Generalized Semi pre Regular Super closed Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract- *In this paper, we introduce the notion of an intuitionistic fuzzy generalized semi pre regular super closed sets and intuitionistic fuzzy generalized semi pre regular super open sets and study some of its properties in Intuitionistic fuzzy topological spaces.*

Keywords- Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi pre regular super closed sets, intuitionistic fuzzy generalized semi pre regular super open sets and intuitionistic fuzzy points.

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I. PRELIMINARIES

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the super closure, the super interior are denoted by cl(A), int(A) and compliment of A is denoted by A^c .

Definition 1.1: Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = {⟨x, $\mu_A(x)$, ν)⟩/ $_A$(xx ∈ X},where the functions μ_A:X→[0,1] and ν_A: X →[0,1] denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x ∈ X to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 1.2: Let (X, τ) be an IFTS and A = ⟨x, μ_A, ν_A⟩ be an IFS in X. Then the intuitionistic fuzzy super interior and an intuitionistic fuzzy super closure are defined by

int(A) = ∪{ G / G is an IFSOS in X and G ≤ A }.
cl(A) = ∩{ K / K is an IFSCS in X and A ≤ K }.

Definition 1.3: An IFS A of an IFTS (X, τ) is an

i. Intuitionistic fuzzy regular super closed set (IFRSCS) if A = cl(int(A)).
ii. Intuitionistic fuzzy regular super open set (IFRSOS) if A = int(cl(A)).
iii. Intuitionistic fuzzy semi super closed set (IFSSCS) if int(cl(A)) ≤ A.
iv. Intuitionistic fuzzy semi super open set (IFSSOS) if A ≤ cl(int(A)).
v. Intuitionistic fuzzy pre super closed set (IFPSCS) if cl(int(A)) ≤ A .
vi. Intuitionistic fuzzy pre-super open set (IFPSOS) if A ≤ int(cl(A)).

Definition 1.4: An IFS A of an IFTS (X, τ) is an

i. Intuitionistic fuzzy semi pre super closed set (IFSPSCS) if there exists an IFPSCS B such that int(B) ≤ A ≤ B,
ii. Intuitionistic fuzzy semi pre-super open set (IFSPSOS) if there exists an IFPSOS B such that B ≤ A ≤ cl(B).

Definition 1.5: An IFS A is an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi pre super closed set (IFGSPSCS) if spcl(A) ≤ U whenever A ≤ U and U is an IFSSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy generalized semi pre-super open set (IFGSPSOS in short) if A^c is an IFGSPCS in (X, τ).

Definition 1.6: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy semi pre generalized super closed set (IFSPGSCS for short) if spcl(A) ≤ U whenever A ≤ U and U is an IFSSOS in (X, τ). Every IFSCS, IFSSCS, IFRSCS, IFPSCS, IFSPSCS, is an IFSPGSCS but the converses are not true in general.

Definition 1.7: The complement A^c of an IFSPGSCS A in an IFTS (X, τ) is called an intuitionistic fuzzy semi pre generalized super open set (IFSPGSOS) in X.

Definition 1.8:Let A be a IFS in an IFTS (X, τ). Then

i. spint (A) = ∪{ G / G is an IFSPSOS in X and G ≤ A },
ii. spcl (A) = ∩{ K / K is an IFSPSCS in X and A ≤ K }.
iii. Note that for any IFS A in (X, τ), we have spcl(A^c) = $(\text{spint}(A))^c$ and spint(A^c) = $(\text{spcl}(A))^c$.

Definition 1.9: Let A be a IFS in an IFTS (X, τ). Then

i. Intuitionistic fuzzy regular generalized super closed set (IFRGSCS) if cl(A) ≤ U whenever A≤ U and U is an intuitionistic fuzzy regular super open in X .

ii. Intuitionistic fuzzy generalized pre regular super closed set (IFGPRSCS) if pcl(A) ≤ U whenever A≤ U and U is an intuitionistic fuzzy regular super open in X .

iii. Intuitionistic fuzzy generalized pre super closed set (IFGPSCS) if pcl(A) ≤ U whenever A≤ U and U is an intuitionistic fuzzy super open in X

An IFS A of an IFTS (X,τ) is called an intuitionistic fuzzy regular generalized super open set, intuitionistic fuzzy generalized pre regular super open set and intuitionistic fuzzy generalized pre-super open set (IFRGSOS, IFGPRSOS and IFGPSOS in short) if the complement A^c is an IFRGSCS, IFGPRSCS and IFGPSCS respectively.

