Solving Fuzzy Transportation Problem Using Symmetric Triangular Fuzzy Number By Vam Method

R.Saravanan[1] ,C.Jaisankar[2], S. Arunvasan[3] and S.Shilpa ivin emimal[4]
[1, 2] Dept of Mathematics
[1] Saradha Gangadharan college, Velrampet Pondicherry-4
[2, 3, 4]AVC College (Autonomous)Mannampandal – 609305, India.

Abstract- *In this paper, We introduce Yager's ranking function to symmetric triangular fuzzy numbers. The main objectives of this paper is to introduce fuzzy transportation problem with symmetric triangular fuzzy numbers. An arithmetic operations on symmetric triangular fuzzy number is performed and based on this we obtain an initial basic feasible solution. The solutions are illustrated with suitable example.*

Keywords- Fuzzy Number, Fuzzy Set, Triangular Fuzzy Number, Symmetric Triangular Fuzzy Number, Fuzzy Transportation Problem.

I. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[11] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [2] in 1951, by means of Zadeh's extension principle [11]. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers. Uthra [10] proposed the optimum assignment schedule of the Fuzzy Assignment Problem is obtained by usual Hungarian Method.

The concept of Rank of a matrix with fuzzy numbers as its elements, which may be used to modern uncertain imprecise aspects of real-word problems. We studied main ideas based on rank of fuzzy matrix and arithmetic operations. We give some necessary and sufficient conditions for algorithm to find rank of fuzzy matrices based on Triangular fuzzy number. In Dubosis and Prade [1] arithmetic operations will be employed for the same purpose but with respect to the inherent difficulties which are derived from the positively restriction on Triangular fuzzy number. The concept of Rank and Nullity of a triangular fuzzy matrix and some of the relevant theorems will be revalued. Finally fuzzifing the defuzzified version of the original problem for introducing fuzzy rank.

The fuzzy matrices introduced first time by Thomson [9] and discussed about the convergence of powers of fuzzy matrix. C.jaisankar and S.Arunvasan [5] presented some important results on rank and nullity of triangular fuzzy number matrices. A.K.Shyamal and M.Pal [7,8] first time introduced triangular fuzzy matrices . In C.Jaisankar and S.Arunvasan [3,4] introduced the concept on Hessenberg of Triangular Fuzzy Matrices. A.Sahaya sudha [6] introduce a new ranking function for symmetric hexagonal fuzzy number.

The paper organized as follows, Firstly in section 2, we recall the definitions of Triangular fuzzy number and some operations on triangular fuzzy numbers (TFNs). In section 3, we defined Fuzzy Transportation problem. In section 4, we defined the procedure. In section 5, we have been presented the example with the aid of notion. Finally in section 6, conclusion is included.

II. DEFINITIONS

2.1 Fuzzy Set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A = \{(x, \mu_A(x)) : x \in X\}$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A (x) is called the membership value of $x \in X$ on the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Fuzzy Number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics *i*. $\tilde{A}$ is normal

ii. $\tilde{A}$ is convex

iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

2.3 Triangular fuzzy number

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & ; a_1 \leq x \leq a_2 \\ 1 & ; x = a_2 \\ \frac{a_3-x}{a_3-a_2} & ; a_2 \leq x \leq a_3 \\ 0 & ; x > a_3 \end{cases}$$

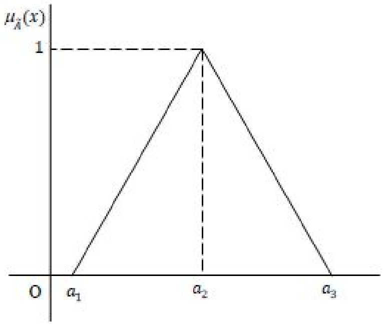

Fig:1 Triangular Fuzzy Number

2.4 Symmetric Triangular Fuzzy Number

If $a_2 = a_3$ then the triangular fuzzy number

$A = (a_1, a_2, a_3)$ is called symmetric Triangular Fuzzy Number. It is denoted by $A = (a_1, a_2)$ where a_1 is a $Core(A)$, a_2 is left width and right width of Core(A).
The parametric form of a Triangular Fuzzy Number is represented by

$$A = [a_1 - a_2(1-r), a_1 + a_2(1-r)].$$

2.4 Ranking of Triangular Fuzzy Number

Yager's ranking technique which satisfy compensation, linearity, additively properties and provides results which consists of human intuition. If $A = (a_1, a_2, a_3)$ is a fuzzy number then the Yager's ranking is defined by

$$R(A) = \int_0^1 0.5(a_\alpha{}^L, a_\alpha{}^U)d\alpha$$

Where

$(a_\alpha{}^L, a_\alpha{}^U) = \{(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha\}$

since $a_3 = a_2$.

