontaining x such that $y \notin U$.By definition of M_I^*-kernel, $y \notin M_I^* - \ker\{x\}$.

Proposition.3.20. Let (X,τ,I) be an ideal space and A be a subset of$X$.Then $M_I^* - \ker(A) = \{x \in X : M_I^*cl(\{x\}) \cap A \neq \emptyset\}$.

Proof. Let $x \in M_I^* - \ker(A)$ and suppose that $M_I^*cl(\{x\}) \cap A = \emptyset$.Then $X\backslash M_I^*cl(\{x\})$ is a M_I^*-open set containing A such that $x \notin X\backslash M_I^*cl(\{x\})$.This is impossible,since $x \in M_I^* - \ker(A)$.Consequently,if $M_I^*cl(\{x\}) \cap A \neq \emptyset$.We claim $x \in M_I^* - \ker(A)$.Suppose that $x \notin M_I^* - \ker(A)$.Then there exist a M_I^*-open set V containing A such that $x \notin V$. Now, let $y \in M_I^*cl(\{x\}) \cap A$.Then $y \in M_I^*cl(\{x\})$ and $y \in A$.If $y \in M_I^*cl(\{x\})$ implies there exist a M_I^*-open set V containing y such that $V \cap \{x\} \neq \emptyset$.Hence $x \in V$.By this contradiction, $x \in M_I^* - \ker(A)$ and the claim is proved.

Proposition.3.21. The following properties hold for the subsets A,B of an ideal space (X,τ,I).

(i) $A \subseteq M_I^* - \ker(A)$.
(ii) $A \subseteq B \Rightarrow M_I^* - \ker(A) \subseteq M_I^* - \ker(B)$.
(iii) If A is M_I^*-open in (X,τ,I),then$A = M_I^* - \ker(A)$.
(iv)$M_I^* - \ker(M_I^* - \ker(A)) = M_I^* - \ker(A)$

Proof.(i) Suppose that A is any subset of X.If $x \notin M_I^* - \ker(A)$,then there exists $U \in M_I^* - O(X)$ such that $A \subseteq U$ and $x \notin U$.Therefore, $x \notin A$.Thus it is proved.
(ii) Let $A \subseteq B$.If suppose $M_I^* - \ker(A) \nsubseteq M_I^* - \ker(B)$,then $x \in M_I^* - \ker(A)$ but $x \notin M_I^* - \ker(B)$.By definition of M_I^*-kernel,there exist a M_I^*-open set U such that $B \subseteq U$ and $x \notin U$.Since $A \subseteq B \subseteq U, x \notin M_I^* - \ker(A)$.By this contradiction, $M_I^* - \ker(A) \subseteq M_I^* - \ker(B)$.
(iii) Obvious from the Definition 3.17 of $M_I^* - \ker(A)$.
(iv) From the subdivisions (i) and (ii) ,we have $M_I^* - \ker(A) \subseteq M_I^* - \ker(M_I^* - \ker(A))$. To prove the other implication, if $x \notin M_I^* - \ker(A)$,then there exists $U \in M_I^* - O(X)$ such that $A \subseteq U$ and $x \notin U$.Hence $M_I^* - \ker(A) \subseteq U$ and so we have $x \notin M_I^* - \ker(M_I^* - \ker(A))$. Thus $M_I^* - \ker(M_I^* - \ker(A)) = M_I^* - \ker(A)$.

Proposition.3.22. If a singleton set $\{x\}$ is a M_I^*-Dset of (X, τ, I), then $M_I^* - \ker(\{x\}) \neq X$.

Proof. Since $\{x\}$ is a M_I^*-Dset of (X, τ, I),then there exists two M_I^*-open subsets U, V such that $\{x\} = U \setminus V$.Then $\{x\} \subseteq U$ and $U \neq X$.Thus, we have that $M_I^* - \ker(\{x\}) \subseteq U \neq X$ and so $M_I^* - \ker(\{x\}) \neq X$.