Definition 1.10: An IFTS (X, τ) is said to be an intuitionistic fuzzy $T_{1/2}$ ($IFT_{1/2}$ for short) space if every intuitionistic fuzzy generalized super closed set in X is an intuitionistic fuzzy super closed set in X.

II. INTUITIONISTIC FUZZY GENERALIZED SEMI PRE REGULAR SUPER CLOSED SETS

In this section we have introduced intuitionistic fuzzy generalized semi pre regular super closed sets and have studied some of its properties.

Definition 2.1: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi pre regular super closed set (IFGSPRSCS) if spcl(A)≤ U whenever A ≤ U and U is an IFRSOS in (X, τ). The family of all IFGSPRSCSs of an IFTS(X, τ) is denoted by IFGSPRSC(X).

For the sake of simplicity, we shall use the notation A= ⟨x, (μ, μ), (ν, ν)⟩ instead of A= ⟨x,(a/μ_a, b/μ_b), (a/ν_a, b/ν_b)⟩ in all the examples used in this paper.

Theorem 2.3: Every IFCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFSCS. Let A≤ U and U be an IFRSOS in (X, τ). Then spcl(A)≤ cl(A) = A ≤ U, by hypothesis. Hence A is an IFGSPRSCS in (X, τ).

Theorem 2.5: Every IFRGSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFRGSCS. Let A ≤ U and U be an IFRSOS in (X, τ). Then spcl(A)≤ cl(A)≤ U, by hypothesis. Hence A is an IFGSPRSCS in (X, τ).

Theorem 2.7: Every IFGPRSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFGPRSCS and A ≤ U, U be an IFRSOS in (X, τ). Then spcl(A) ≤ pcl(A), since pcl(A) ≤ U. We have spcl(A)≤ U. Hence A is an IFGSPRSCS in (X, τ).

Theorem 2.9: Every IFPSCS in (X, τ) is an IFGSPRCS in (X, τ) but not conversely.

Proof: Let A be an IFPSCS in (X, τ) and let A≤ U, U be an IFRSOS in (X, τ). Then spcl(A) ≤ pcl(A) = A ≤ U, by hypothesis. Hence is an IFGSPRSCS in (X, τ).

Theorem 2.11: Every IFGPSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFGPSCS in (X, τ). Let A≤ U and U be an IFRSOS in (X, τ). Since every IFRSOS in (X, τ) is an IFSOS in (X, τ). Then spcl(A) ≤ pcl(A) ≤ U. Hence is an IFGSPRSCS in (X, τ).

Theorem 2.13: Every IFRSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFRSCS in (X, τ). Since every IFRSCS is an IFSCS, by then A is an IFGSPRSCS in (X, τ).

Theorem 2.15: Every IFSSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFSSCS in (X, τ) . Let A≤ U and U be an IFRSOS in (X, τ). Since spcl(A) ≤ scl(A) = A ≤ U. Hence spcl(A) ≤ U. Therefore A is an IFGSPRSCS in (X, τ).

Theorem 2.17: Every IFSPSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFSPSCS and A ≤ U, U be an IFRSOS in (X, τ). Then since spcl(A)= A and A≤ U, we have spcl(A)≤ U. Hence A is an IFGSPRSCS in (X, τ).

Theorem 2.19: Every IFSPGSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFSPGSCS in (X, τ). Let A≤U and U is an IFRSOS in (X, τ). Since every IFRSOS in (X, τ) is an IFSOS in (X, τ) and every IFSOS in (X, τ) is an IFSSOS in (X, τ). We have spcl(A) ≤ U, by hypothesis. Hence A is an IFGSPRSCS in (X, τ).