Definition 2.7 Arithmetic Operations of Symmetric Triangular Fuzzy Numbers

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B}=(b_1, b_2, b_3)$ be triangular fuzzy numbers (TFNs) then we defined,

Addition

$$\tilde{A}+\tilde{B} = (a_1+b_1, a_2 + b_2, a_3 + b_3)$$

Subtraction

$$\tilde{A}-\tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

Multiplication

$$\tilde{A}\times\tilde{B} = (a_1\Re(B), a_2\Re(B), a_3\Re(B))$$

where $\Re(\tilde{B}) = \left(\frac{b_1+b_2+b_3}{3}\right)$ or $\Re(\tilde{b}) = \left(\frac{b_1+b_2+b_3}{3}\right)$

Division

$$\tilde{A}\div\tilde{B} = \left(\frac{a_1}{\Re(\tilde{B})}, \frac{c}{\Re(\tilde{B})}, \frac{a_3}{\Re(\tilde{B})}\right)$$

where $\Re(\tilde{B}) = \left(\frac{b_1+b_2+b_3}{3}\right)$ or $\Re(\tilde{b}) = \left(\frac{b_1+b_2+b_3}{3}\right)$

Scalar Multiplication

$$k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3) \; if \; K \geq 0 \\ (ka_3, ka_2, ka_1) \; if k < 0 \end{cases}$$

III. FUZZY TRANSPORTATION PROBLEM

Let a_i be the quantity of commodity available at origin i. b_j be the quantity of commodity needed at destination j. C_{ij} be the fuzzy cost of transporting one unit of commodity from origin i to destination j. x_{ij} be the quantity transported from origin i to destination j. Then the problem is to determine the transportation schedule so as to minimize the total fuzzy transportation cost satisfy supply and demand constraints.

The mathematical model of the Fuzzy Transportation Problem is given by

$$Minimize\ \ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} C_{ij}$$
$$subject\ to$$
$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad for\ j = 1,2,\ldots\ldots,n$$
$$\sum_{j=1}^{m} x_{ij} = b_j, \qquad for\ j = 1,2,\ldots\ldots,n$$
$$x_{ij} \geq 0\ for\ all\ i\ and\ j$$
$$C^*_{ij} = C^1_{ij}, C^2_{ij}, C^3_{ij}$$

IV. PROCEDURE

Step 1: Find the difference (penality) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step 2: Identify the row or column with largest penality. If a tie occurs, break the tie arbitrary. Choose the cell with smallest costs in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step 3.

Step 3: Again compute the column and roe penalities for the reduced transportation table and then go to step2 . Repeat the procedure until all the rim requirement are satisfy.

V. EXAMPLE

Consider the following fuzzy transportation problem. A company has three Sources O_1, O_2, O_3 and four destinations D1, D2,D3, and D4. The fuzzy transportation cost for unit quantity of the product form i[th] source to jth destinations is where

				Supply
	(0,1,1)	(0,2,2)	(4,6,6)	(5,7,7)
	(0,0,0)	(2,4,4)	(0,2,2)	(10,12,12)
	(1,3,3)	(0,1,1)	(3,5,5)	(9,11,11)
Demand	(8,10,10)	(8,10,10)	(8,10,10)	

After Yager's ranking, we get

0.75	1.5	5.5	6.5
0	3.5	1.5	11.5
2.5	0.75	4.5	12.5
9.5	9.5	9.5	

Total Supply= 28.5
Total Demand= 28.5
Total Supply=Total Demand
∴The given TP is balanced TP.

0.75	1.5	5.5	6.5 (0.75)
0	3.5	9.5 1.5	11.5 (1.5)
2.5	0.75	4.5	10.5 (1.75)
9.5 (0.75)	9.5 (0.75)	9.5 (3)	

0.75	1.5	6.5 (0.75)
2 ~~0~~	~~3.5~~	~~2~~ (3.5)
2.5	0.75	10.5 (1.75)
~~9.5~~ (0.75)	9.5 (0.75)	

6.5 0.75	1.5	6.5 (0.75)
2.5	0.75	10.5 (1.75)
~~7.5~~ (1.75)	9.5 (0.5)	

2.5	0.75	9.5 ~~10.5~~ (1.75)
1 (2.5)	9.5 (0.75)	

