Fuzzy Linear Complementary Pivot Algorithm without Introducing Any Artificial Variables

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Abstract- *This paper represents a novel design and control architecture In this paper, a ranking procedure based on hexagonal fuzzy numbers, is applied to a linear complementarity problem (LCP) with fuzzy coefficients. By this ranking method, any linear complementarity problem can be converted into a crisp value problem to get an optimal solution. We solve the fuzzy linear complementarity problem; a method of carrying out the complementary pivot algorithm without introducing any artificial variables, under certain conditions is illustrated by means of an example.*

Keywords- Fuzzy Set, Fuzzy Number, Hexagonal Fuzzy Numbers, Fuzzy Linear Complementarity Problem, Centroid points

I. INTRODUCTION

A fuzzy set is a class of objects with a continuum of grades of membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.R. E. Bellman and L. A. Zadeh proposed by decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined [1].The fuzzy set theory has been applied in many fields such as operation research, control theory, management sciences, engineering, etc. Fuzzy sets have been introduced by Lotfi. A. Zadeh (1965)[9]. Cottle, R.W., Dantzig. G.B proposed by linear programming, quadratic programming and bimartix (two-person, non-zero sum) games lead to the consideration of the following fundamental problem with some constructive procedures for solving the fundamental problem under various assumptions on the data [4].. Ludo Van der Heyden proposed by algorithm solves a sequence of sub problems of different dimensions, the sequence being possible non monotonic in the dimension of the sub problem solved. Every sub problem is the linear complementarity problem defined by a leading principal minor of the matrix M. Index-theoretic arguments characterizes the points at which non monotonic behaviour occurs [8]. In fuzzy environment ranking fuzzy numbers is a very important in decision making procedure. Ranking fuzzy numbers were first proposed by Jain [5] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [2], and more recently by Chen and Hwang [3].Lee and Li [6] proposed the comparison of fuzzy numbers. Liou and Wang [7] presented ranking fuzzy numbers with interval values.

In section 2, discusses the basic definition. In section 3, concepts of triangular, trapezoidal and hexagonal fuzzy numbers is reviewed. In section 4, ranking function is discussed. In section 5, solving the fuzzy linear complementarity problem and algorithm to solve the problem is discussed. To solve this method, a numerical example is solved in section 6. Conclusion is discussed in section 7.

II. PRELIMINARIES

2.1 DEFINITION:

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \epsilon A, \mu_A(x) \epsilon [0,1]\}$. In this pair $(x, \mu_A(x))$, the first element x belongs to the classical set A and the second element $\mu_A(x)$ belongs to the interval [0,1] called membership function.

2.2 DEFINITION:

A fuzzy set $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}(x)$: R→ [0,1] is called fuzzy number if

i. $\tilde{A}$ must be normal fuzzy set.
ii. $\alpha_{\tilde{A}}$ must be closed interval for every $\alpha \in [0,1]$.
iii. The support of $\tilde{A}$ must be bounded.

2.3 DEFINITON:

A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 *and* a_3 are real numbers and its membership function is given below

$$\mu_A(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \frac{(a_3-x)}{(a_3-a_1)} & for\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

2.4 DEFINITION:

A fuzzy set $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be trapezoidal fuzzy number where $a_1 \le a_2 \le a_3 \le a_4$ if its membership function is given by

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

III. CONCEPT OF HEXAGONAL FUZZY NUMBERS

3.1 DEFINITION:

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by

$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$

where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)} & for\ a_2 \le x \le a_3 \\ 1 & for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\frac{(x-a_4)}{(a_5-a_4)} & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\frac{(a_6-x)}{(a_6-a_5)} & for\ a_5 \le x \le a_6 \\ 0 & for\ x < a_1 \end{cases}$$

REMARK:

When $w = 1$, the hexagonal fuzzy number is a normal hexagonal fuzzy number.