IV. $M_I^* R_k$-SPACE (k=0,1)

In this section new classes of an ideal spaces called $M_I^* R_0$ and $M_I^* R_1$ spaces are introduced and found the necessary and sufficient conditions for a space (X, τ, I) to be $M_I^* R_0$ and $M_I^* R_1$.

Definition.4.1. An ideal space (X, τ, I) is said to be
(i) $M_I^* R_0$, if every M_I^*-open set contains the M_I^*-closure of each of its singletons. That is, for any M_I^*-open set U in X, we have $M_I^* cl(\{x\}) \subseteq U$ for every $x \in U$.
(ii) $M_I^* R_1$,if for any $x, y \in X$ such that $M_I^* cl(\{x\}) \neq M_I^* cl(\{y\})$, there exists disjoint M_I^*-open sets U and V such that $M_I^* cl(\{x\}) \subseteq U$ and $M_I^* cl(\{y\}) \subseteq V$

Example.4.2. Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then M_I^*-open sets of an ideal space are $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\{a, c\}\}$.Clearly the ideal space (X, τ, I) is $M_I^* R_0$ as well as $M_I^* R_1$

Proposition.4.3. For an ideal space (X, τ, I), the following properties are equivalent:
(i) (X, τ, I) is $M_I^* R_0$
(ii) For any $F \in M_I^* - C(X), x \notin F$ implies $F \subseteq U$ and $x \notin U$ for some $U \in M_I^* - O(X)$
(iii) For any $F \in M_I^* - C(X), x \notin F$ implies $F \cap M_I^* cl(\{x\}) = \emptyset$
(iv)For any distinct points x and y of X, either $M_I^* cl(\{x\}) = M_I^* cl(\{y\})$ or $M_I^* cl(\{x\}) \cap M_I^* cl(\{y\}) = \emptyset$.

Proof: $(i) \Rightarrow (ii)$.Let $F \in M_I^* - C(X)$ and $x \notin F$. Then by (i), $M_I^* cl(\{x\}) \subseteq X \setminus F$.Set $U = X \setminus M_I^* cl(\{x\})$, then U is a M_I^*-open set such that $F \subseteq U$ and $x \notin U$.
$(ii) \Rightarrow (iii)$. Let $F \in M_I^* - C(X)$ and $x \notin F$. Then there exists $U \in M_I^* - O(X)$ such that $F \subseteq U$and $x \notin U$. Since $U \in M_I^* - O(X), U \cap M_I^* cl(\{x\}) = \emptyset$ and $F \cap M_I^* cl(\{x\}) = \emptyset$.
$(iii) \Rightarrow (iv)$ Suppose that $M_I^* cl(\{x\}) \neq M_I^* cl(\{y\})$ for distinct points $x, y \in X$. There exists $z \in M_I^* cl(\{x\})$ such that $z \notin M_I^* cl(\{y\})$ ($or\ z \in M_I^* cl(\{y\})$ such that $z \notin M_I^* cl(\{x\})$). There exists $V \in M_I^* - O(X)$ such that $y \notin V$ and $z \in V$;hence $x \in V$. Therefore, we have $x \notin M_I^* cl(\{y\})$. By (iii) we obtain $M_I^* cl(\{x\}) \cap M_I^* cl(\{y\}) = \emptyset$. $(iv) \Rightarrow (i)$. Let $V \in M_I^* - O(X)$and $x \in V$. For each $y \notin V$ and $x \neq y$and $x \notin M_I^* cl(\{y\})$. This shows that $M_I^* cl(\{x\}) \neq x \notin M_I^* cl(\{y\})$. By (iv), $M_I^* cl(\{x\}) \cap M_I^* cl(\{y\}) = \emptyset$ for each $y \in X \setminus V$ and hence $M_I^* cl(\{x\}) \cap \bigcup_{y \in X \setminus V} M_I^* cl(\{y\}) = \emptyset$.
On the other hand, since $V \in M_I^* - O(X)$and $y \in X \setminus V$, we have $M_I^* cl(\{y\}) \in X \setminus V$ and hence $X \setminus V = \bigcup_{y \in X \setminus V} M_I^* cl(\{y\})$. Therefore we obtain $(X \setminus V) \cap M_I^* cl(\{x\}) = \emptyset$and $M_I^* cl(\{x\}) \subseteq V$. This shows that (X, τ, I) is a $M_I^* R_0$ space.