9.5 0.75	~~9.5~~
~~9.5~~	

[6.5] 0.75	1.5	5.5	6.5
[2] 0	3.5	[9.5] 1.5	11.5
[1] 2.5	[9.5] 0.75	4.5	12.5
9.5	9.5	9.5	

The initial fuzzy transportation cost according to the above is given by

$$= 6.5 \times 0.75 + 2 \times 0 + 9.5 \times 1.5 + 1 \times 2.5 + 9.5 \times 0.75$$
$$= 28.75 \cong 29$$

To find the optimal solution

Consider the above fuzzy transportation table
For occupied cell

[6.5] 0.75	1.5	5.5	6.5
[2] 0	3.5	[9.5] 1.5	11.5
[1] 2.5	[9.5] 0.75	4.5	12.5
9.5	9.5	9.5	

$$u_1 + v_1 = 0.75 \Rightarrow u_1 = 0 \Rightarrow v_1 = 0.75$$
$$u_2 + v_1 = 0 \Rightarrow v_1 = 0.75 \Rightarrow u_2 = -0.75$$
$$u_2 + v_3 = 1.5 \Rightarrow u_2 = 0.75 \Rightarrow v_3 = 2.25$$
$$u_3 + v_1 = 2.5 \Rightarrow v_1 = 0.75 \Rightarrow u_3 = 1.75$$
$$u_3 + v_2 = 0.75 \Rightarrow u_3 = 1.75 \Rightarrow v_2 = -1$$

For unoccupied cells

$$d_{ij} = c_{ij} - (u_i + v_j)$$
$$d_{12} = 2.5 > 0$$
$$d_{13} = 3.25 > 0$$
$$d_{22} = 5.25 > 0$$
$$d_{33} = 0.5 > 0$$

Since all $d_{ij} > 0$, the solution under the test is optimal and unique the optimal fuzzy transportation cost is

$$= 6.5 \times 0.75 + 2 \times 0 + 9.5 \times 1.5 + 1 \times 2.5 + 9.5 \times 0.75$$
$$= 28.75 \cong 29$$

VI. CONCLUSION

In this paper, a Yager's ranking function for symmetric triangular fuzzy number is arrived. The transportation cost source and demands are represented by symmetric triangular fuzzy numbers and using the Yager's ranking function and it is converted into crisp values. Here, the Vogel's Approximation Method(**VAM**) and Yager's ranking to transform the fuzzy transportation in to a crisp transportation problem is used and finally the optimal cost is arrived.

REFERENCES

[1] Dubosis.D and Prade.H, Operation on fuzzy numbers, International journal of systems, 9(6) (1978), 613-626.

[2] Dwyer., P.S.Fuzzy sets information and control no.8. (1965),338 – 353.

[3] Jaisankar.C, Arunvasan.S and Mani.R, On Hessenberg of Triangular Fuzzy Number Matrices, International Journal of Scientific Research and Engineering Technology, 5(12),(2016),589-591.

[4] Jaisankar.C and Arunvasan.S ,An Approach to Relate the Triangular and Hessenberg TFNM, Bulletin of Mathematical Statistics and Research,5(1),(2017),64-74.

[5] Jaisankar.C and Arunvasan.S, Rank and Nullity of Triangular Fuzzy Number Matrices,International Journal of Mathematical Archieve (IJMA),Volume 8, Issue:6, Jun-2017,173 -180.

[6] A.Sahaya sudha and K.R.Vijayalakshmi, A modern approach in ranking of symmetric hexagonal fuzzy number, Arhyabatta Journal of Mathematics and Informatics, 8(2),(2016),42-49.

[7] Shyamal.A.K, and Pal.M, Distance Between Fuzzy Matrices and its Application-I, J.Natural and Physical Sciences, 19(1),(2005),39-58.

[8] Shyamal.A.K and Pal.M, Triangular Fuzzy Matrices, Iranian Journal of Fuzzy Systems,4(1),(2007),75-87.

[9] Thomson.M.G, Converges of Power of the Fuzzy Matrix, J.Math Anal.Appl., 57,(1977),476-480.

[10] Uthra.G, Thangavelu.k and B.Amutha, An Approach of Solving Fuzzy Assignment Problem using Symmetric Triangular Fuzzy Number, International Journal of Pure and Applied Mathematics,113(7),(2017),16-24

[11] Zadeh.L.A., Fuzzy sets, Information and Control.,8,(1965),338-353.

[12] Zadeh.L.A., Fuzzy sets as a basis for a theory of Possibility, Fuzzy sets and Systems,1,(1978),3-28