IV. RANKING FUNCTION

The centroid of a hexagonal fuzzy number is considered to be the balancing point of the hexagon (figure.1)

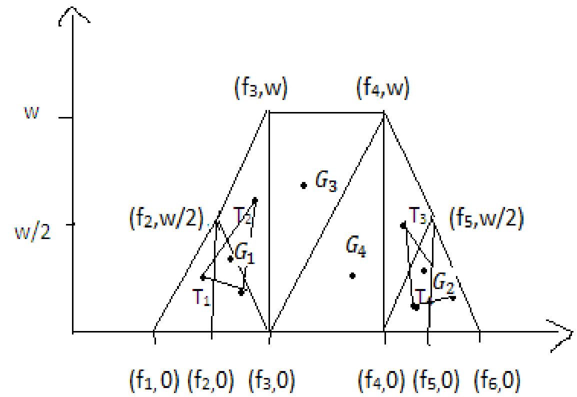

figure.1

Divide the hexagonal into eight triangles. By using the centroid formula of triangle, calculate the ranking as centroid of triangle T_1, T_2 and T_3 are as follows

$$T_1 = \left(\frac{f_1+2f_2}{3}, \frac{w}{6}\right);\ T_2 = \left(\frac{2f_2+f_3}{3}, \frac{w}{6}\right);\ T_1 = \left(\frac{2f_3+f_2}{3}, \frac{w}{2}\right)$$

Centroid of the centroid of T_1, T_2 and T_3 are $G_1 = \left(\frac{f_1+5f_2+3f_3}{9}, \frac{5w}{18}\right)$

Centroid of triangle T_4, T_5 and T_6 are as follows

$$T_1 = \left(\frac{2f_4+f_5}{3}, \frac{w}{2}\right);\ T_2 = \left(\frac{f_4+2f_5}{3}, \frac{w}{6}\right);\ T_1 = \left(\frac{2f_5+f_6}{3}, \frac{w}{6}\right)$$

Centroid of the centroid of T_4, T_5 and T_6 are $G_2 = \left(\frac{3f_4+5f_5+f_6}{9}, \frac{5w}{18}\right)$

Centroid of triangle G_3 and G_4 is

$G_3 = \left(\frac{2f_3+f_4}{3}, \frac{2w}{3}\right)$ and $G_4 = \left(\frac{f_3+2f_4}{3}, \frac{w}{3}\right)$

The centroid of centroid of the triangles G_1, G_2, G_3 and G_4 are

$$R(\tilde{A}_H) = \left(\frac{f_1+5f_2+12f_3+12f_4+5f_5+f_6}{36}, \frac{14w}{36}\right)$$

The ranking function of the generalized hexagonal fuzzy number

$\tilde{A}_H = (f_1, f_2, f_3, f_4, f_5, f_6; w)$ Which maps the set of all fuzzy numbers to a set of real numbers is defined by

$$R(\tilde{A}_H) = (\bar{x}_0, \bar{y}_0) = \left(\frac{f_1 + 5f_2 + 12f_3 + 12f_4 + 5f_5 + f_6}{36}\right)\left(\frac{14w}{36}\right)$$

V. LINEAR COMPLEMENTARITY PROBLEM (LCP)

Given a real $n \times n$ square matrix M and a $n \times 1$ real vector q, then the linear complementarity problem denoted by LCP (q, M) is to find real $n \times 1$ vector W, Z such that

$$W - MZ = q \qquad (1)$$

$$W_j \geq 0,\ Z_j \geq 0,\ for\ \ j = 1, 2, \ldots, n \qquad (2)$$

$$W_j Z_j = 0,\ forj = 1, 2, \ldots, n \qquad (3)$$

Here the pair (W_j, Z_j) is said to be a pair of complementarity variables.

5.1 FUZZY LINEAR COMPLEMENTARITY PROBLEM (FLCP):

Assume that all parameters in (1) – (3) are fuzzy and are described by triangular fuzzy numbers. Then the following fuzzy Linear Complementarity Problem can be obtained by replacing crisp parameters with triangular fuzzy numbers.

$$\tilde{w} - \tilde{M}\tilde{z} = \tilde{q} \qquad (4)$$

$$\tilde{w}_i, \tilde{z}_i \geq 0, for\ i = 1,2, \ldots, n \qquad (5)$$

$$\tilde{w}_i \tilde{z}_i = 0, for\ i = 1,2, \ldots, n \qquad (6)$$

The pair $(\tilde{w}_i, \tilde{z}_i)$ is said to be a pair of fuzzy complementarity variables.