Proposition.4.4. If an ideal space (X, τ, I) is $M_I^* T_0$ and $M_I^* R_0$space, then it is $M_I^* T_1$.

Proof: Let x and y be any points of X. Since $(X, \tau, I)$is $M_I^* T_0$, there exists a M_I^* -open set U such that $x \in U$ and $y \notin U$. Since (X, τ, I) is $M_I^* R_0, x \in U$ implies that $M_I^* cl(\{x\}) \subseteq U$. Since $y \notin U$, so $y \notin M_I^* cl(\{x\})$. Hence $y \in V = X \setminus M_I^* cl(\{x\})$ and it is clear that $x \notin V$. Hence it follows that, there exist two M_I^* -open sets U and V containing x and y respectively, such that $y \notin U$and $x \notin V$. This implies that $(X, \tau, I)$is $M_I^* T_1$.

Proposition.4.5. For an ideal space (X, τ, I), the following statements are equivalent:

(i) $(X, \tau, I)$is a $M_I^* R_0$ -space
(ii) $x \in M_I^* cl(\{y\})$if and only if $y \in M_I^* cl(\{x\})$for any points $x, y \in X$.

Proof. $(i) \Rightarrow (ii)$. Assume that $(X,\tau,I)$is $M_I{}^*R_0$. Let $x \in M_I{}^*cl(\{y\})$ and V be any $M_I{}^*$ -open set such that $y \in V$. Now by hypothesis, $x \in V$. Therefore, every $M_I{}^*$ -open set which contain $y$contains x. Hence $y \in M_I{}^*cl(\{x\})$. Similarly the converse is obvious.

$(ii) \Rightarrow (i)$. Let U be any $M_I{}^*$ -open set in $(X,\tau,I)$and $x \in U$. If $y \notin U$, then $x \notin M_I{}^*cl(\{y\})$ and hence $y \notin M_I{}^*cl(\{x\})$. This implies that $M_I{}^*cl(\{y\}) \subseteq U$. Hence $(X,\tau,I)$is $M_I{}^*R_0$.

Proposition.4.6. For any point x,y in an ideal space (X,τ,I), the following properties are equivalent:

(i) $M_I{}^* - ker(\{x\}) \neq M_I{}^* - ker(\{y\})$.
(ii)$M_I{}^* - cl(\{x\}) \neq M_I{}^* - cl(\{y\})$

Proof. $(i) \Rightarrow (ii)$. Suppose that $M_I{}^* - ker(\{x\}) \neq M_I{}^* - ker(\{y\})$, then there exists a point z in X such that $z \in M_I{}^* - ker(\{x\})$ and $z \notin M_I{}^* - ker(\{y\})$. From $z \in M_I{}^* - ker(\{x\})$, it follows that $\{x\} \cap M_I{}^*cl(\{z\}) \neq \emptyset$ which implies $x \in M_I{}^*cl(\{z\})$. By $z \notin M_I{}^* - ker(\{y\})$, we have $\{y\} \cap M_I{}^*cl(\{z\}) = \emptyset$. Since $x \in M_I{}^*cl(\{z\})$, $M_I{}^*cl(\{x\}) \subseteq M_I{}^*cl(\{z\})$ and $\{y\} \cap M_I{}^*cl(\{x\}) = \emptyset$. Therefore, it follows that $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. Now $M_I{}^* - ker(\{x\}) \neq M_I{}^* - ker(\{y\})$ implies that $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. $(ii) \Rightarrow (i)$. Suppose that $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. Then there exists a point z in X such that $z \in M_I{}^*cl(\{x\})$and $z \notin M_I{}^*cl(\{y\})$. Then there exists a $M_I{}^*$ -open set containing z and therefore x but not y, namely $y \notin M_I{}^*ker(\{x\})$ and thus $M_I{}^* - ker(\{x\}) \neq M_I{}^* - ker(\{y\})$.

