3-Modulo Cordial Graphs on Cycle Related Graphs

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Abstract- *Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have 3 –modulo cordial labeling if there is a injective map* $f: V(G) \to \{0,1,2,3,\ldots\ldots\ldots,3n\}$ *such that for every edge* uv *, the induced labeling* f^* *is defined as f*(uv) = 1 if f(u)+f(v)* $\equiv$ *0 (mod 3) and 0 elsewhere with the condition that* $|e_f(0) - e_f(1)| \le 1$, *where* $e_f(0)$ *is the number of edges with label 0 and* $e_f(1)$ *is the number of edges with label 1.If G admits 3-modulo cordial labeling then G is a 3-modulo cordial graph.In this paper,we proved that cycle related graphs* $T_n, C_2(P_n), K_{1,n} \times K_2, fl_n$ *are 3-modulo cordial graphs.*

Keywords- 3-modulo cordial graph, 3-modulo cordial labeling.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. Each pair e={u,v} of vertices in E is called edges or a line of G. In this paper,we proved that cycle related graphs $T_n, C_2(P_n), K_{1,n} \times K_2, fl_n$ are 3-modulo cordial graphs. For graph theoretic terminology we follow [2].

II. PRELIMINARIES

Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have 3–modulo cordial labeling if there is a injective map $f: V(G) \to \{0,1,2,3,\ldots\ldots\ldots,3n\}$ such that for every edge uv , the induced labeling f^* is defined as follows with the condition that $|e_f(0) - e_f(1)| \le 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.If G admits 3-modulo cordial labeling then G is a 3-modulo cordial graph. In this paper, we proved that cycle related graphs $T_n, C_2(P_n), K_{1,n} \times K_2, fl_n$ are 3-modulo cordial graphs.

DEFINITION 2.1

The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

DEFINITION 2.2

A double triangular snake $C_2(P_n)$ consists of two triangular snakes that have a common path.

DEFINITION 2.3

The graph $B_m = K_{1,m} \times K_2$ is called a Book.

DEFINITION 2.4

The flower fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm.

III. MAIN RESULT

THEOREM 3.1:

The triangular snake (T_n) is a 3 – modulo cordial graph

Proof:

Let G be T_n

Let V(G) = $\{u_1, u_2, u_3, \ldots\ldots\ldots u_n, v_1, v_2, v_3, \ldots\ldots\ldots v_{n-1}\}$

E(G) = $\{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n-1\} \cup$

$\{u_{i+1} v_i / 1 \le i \le n-1\}$

Then $|V(G)| = 2n-1$ and $|E(G)| = 3n-3$

Define $f: V(G) \to \{0,1,2,3,\dots\dots\dots,6n-3\}$

Case 1: Suppose n is odd , say n = 2k + 1

The vertex labels are defined as follows,

$$f(u_i) = \begin{cases} 3i-1 & , \quad 1 \le i \le k \\ 3i & , \quad k+1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i)-1 & , \quad 1 \le i \le k \\ 3(k+i)+3 & , \quad k+1 \le i \le n-1 \end{cases}$$

The induced edge labels are,

For

$1 \le i \le k-1, f^*(u_iu_{i+1}) = 6i+1 \equiv 1(mod\,3)$

For $+1 \le i \le n-1$,

$f^*(u_iu_{i+1}) = 6i+3 \equiv 0(mod\,3)$

$f^*(u_ku_{k+1}) = 6k+2 \equiv 2(mod\,3)$

For

$1 \le i \le k, f^*(u_iv_i) = 6i+3k-2 \equiv 1(mod\,3)$

For

$k+1 \le i \le n-1, f^*(u_iv_i) = 3(2i+k+1) \equiv 0(mod\,3)$

For

$1 \le i \le k-1, f^*(u_{i+1}v_i) = 3(k+2i)+1 \equiv 1(mod\,3)$

For

$k+1 \le i \le n-1, f^*(u_{i+1}v_i) = 3(k+2i+2) \equiv 0(mod\,3)$

$f^*(u_{k+1}v_k) = 9k+2 \equiv 2(mod\,3)$

It is observed that $e_f(0) = 3k$ and $e_f(1) = 3k$

Case 2: Suppose n is even, say n = 2k.