5.2 ALGORITHM FOR FUZZY LINEAR COMPLEMENTARITY PROBLEM WITHOUT USING ANY ARTIFICIAL VARIABLE:

Consider the FLCP $(\tilde{q}, \tilde{M})$ of order n, suppose the matrix $\tilde{M}$ satisfies the condition:

There exists a column vector of $\tilde{M}$ in which all the entries are strictly positive.

Then a variant of the complementary pivot algorithm which uses no artificial variable at all, can be applied on the FLCP $(\tilde{q}, \tilde{M})$. We discuss it here. the original tableau for this version of the algorithm is :

w	z	
I	$-M$	q

$w \geq 0, z \geq 0$

Step: 1

First we assume that $q < 0$. Let s be such that $M_s > 0$. So the column vector associated with z_s is strictly negative in the table. Hence the variable z_s can be made to play the same role as that of the artificial variable z_0 in versions of the complementary pivot algorithm and there is no need to introduce the artificial variableand go to step: 2.

Step: 2

Determine t to satisfy

$$\left(\frac{q_t}{m_{ts}}\right) = \min imum\left\{\left(\frac{q_i}{m_{is}}\right) : i = 1ton\right\}$$

ties fro t can be broken arbitrarily. When a pivot step is carried out with the column of z_s as the pivot column and row t as the pivot row, the right hand side constants vector becomes non-negative after this pivot step. Hence, $(w_1, \ldots, w_{t-1}, z_s, w_{t+1}, \ldots, w_n)$ is a feasible basic vector for the table and go to step: 3.

Step: 3

If $s = t$, it is a complementary feasible basic vector and the solution corresponding to it is a solution of the FLCP ($\tilde{q}, \tilde{M}$), terminate.

If $s \neq t$, the feasible basic vector $(w_1, \ldots, w_{t-1}, z_s, w_{t+1}, \ldots, w_n)$ for the table satisfies the following properties:

i. It contains exactly one basic variable from the complementary pair (w_i, z_i) r n-2 values of i (namely $i \neq s, t\ here)$.

ii. It contains both the variables form a fixed complementary pair (namely (w_s, z_s) *here*), as basic variables.

iii. There exists exactly one complementary pair both the variables in which are not contained in this basic vector (namely (w_t, z_t) *here*).

The complementary pair of variables identified by property (iii), both of which are contained in the basic vector, is known as the **left out complementary pair of variables** in the present basic vector.

For carrying out this version of the complementary pivot algorithm, any feasible basic vector satisfying (i), (ii), (iii) is known as an **almost complementary feasible basic vector**.

Step: 4

In all subsequent steps, the entering variable is uniquely determined by the complementary pivot rule, that is, the entering variable in a step is the complement of the dropping variable in the previous step.

Step: 5

Stop with ray termination.

The algorithm can terminate in two possible ways:

1. At some stage on of the variables from the complementary pair (w_s, z_s) (this is the pair specified in property (ii) of the almost complementary feasible basic vectors obtained during the algorithm) drops out of the basic vector, or becomes equal to zero in the BFS of the table. The BFS of the table at that stage is a solution of the FLCP $(\tilde{q}, \tilde{M})$.
2. At some stage of the algorithm both the variables in the complementary pair (w_s, z_s) may be strictly positive in the BFS and the pivot column in that stage may turn out to be non-positive, and in this case the algorithm terminates with another almost complementary ray. This is **ray termination.**

When ray termination occurs, the algorithm has been unable to solve the FLCP $(\tilde{q}, \tilde{M})$.