Theorem.4.7. Let (X,τ,I)be an ideal space. Then $\bigcap\{M_I{}^*cl(\{x\}) : x \in X\} = \emptyset$ and only if $M_I{}^* - ker(\{x\}) \neq X$ for every $x \in X$.

Proof. Necessity: Suppose that $\bigcap\{M_I{}^*cl(\{x\}) : x \in X\} = \emptyset$. Assume that there is a point $y \in X$ such that $M_I{}^* - ker(\{y\}) = X$. Let x be any point of X, then $x \in V$ for every $M_I{}^*$ -open set V containg y and hence $y \in M_I{}^*cl(\{x\})$ for any $x \in X$. This implies that $y \in \bigcap\{M_I{}^*cl(\{x\}) : x \in X\}$. But this isa contradiction.

Sufficiency: Assume that $M_I{}^* - ker(\{x\}) \neq X$ for every $x \in X$. If there exist apoint y in X such that $y \in \bigcap\{M_I{}^*cl(\{x\}) : x \in X\}$, then every $M_I{}^*$ -open set containing y must contain every point of X. This implies that the space (X,τ,I) is the unique $M_I{}^*$ -open set containing y. Hence $M_I{}^* - ker(\{y\}) = X$, which isa contradiction. Therefore $\bigcap\{M_I{}^*cl(\{x\}) : x \in X\} = \emptyset$.

Theorem.4.8. An ideal space (X,τ,I) is $M_I{}^*R_0$ if and only if for every $x,y \in X, M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\}) \Rightarrow M_I{}^*cl(\{x\}) \cap M_I{}^*cl(\{y\}) = \emptyset$
.

Proof. Necessity: Suppose that (X,τ,I) is $M_I{}^* - R_0$ and $x,y \in X$ such that $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. Then there exists some $z \in M_I{}^*cl(\{x\})$ such that $z \notin M_I{}^*cl(\{y\})$ ($z \in M_I{}^*cl(\{y\})$ such that $z \notin M_I{}^*cl(\{x\})$. Then there exists a $M_I{}^*$ -open set V such that $y \notin V$and $z \in V$, hence $x \in V$. Therefore we have $x \notin M_I{}^*cl(\{y\})$. Thus $x \in X\backslash M_I{}^*cl(\{y\})$ which is a $M_I{}^*$ -open set containing x. Since $(X,\tau,I)$is $M_I{}^* - R_0$,$M_I{}^*cl(\{x\}) \subseteq X\backslash M_I{}^*cl(\{y\})$ and hence $M_I{}^*cl(\{x\}) \cap M_I{}^*cl(\{y\}) = \emptyset$.

Sufficiency: Let $V \in M_I{}^* - O(X)$ and let $x \in V$. We claim that $M_I{}^*cl(\{x\}) \subseteq V$. Let $y \notin V$, that is $y \in X\backslash V$. Then $x \neq y$and $x \notin M_I{}^*cl(\{y\})$. This shows that $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. By assumption, $M_I{}^*cl(\{x\}) \cap M_I{}^*cl(\{y\}) = \emptyset$. Hence $y \notin M_I{}^*cl(\{x\})$ and therefore $M_I{}^*cl(\{x\}) \subseteq V$. Hence $(X,\tau,I)$is $M_I{}^* - R_0$.

Theorem.4.9. An ideal space (X,τ,I) is $M_I{}^*R_0$ if and if only for every $x,y \in X, M_I{}^*ker(\{x\}) \neq M_I{}^*ker(\{y\}) \Rightarrow M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{y\}) = \emptyset$
.