The vertex labels are,

$$f(u_i) = \begin{cases} 3i-1 & , \quad 1 \le i \le k \\ 3i & , \quad k+1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i)-1 & , \quad 1 \le i \le k-1 \\ 3(k+i)+3 & , \quad k \le i \le n-1 \end{cases}$$

The induced edge labels are,

For

$1 \le i \le k-1, f^*(u_iu_{i+1}) = 6i+1 \equiv 1(mod\,3)$

For

$k+1 \le i \le n-1, f^*(u_iu_{i+1}) = 6i+3 \equiv 0(mod\,3)$

$f^*(u_ku_{k+1}) = 6k+2 \equiv 2(mod\,3)$

For

$1 \le i \le k-1, f^*(u_iv_i) = 3k+6i-2 \equiv 1(mod\,3)$

For

$k+1 \le i \le n-1, f^*(u_iv_i) = 3k+6i+3 \equiv 0(mod\,3)$

$f^*(u_kv_k) = 9k+2 \equiv 2(mod\,3)$

For

$1 \le i \le k-1, f^*(u_{i+1}v_i) = 3(k+2i)+1 \equiv 1(mod\,3)$

For

$k+1 \le i \le n-1, f^*(u_{i+1}v_i) = 3(k+2i+2) \equiv 0(mod\,3)$

$f^*(u_{k+1}v_k) = 9k+6 \equiv 0(mod\,3)$

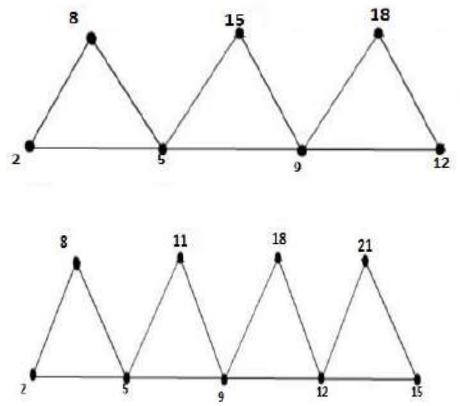
It is observed that $e_f(0) = 3k-1$ and $e_f(1) = 3k-2$

Clearlyinboththecases

Then*f*isa3–modulocordiallabeling.

Henceisa3–modulocordialgraph.

Example 3.1:



$T_4 T_5$

Fig .1

THEOREM 3.2:

The double triangular snake ($C_2(P_n)$) is a 3 – modulo cordial graph.

Proof:

Let G be $C_2(P_n)$

Let V(G) ={ $u_1, u_2, u_3, \dots\dots\dots u_n, v_1, v_2, v_3, \dots\dots\dots v_{n-1}, w_1, w_2, w_3, \dots\dots\dots w_{n-1}$}

E(G) = { $u_i u_{i+1} / 1 \le i \le n-1\} \cup \{ u_i v_i / 1 \le i \le n-1\} \cup$

$\{u_{i+1} v_i / 1 \le i \le n-1\} \cup \{ u_i w_i / 1 \le i \le n-1\} \cup \{u_{i+1} w_i / 1 \le i \le n-1\}$

Then $|V(G)| = 3n-2$ and $|E(G)| = 5n-5$

Define $f: V(G) \to \{0,1,2,3, \dots\dots\dots, 9n-6\}$

Case 1: Suppose n is odd , say n = 2k + 1

The vertex labels are defined as follows,

$$f(u_i) = \begin{cases} 3i-1 & , \quad 1 \le i \le k \\ 3i & , \quad k+1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i)-1 & , \quad 1 \le i \le k \\ 3(k+i)+3 & , \quad k+1 \le i \le n-1 \end{cases}$$

$$f(w_i) = \begin{cases} 3(2k+i)-1 & , \quad 1 \le i \le k \\ 3(2k+i)+3 & , \quad k+1 \le i \le n-1 \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1, f^*(u_i u_{i+1}) = 6i+1 \equiv 1 (mod\, 3)$

For $k+1 \le i \le n-1, f^*(u_i u_{i+1}) = 6i+3 \equiv 0 (mod\, 3)$

$f^*(u_k u_{k+1}) = 6k+2 \equiv 2 (mod\, 3)$

For $1 \le i \le k, f^*(u_i v_i) = 6i+3k-2 \equiv 1 (mod\, 3)$

For $k+1 \le i \le n-1, f^*(u_i v_i) = 3(2i+k+1) \equiv 0 (mod\, 3)$

For $1 \le i \le k-1, f^*(u_{i+1} v_i) = 3(k+2i)+1 \equiv 1 (mod\, 3)$

For $k+1 \le i \le n-1, f^*(u_{i+1} v_i) = 3(k+2i+2) \equiv 0 (mod\, 3)$

$f^*(u_{k+1} v_k) = 9k+2 \equiv 2 (mod\, 3)$

For $1 \le i \le k, f^*(u_i w_i) = 6i+6k-2 \equiv 1 (mod\, 3)$

For $k+1 \le i \le n-1, f^*(u_i w_i) = 3(2i+2k+1) \equiv 0 (mod\, 3)$

For 1

$\leq i \leq k-1, f^*(u_{i+1}w_i) = 6(k+i)+1 \equiv 1 (mod\ 3)$

For
$k+1 \leq i \leq n-1, f^*(u_{i+1}w_i) = 6(k+i+1) \equiv 0 (mod\ 3)$

$f^*(u_{k+1}w_k) = 12k+2 \equiv 2 (mod\ 3)$

It is observed that $e_f(0) = 5k$ and $e_f(1) = 5k$

Case 2: Suppose n is even, say n = 2k.
The vertex labels are defined as follows,

$$f(u_i) = \begin{cases} 3i-1 & , 1 \leq i \leq k \\ 3i & , k+1 \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i)-1 & , 1 \leq i \leq k-1 \\ 3(k+i)+3 & , k \leq i \leq n-1 \end{cases}$$