VI. NUMERICAL EXAMPLE

Consider the following FLCP ($(\tilde{q}, \tilde{M})$where

$$\tilde{M} = \begin{pmatrix} (-8,-1,2,3,4,5;1) & (-4,-3,1,2,3,4;1) & (-4,-3,1,2,3,4;1) \\ (-4,-3,1,2,3,4;1) & (-8,-1,2,3,4,5;1) & (-4,-3,1,2,3,4;1) \\ (-4,-3,1,2,3,4;1) & (-4,-3,1,2,3,4;1) & (-8,-1,2,3,4,5;1) \end{pmatrix},$$

$$\tilde{q} = \begin{pmatrix} (-7,-6,-5,-4,-1,6;1) \\ (-8,-7,-6,-5,-2,5;1) \\ (-4,-3,-2,-1,3,4;1) \end{pmatrix}$$

Solution: The fuzzy linear complementarity problem is converted into hexagonal fuzzy linear complementarity problem as

By using our ranking function, we obtained

$$R(-4,-3,1,2,3,4;1) = 0.389$$
$$R(-8,-1,2,3,4,5;1) = 0.778$$
$$R(-7,-6,-5,-4,-1,6;1) = -1.556$$
$$R(-8,-7,-6,-5,-2,5;1) = -1.945$$
$$R(-4,-3,-2,-1,3,4;1) = -0.389$$

Now, we have

$$\tilde{M} = \begin{pmatrix} 0.778 & 0.389 & 0.389 \\ 0.389 & 0.778 & 0.389 \\ 0.389 & 0.389 & 0.778 \end{pmatrix} \quad \tilde{q} = \begin{pmatrix} -1.556 \\ -1.945 \\ -0.389 \end{pmatrix}$$

All the column vectors of $\tilde{M}$ are strictly positive here. We will illustrate the algorithm on this problem using s=3.The above problem can be written in the simplex table format as below:

Basic variables	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{q}$
$\tilde{w}_1$	1	0	0	-0.778	-0.389	-0.389	-1.556
$\tilde{w}_2$	0	1	0	-0.389	-0.778	**-0.389**	-1.945
$\tilde{w}_3$	0	0	1	-0.389	-0.389	-0.778	-0.389

$\therefore \tilde{w}_2$ leaves from the basic variable.

Here bold letters denotes the pivot element and the corresponding column is the leaving variable.

$-\tilde{M}_{.3} < 0.$ The minimum of $\{-1.556, -1.945, -0.389\} = -1.945,$ and hence t=2 here. So the pivot row is row 2, and the pivot element for the pivot operation to get the initial almost complementary feasible basic vector is here bold letters in the original tableau. Applying the algorithm we get the following canonical tableaus:

Basic variable	$\tilde{\tilde{w}}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{q}$	Ratio
$\tilde{w}_1$	1	-1	0	-0.389	**0.389**	0	0.389	**1**
$\tilde{z}_3$	0	-2.571	0	1	2	1	5	2.5
$\tilde{w}_3$	0	-2	1	0.389	1.167	0	3.501	3

Basic variable	$\tilde{\tilde{w}}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{q}$	Ratio
$\tilde{z}_2$	2.571	-2.571	0	-1	1	0	1	-
$\tilde{z}_3$	-5.142	2.571	0	**3**	0	1	3	**1**
$\tilde{w}_3$	-3	1	1	1.556	0	0	2.334	1.5

Basic variable	$\tilde{\tilde{w}}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{q}$	Ratio
$\tilde{z}_2$	0.857	-1.714	0	0	1	0.333	2	
$\tilde{z}_1$	-1.714	0.857	0	1	0	0.333	1	
$\tilde{w}_3$	-0.333	0.334	1	0	0	-0.519	0.778	

∴ The corresponding solution of the FLCP is

$$\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \tilde{w}_3) = (0,0,0.778);$$

$$\tilde{z} = (\tilde{z}_1, \tilde{z}_2, \tilde{z}_3) = (1,2,0).$$

VII. CONCLUSION

In this paper, a hexagonal fuzzy ranking method is proposed by using centroid of centroid of a triangle. An algorithm is also used to solve the fuzzy linear complementarity problem by using proposed hexagonal ranking method. It has been explained with numerical examples. This ranking procedure can be applied in various decision making problems.

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