Proof. Necessity: Suppose that (X,τ,I) is $M_I{}^* - R_0$ .Thus by Proposition 4.6, for any points $x,y \in X$, if $M_I{}^*ker(\{x\}) \neq M_I{}^*ker(\{y\})$ then $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. Now we prove that $M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{y\}) = \emptyset$. Assume that $z \in M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{y\})$. By Proposition 3.19,and $z \in M_I{}^*ker(\{x\})$, it follows that $x \in M_I{}^*cl(\{z\})$. Since $x \in M_I{}^*cl(\{z\})$ and by Proposition 4.3, $M_I{}^*cl(\{x\}) = M_I{}^*cl(\{z\})$. Similarly, we have $M_I{}^*cl(\{y\}) = M_I{}^*cl(\{z\}) = M_I{}^*cl(\{x\})$. This is a contradiction. Therefore, we have $M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{y\}) = \emptyset$.

Sufficiency: Let (X,τ,I) be an ideal space such that for any points x and y in $X, M_I{}^*ker(\{x\}) \neq M_I{}^*ker(\{y\})$implies $M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{y\}) = \emptyset$. By Proposition 4.6, if $M_I{}^*ker(\{x\}) \neq M_I{}^*ker(\{y\})$then $M_I{}^*cl(\{x\}) \neq M_I{}^*cl(\{y\})$. Hence $M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{y\}) = \emptyset$ which implies $M_I{}^*cl(\{x\}) \cap M_I{}^*cl(\{y\}) = \emptyset$. Because $z \in M_I{}^*cl(\{x\})$ implies that $x \in M_I{}^*ker(\{z\})$ and therefore $M_I{}^*ker(\{x\}) \cap M_I{}^*ker(\{z\}) = \emptyset$. By hypothesis, we have $M_I{}^* - ker(\{x\})$ =$M_I{}^* - ker(\{z\})$.Then $z \in M_I{}^*cl(\{x\}) \cap M_I{}^*cl(\{y\})$implies that $M_I{}^* - ker(\{x\})$ =$M_I{}^* - ker(\{z\}) = M_I{}^* - ker(\{y\})$.This is a contradiction.Therefore,$M_I{}^*cl(\{x\}) \cap M_I{}^*cl(\{y\}) = \emptyset$ and by Proposition 4.3,(X,τ,I) is a $M_I{}^*R_0$–space.

Proposition.4.10. In an ideal space (X,τ,I),the following statements are equivalent:

(i) $(X,\tau,I)$is a $M_I{}^*R_0$-space.
(ii) For any non-empty set A and $G \in M_I{}^* - O(X)$ such that $A \cap G \neq \emptyset$,there exists $F \in M_I{}^* - C(X)$ such that $A \cap F \neq \emptyset$ and $F \subseteq G$.
(iii) $G = \cup\{F \in M_I{}^* - C(X) : F \subseteq G\}$for any $G \in M_I{}^* - O(X)$.
(iv) $F = \cap\{G \in M_I{}^* - O(X) : F \subseteq G\}$for any $F \in M_I{}^* - C(X)$.
(v) $M_I{}^*cl(\{x\}) \subseteq M_I{}^* - \ker(\{x\})$for every $x \in X$.