$$f(w_i) = \begin{cases} 3(2k+i)-4 & , 1 \leq i \leq k-1 \\ 3(2k+i)+3 & , k \leq i \leq n-1 \end{cases}$$

The induced edge labels are,
For
1
$\leq i \leq k-1, f^*(u_iu_{i+1}) = 6i+1 \equiv 1 (mod\ 3)$

For
$k+1 \leq i \leq n-1, f^*(u_iu_{i+1}) = 6i+3 \equiv 0 (mod\ 3)$

$f^*(u_ku_{k+1}) = 6k+2 \equiv 2 (mod\ 3)$
For
1
$\leq i \leq k-1, f^*(u_iv_i) = 3k+6i-2 \equiv 1 (mod\ 3)$

For
$k+1 \leq i \leq n-1, f^*(u_iv_i) = 3k+6i+3 \equiv 0 (mod\ 3)$

$f^*(u_kv_k) = 9k+2 \equiv 2 (mod\ 3)$
For
1
$\leq i \leq k-1, f^*(u_{i+1}v_i) = 3(k+2i)+1 \equiv 1 (mod\ 3)$

For
$k \leq i \leq n-1, f^*(u_{i+1}v_i) = 3(k+2i+2) \equiv 0 (mod\ 3)$

For
1
$\leq i \leq k-1, f^*(u_iw_i) = 6(k+i)-5 \equiv 1 (mod\ 3)$

For
$k+1 \leq i \leq n-1, f^*(u_iw_i) = 3(2k+2i+1) \equiv 0 (mod\ 3)$

$f^*(u_kw_k) = 12k+2 \equiv 2 (mod\ 3)$
For
1
$\leq i \leq k-1, f^*(u_{i+1}w_i) = 6(k+i)-2 \equiv 1 (mod\ 3)$

For
$k \leq i \leq n-1, f^*(u_{i+1}w_i) = 6(k+i+1) \equiv 0 (mod\ 3)$

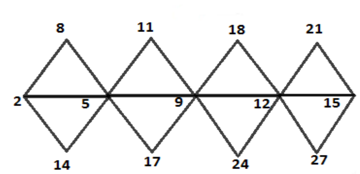
It is observed that $e_f(0) = 5k-2$ and $e_f(1) = 5k-3$

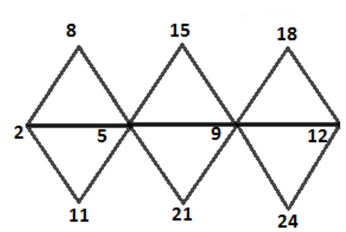
Clearly in both the cases

Then*f*isa3–modulocordiallabeling.

Henceisa3–modulocordialgraph.

Example 3.2:





$C_2(P_5)\ C_2(P_4)$

Fig .2

THEOREM 3.3:

Book-$B_n = K_{1,n} \times K_2$ is a 3 – modulo cordial graph.

Proof:

Let G be $K_{1,n} \times K_2$

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, u_3, \dots \dots \dots u_n, u, v\}$

E(G) = { $uu_i/1 \le i \le n\} \cup \{vu_i/1 \le i \le n\} \cup \{u_iv_i/1 \le i \le n\}$

Then $|V(G)| = n + 2$ and $|E(G)| = 3n$

Define $f: V(G) \to \{0,1,2,3, \dots \dots \dots, 6n+6\}$

The vertex labels are defined as follows,

$$f(u_i) = \begin{cases} 3i - 1 , & 1 \le i \le k \\ 3i , & k+1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i) - 1 , & 1 \le i \le k \\ 3(k+i) + 3 , & k+1 \le i \le n \end{cases}$$