Theorem 2.21: Every IFGSPSCS in (X, τ) is an IFGSPRSCS in (X, τ) but not conversely.

Proof: Let A be an IFGSPSCS in (X, τ). Let A≤U and U is an IFRSOS in (X, τ). Since every IFRSOS in (X, τ) is an IFOS in (X, τ). We have spcl(A)≤ U, by hypothesis. Hence A is an IFGSPRSCS in (X, τ).

Theorem 2.23: Let (X, τ) be an IFTS. Then for every A ∈ IFGSPRSC(X) and for every IFS B ∈ IFS(X), A≤ B ≤ spcl(A) implies B ∈ IFGSPRSC(X).

Proof: Let B ≤ U and U is an IFRSOS in (X, τ). Then since A ≤ B, A ≤ U. Since A is an IFGSPRSCS, it follows that spcl(A) ≤ U. Now B ≤ spcl(A) implies spcl(B) ≤ spcl(spcl(A)) = spcl(A). Thus, spcl(B) ≤ U. This proves that B ∈ IFGSPRSC(X).

Theorem 2.24: If A is an IFRSOS and an IFGSPRSCS in (X, τ), then A is an IFSPSCS in (X, τ).

Proof: Since A ≤ A and A is an IFRSOS in (X, τ), by hypothesis, spcl(A) ≤ A. But since A ≤ spcl(A). Therefore spcl(A) = A. Hence A is an IFSPSCS in (X, τ).

Theorem 2.25: Let (X, τ) be an IFTS. Then for every A ∈ IFSPC(X) and for every IFS B in X, int(A) ≤ B ≤ A B ∈ IFGSPRSC(X).

Proof: Let A be an IFSPSCS in X. Then there exists an IFPSCS, say C such that int(C) ≤ A ≤ C. By hypothesis,B≤ A .Therefore B ≤ C. Since int(C) ≤ A, int(C) ≤ int(A) and int(C) ≤ B. Thus int(C)≤B≤C and by definition 2.6,B∈ IFSPSC(X).Hence by the previous result B ∈ IFGSPRSC(X).

III. INTUITIONISTIC FUZZY GENERALIZED SEMIPRE REGULAR SUPER OPEN SETS

In this section we have introduced intuitionistic fuzzy generalized semi pre regular super open sets and have studied some of its properties.

Definition 3.1: An IFS A is said to be an intuitionistic fuzzy generalized semi pre regular super open set (IFGSPRSOS for short) in (X, τ) if the complement Ac is an IFGSPRSCS in The family of all IFGSPRSOS of an IFTS (X, τ) is denoted by IFGSPRSO(X).

Theorem 3.1: Let (X, τ) be a IFTS. Then for every A ∈ IFGSPRSO(X) and for every B ∈ IFSS(X), spint(A) ≤ B ≤ A implies B ∈ IFGSPRSO(X).

Proof: Let A be any IFGSPRSOS of X and B be any IFS of X. By hypothesis spint(A) ≤ B ≤ A. Then A^c is an IFGSPRSCS in X and A^c≤B^c≤spcl(A^c) then B^c is an IFGSPRSCS in (X, τ). Therefore B is an IFGSPRSOS in (X, τ).Hence B ∈ IFGSPRSO(X).

Theorem 3.2: An IFS A of an IFTS (X, τ) is an IFGSPROS in (X, τ) if and only is F ≤ spint (A) whenever F is an IFRCS in (X, τ) and F ≤ A.

Proof: Necessity: Suppose A is an IFGSPRSOS in (X, τ). Let F be an IFRSCS in (X, τ) such that F ≤ A. Then Fc is an IFRSOS and A^c ≤ F^c. By hypothesis Ac is an IFGSPRSCS in (X, τ), we have spcl(A^c) ≤ F^c. Therefore F≤ spint (A).