Proof.$(i) \Rightarrow (ii)$.Let A be any non-empty subset of X and $G \in M_I{}^* - O(X)$ such that $A \cap G \neq \emptyset$.Then choose $x \in A \cap G$.Since (X,τ,I) is a $M_I{}^*R_0$-space,$x \in G \in M_I{}^* - O(X)$,$M_I{}^*cl(\{x\}) \subseteq G$.Suppose $F = M_I{}^*cl(\{x\})$,then $F \in M_I{}^* - C(X)$ such that $A \cap F \neq \emptyset$ and $F \subseteq G$.
$(ii) \Rightarrow (iii)$.Let $G \in M_I{}^* - O(X)$.Then for every $x \in F \in M_I{}^* - C(X), F \subseteq G, x \in G$, and hence $\cup\{F \in M_I{}^* - C(X): F \subseteq G\} \subseteq G$.On the other hand, suppose that x is an arbitrary point of$G$.If we define $A = \{x\}$,then by hypothesis,there exists $F \in M_I{}^* - C(X)$ such that $x \in F$ and $F \subseteq G$.Therefore,$G \subseteq \cup\{F \in M_I{}^* - C(X) : F \subseteq G\}$ and hence $G = \cup\{F \in M_I{}^* - C(X) : F \subseteq G\}$.
$(iii) \Rightarrow (iv)$.It is obvious.
$(iv) \Rightarrow (v)$.Let $x \in X$ be arbitrary and $y \notin M_I{}^* - \ker(\{x\})$.There exists $V \in M_I{}^* - O(X)$ such that $x \in V$ and $y \notin V$,hence $M_I{}^*cl(\{y\}) \cap V = \emptyset$.By (iv),$\cap\{G \in M_I{}^* - O(X): M_I{}^*cl(\{y\}) \subseteq G\} \cap V = \emptyset$ and hence there exists $G \in M_I{}^* - O(X)$ such that $x \notin G$ and $M_I{}^*cl(\{y\}) \subseteq G$.Therefore $M_I{}^*cl(\{y\}) \cap G = \emptyset$ and$y \notin M_I{}^*cl(\{x\})$.Consequently,we obtain $M_I{}^*cl(\{x\}) \subseteq M_I{}^* - \ker(\{x\})$.
$(v) \Rightarrow (i)$.Let G be any $M_I{}^*$-open set and $x \in G$ be any arbitrary point.Let $y \in M_I{}^* - \ker(\{x\})$,then$x \in M_I{}^*\mathrm{cl}(\{y\})$ and $y \in G$.This implies that $M_I{}^* - \ker(\{x\}) \subseteq G$.Therefore,we obtain $x \in M_I{}^*cl(\{x\}) \subseteq M_I{}^* - \ker(\{x\}) \subseteq G$.This shows that (X,τ,I) is a $M_I{}^*R_0$-space.

Corollary.4.11. For an ideal space (X,τ,I) the following statements are equivalent:

(i) $(X,\tau,I)$is a $M_I{}^*R_0$-space.
(ii) $M_I{}^*cl(\{x\}) = M_I{}^* - \ker(\{x\})$for all $x \in X$.

Proof. $(i) \Rightarrow (ii)$.Suppose that (X,τ,I) is a $M_I{}^*R_0$-space.By Proposition 4.10 (v),$M_I{}^*cl(\{x\}) \subseteq M_I{}^* - \ker(\{x\})$ for each $x \in X$.Let $y \in M_I{}^* - \ker(\{x\})$,then byProposition 3.19, $x \in M_I{}^*\mathrm{cl}(\{y\})$ and by Proposition 4.3 (iv),$M_I{}^*cl(\{x\}) = M_I{}^*cl(\{y\})$.Therefore, $y \in M_I{}^*cl(\{x\})$ and hence $M_I{}^* - \ker(\{x\}) \subseteq M_I{}^*cl(\{x\})$.This shows that $M_I{}^*cl(\{x\}) = M_I{}^* - \ker(\{x\})$.
$(ii) \Rightarrow (i)$.It follows from Proposition 4.10.

Proposition.4.12. For an ideal space (X,τ,I) the following statements are equivalent.

(i) $(X,\tau,I)$is a $M_I{}^*R_0$-space.
(ii) If F is $M_I{}^*$-closed, then $F = M_I{}^* - \ker(F)$.
(iii) If F is $M_I{}^*$-closed and $x \in F$, then $M_I{}^* - \ker(\{x\}) \subseteq F$.
(iv) If $x \in X$,then$M_I{}^* - \ker(\{x\}) \subseteq M_I{}^*cl(\{x\})$.