$$f(u) = 0$$

$$f(v) = 1$$

The induced edge labels are,

For

$1 \le i \le k, f^*(uu_i) = 3i - 1 \equiv 2(mod\ 3)$

For

$k+1 \le i \le n, f^*(uu_i) = 3i \equiv 0(mod\ 3)$

For

1

$\le i \le k, f^*(vu_i) = 3(k+i) \equiv 0(mod\ 3)$

For

$k+1 \le i \le n, f^*(vu_i) = 3(k+i) + 4 \equiv 1(mod\ 3)$

For

1

$\le i \le k, f^*(u_iv_i) = 3(k+2i) - 2 \equiv 1(mod\ 3)$

For

$k+1 \le i \le n, f^*(u_iv_i) = 3(k+2i+1) \equiv 0(mod\ 3)$

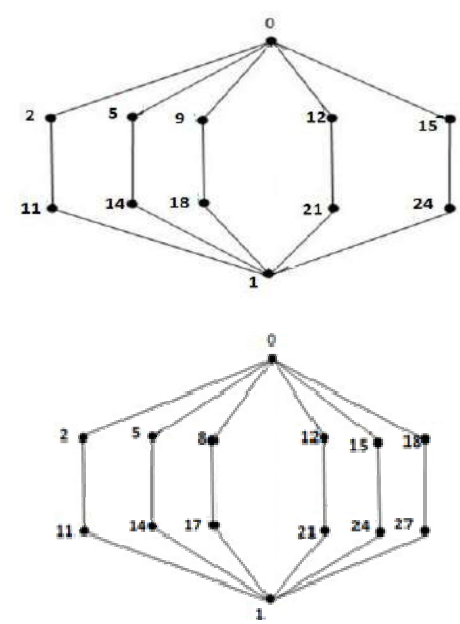
It is observed that $e_f(0) = n + k$ and $e_f(1) = 2n - k$

Clearly in both the cases

Then *f* is a3– modulo cordial labeling.

Hence is a3–modulo cordial graph.

Example 3.3 :



,

$K_{1,5} \times K_2\ K_{1,6} \times K_2$

Fig 3

THEOREM 3.4:

The flower (fl_n) is a 3 – modulo cordial graph.

Proof:

Let G be fl_n

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = $\{u_1, u_2, u_3, \dots\dots\dots u_n, v_1, v_2, v_3, \dots\dots\dots v_n, u\}$

E(G) = $\{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\} \cup \{u_n u_1\} \cup \{u u_i / 1 \le i \le n\} \cup \{u v_i / 1 \le i \le n\}$

Then $|V(G)| = 2n + 1$ and $|E(G)| = 4n$

Define $f: V(G) \to \{0,1,2,3,\dots\dots\dots,6n+3\}$

$f(u_i) = 3i \quad , \quad 1 \le i \le n$

The vertex labels are defined as follows,

$$f(v_i) = \begin{cases} 3i - 1 & , \quad 1 \le i \le k+1 \\ 3(k+i) & , \quad k+2 \le i \le n \end{cases}$$

$f(u) = 1$

The induced edge labels are,

For $1 \le i \le n-1, f^*(u_i u_{i+1}) = 6i + 3 \equiv 0 (mod\, 3)$

$f^*(u_n u_1) = 3n + 3 \equiv 0 (mod\, 3)$

For $1 \le i \le n, f^*(u u_i) = 3i + 1 \equiv 1 (mod\, 3)$

For $1 \le i \le k+1, f^*(u_i v_i) = 6i - 1 \equiv 2 (mod\, 3)$

For $k+2 \le i \le n, f^*(u_i v_i) = 3(k + 2i) \equiv 0 (mod\, 3)$

For $1 \le i \le k+1, f^*(u v_i) = 3i \equiv 0 (mod\, 3)$

For $k+2 \le i \le n, f^*(u v_i) = 3(k+i) + 1 \equiv 1 (mod\, 3)$

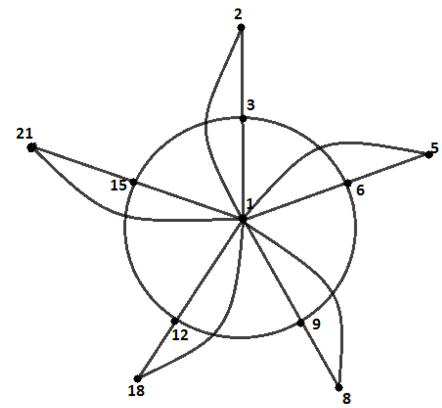
It is observed that $e_f(0) = 2n$ and $e_f(1) = 2n$

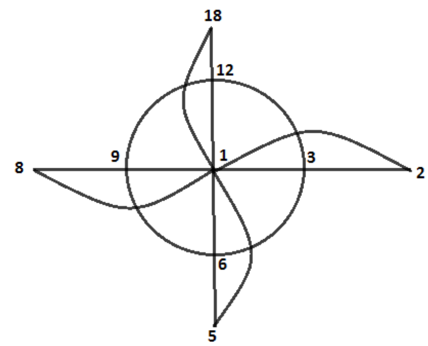
Clearly in both the cases

Then*f*isa3–modulo cordial labeling.

Henceisa3–modulo cordial graph.

Example 3.4





$fl_5 fl_4$

Fig 4

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