Sufficiency: Let U be an IFROS in (X, τ) such that A^c ≤ U. By hypothesis, U^c≤ spint (A). Therefore spcl(A^c) ≤ U and A^c is an IFGSPRSCS in (X, τ). Hence A is an IFGSPRSOS in (X, τ).

Theorem 3.3: Let (X, τ) be an IFTS then for every A ∈ IFSPSO(X) and for every IFS B in X, A≤ B ≤ cl(A) implies B ∈ IFGSPRSO(X).

Proof: Let A be an IFSPOS in X. Then by Definition 2.6., there exists an IFPSOS, say C such that C ≤A≤cl(C).By hypothesis A≤B .Therefore C≤B. Since A≤cl(C), cl(A)≤cl(C) and B≤cl(C).Thus C≤ B≤ cl(C). This implies that B is an IFSPSOS in X. Then By Theorem 4.2., B is an IFGSPROS. That is B ∈ IFGSPRO(X).

IV. APPLICATIONS OF INTUITIONISTIC FUZZY GENERALIZED SEMIPRE REGULAR SUPER CLOSED SETS

In this section we have provided some applications of intuitionistic fuzzy generalized semi pre regular super closed sets in intuitionistic fuzzy topological spaces.

Definition 4.1: If every IFGSPRSCS in (X, τ) is an IFSPSCS in (X, τ), then the space can be called as an intuitionistic fuzzy semi pre regular $T_{1/2}$ (IFSPR$T_{1/2}$ for short) space.

Theorem 4.1: An IFTS (X, τ) is an IFSPRS$T_{1/2}$ space if and only if IFSPSOS(X) = IFGSPRSO(X).

Proof: Necessity: Let (X, τ) be an IFSPR$T_{1/2}$ space. Let A be an IFGSPROS in (X, τ). By hypothesis, A^c is an IFGSPRSCS in (X,τ) and therefore A is an IFSPSOS in (X,τ). Hence IFSPSO(X) = IFGSPRSO(X).

Sufficiency: Let IFSPSO(X, τ) = IFGSPRSO (X, τ). Let A be an IFGSPRSCS in (X, τ). Then Ac is an IFSPSOS in (X, τ). By hypothesis, Ac is an IFSPSOS in (X, τ) and therefore A is an IFSPSCS in (X, τ). Hence (X, τ) is an $IFSPRT_{1/2}$ space.

Remark 4.1: Not every $IFSPRT_{1/2}$ space is an $IFT_{1/2}$ space. This can be seen easily by the following example.

Theorem 4.2: For any IFS A in (X, τ) where X is an $IFSPRT_{1/2}$ space, A ∈ IFGSPRSO(X) if and only if for every IFP p(α, β) ∈ A, there exists an IFGSPRSOS B in X such that p(α, β) ∈ B ≤ A.

Proof: Necessity: If A ∈ IFGSPRSO(X), then we can take B= A so that p(α, β) ∈ B ≤ A for every IFP p(α, β) ∈ A.

Sufficiency: Let A be a IFS in (X, τ) and assume that there exists B ∈ IFGSPRSO(X) such that p(α,β) ∈ B ≤ A. Since X is an $IFSPRT_{1/2}$ space, B is an IFSPSOS. Then A= ⋃ * (α β)+ (α β) ∈ ≤ ⋃ (α β) ∈ ≤ A. Therefore A is IFSPSOS. Then A is an IFGSPRSOS in (X, τ).

Definition 4.2: An IFTS(X,τ) is said to be an intuitionistic fuzzy semi pre regular $T^*_{1/2}$ space ($IFSPRT^*_{1/2}$ space for short) if every IFGSPRSCS is an IFCS in (X, τ).

Remark 4.2: Every $IFSPRT^*_{1/2}$ space is an $IFSPRT_{1/2}$ space but not conversely.

Proof: Assume be an $IFSPRT^*_{1/2}$ space. Let A be an IFGSPRSCS in (X, τ). By hypothesis, A is an IFCS. Since every IFSCS is an IFSPSCS, A is an IFSPCS in (X, τ). Hence (X, τ) is an $IFSPRT_{1/2}$ space.

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