Proof.$(i) \Rightarrow (ii)$.Let F be a $M_I{}^*$-closed set and $x \notin F$.Thus $X\backslash F$ is a $M_I{}^*$-open set containing$x$.Since (X,τ,I) is $M_I{}^*$-R_0, $M_I{}^*cl(\{x\} \subseteq X\backslash F$.Thus $M_I{}^*cl(\{x\}) \cap F = \emptyset$ and byProposition 3.20,,$x \notin M_I{}^* - \ker(F)$.Therefore $F = M_I{}^* - \ker(F)$.
$(ii) \Rightarrow (iii)$.In general by Proposition 3.21, $A \subseteq B \Rightarrow M_I{}^* - \ker(A) \subseteq M_I{}^* - \ker(B)$.Therefore it follows from (ii) that $M_I{}^* - \ker(\{x\}) \subseteq M_I{}^* - \ker(F) = F$.
$(iii) \Rightarrow (iv)$.Since $x \in M_I{}^*cl(\{x\})$ and $M_I{}^*cl(\{x\})$ is $M_I{}^*$-closed, by(iii), $M_I{}^* - \ker(\{x\}) \subseteq M_I{}^*cl(\{x\})$.
$(iv) \Rightarrow (i)$.We show that the implication by using Proposition 4.5.Let $x \in M_I{}^*cl(\{y\})$.Then by Proposition 3.19, $y \in M_I{}^* - \ker(\{x\})$.Since $x \in M_I{}^*cl(\{x\})$ and $M_I{}^*cl(\{x\})$ is $M_I{}^*$-closed, by (iv), we obtain$y \in M_I{}^* - \ker(\{x\}) \subseteq M_I{}^*cl(\{x\})$.Therefore $x \in M_I{}^*cl(\{y\})$ implies $y \in M_I{}^*cl(\{x\})$.The converse is obvious and (X,τ,I) is $M_I{}^*R_0$.

Proposition.4.13. (i) Every $\alpha - R_0$ space is $M_I{}^*R_0$ but not conversely.
(ii) Every $\alpha - R_1$ space is $M_I{}^*R_1$ but not conversely.

Proof. The proof is follows from the fact that every α-open set is $M_I{}^*$-open.

Example.4.14. Let $X = \{a,b,c\}, \tau = \{\emptyset, X, \{a\},\{b\},\{a,b\}\}$and $I = \{\emptyset,\{a\},\{b\},\{a,b\}\}$.Then $M_I{}^*$-open sets of an ideal space are $\emptyset, X, \{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}$ and α-open sets of X are $\emptyset, X, \{b\},\{c\},\{b,c\}$.Here the ideal space (X,τ,I) is $M_I{}^*R_0$ and $M_I{}^*R_1$ but not $\alpha - R_0$and $\alpha - R_1$.

Proposition.4.15. An ideal space (X,τ,I) is $M_I{}^*R_1$ if it is $M_I{}^*T_2$.

Proof. Let x and y be any points of X such that $M_I^*cl(\{x\}) \neq M_I^*cl(\{y\})$.By Proposition 3.8(ii),every $M_I^*T_2$-space is $M_I^*T_1$.Therefore,by Proposition 3.5,$M_I^*cl(\{x\}) = \{x\}, M_I^*cl(\{y\}) = \{y\}$ and hence $x \neq y$.Since (X,τ,I) is $M_I^*T_2$,there exists disjoint M_I^*-open sets U and V such that $M_I^*cl(\{x\}) = \{x\} \subseteq U$ and $M_I^*cl(\{y\}) = \{y\} \subseteq V$.This shows that (X,τ,I) is $M_I^*R_1$.

Proposition.4.16. In an ideal space (X,τ,I) is M_I^*-symmetric, then the following statements are equivalent.
(i) $(X,\tau,I)$is $M_I^*T_2$.
(ii) $(X,\tau,I)$is$M_I^*R_1$ and $M_I^*T_1$.
(iii) $(X,\tau,I)$is$M_I^*R_1$ and $M_I^*T_0$.
Proof.$(i) \Rightarrow (ii)$.Follows from Proposition 3.8(ii) and Proposition 4.15.
$(ii) \Rightarrow (iii)$.Follows from Proposition 3.8(i).
$(iii) \Rightarrow (i)$.FromTheorem 3.5,(X,τ,I) is $M_I^*T_0$ implies M_I^*-closures of distinct points are distinct. Since $(X,\tau,I)$is $M_I^*R_1$,there exists disjoint M_I^*-open sets U and V such that $M_I^*cl(\{x\}) \subseteq U$ and $M_I^*cl(\{y\}) \subseteq V$ and so $x \in U$ and $y \in V$.Hence (X,τ,I) is $M_I^*T_2$. **Proposition.4.17.** For an ideal space (X,τ,I), the following statements are equivalent
(i)$(X,\tau,I)$is$M_I^*R_1$
(ii) If $x,y \in X$ such that $M_I^*cl(\{x\}) \neq M_I^*cl(\{y\})$, then there exists two M_I^*-closed sets F_1 and F_2 such that $x \in F_1, y \notin F_1$ $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.

Proof. Proof is obvious.

Proposition.4.18. Every $M_I^*R_1$ space is $M_I^*R_0$.

Proof. Let U be M_I^* -open set such that $x \in U$. We have to prove, $M_I^*cl(\{x\}) \subseteq U$
If $y \notin U$ and since $x \notin M_I^*cl(\{y\})$, we have $M_I^*cl(\{x\}) \neq M_I^*cl(\{y\})$. So there exists a M_I^* -open set V such that $M_I^*cl(\{y\}) \subseteq V$and $x \notin V$, which implies $y \notin M_I^*cl(\{x\})$. Hence $M_I^*cl(\{x\}) \subseteq U$. Therefore (X,τ,I) is $M_I^*R_0$

Corollary.4.19. An ideal space (X,τ,I) is $M_I^*R_1$ if and only if for$x,y \in X$,
$M_I^* - ker(\{x\}) \neq M_I^* - ker(\{y\})$, there exists disjoint M_I^* -open sets U and V such that $M_I^*cl(\{x\}) \subseteq U$and $M_I^*cl(\{y\}) \subseteq V$.

Proof. Follows from Proposition 4.6.

Theorem.4.19. An ideal space (X,τ,I) is $M_I^*R_1$ if and only if $x \in X\backslash M_I^*cl(\{y\})$ implies that x and y have disjoint M_I^* -open neighbourhoods.

Proof. Necessity: Let $x \in X\backslash M_I^*cl(\{y\})$. Since $(X,\tau,I)$is $M_I^*R_1$, then $M_I^*cl(\{x\}) \neq M_I^*cl(\{y\})$, so x and y have disjoint M_I^* -open neighbourhoods.

Sufficiency: First we show that $(X,\tau,I)$is $M_I^*R_0$. let$U$ be a M_I^* -open set and $x \in U$. Suppose that $y \notin U$. Then $M_I^*cl(\{y\}) \cap U = \emptyset$and $x \notin M_I^*cl(\{y\})$. By hypothesis, there exists two M_I^* -open sets U_x and U_y such that $\in U_x$, $y \in U_y$ and $U_x \cap U_y = \emptyset$. Hence $M_I^*cl(\{x\}) \subseteq M_I^*cl(\{U_x\})$and $M_I^*cl(\{x\}) \cap U_y \subseteq M_I^*cl(\{U_x\}) \cap U_y = \emptyset$. Therefore, $y \notin M_I^*cl(\{x\})$. Consequently, $M_I^*cl(\{x\}) \subseteq U$ and $(X,\tau,I)$is $M_I^*R_0$. Next, we show that $(X,\tau,I)$is $M_I^*R_1$. Suppose that $M_I^*cl(\{x\}) \neq M_I^*cl(\{y\})$. Then, we can assume that there exists $z \in M_I^*cl(\{x\})$ such that $z \notin M_I^*cl(\{y\})$. There exists M_I^* -open sets V_y and V_z such that $\in V_y$, $z \in V_z$ and $V_y \cap V_z = \emptyset$. Since $z \in M_I^*cl(\{x\})$, $x \in V_z$. Since $(X,\tau,I)$is $M_I^*R_0$, we obtain $M_I^*cl(\{x\}) \subseteq V_z$, $M_I^*cl(\{y\}) \subseteq V_y$ and $V_y \cap V_z = \emptyset$. This shows that (X,τ,I) is $M_I^*R_1$

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