Comparative Study For Cost And Work Minimization In Assignment Problem

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Abstract- *The main effort of this paper to produce optimal efficiently and effectively estimation model by means of mathematical model to estimate assignment problem with less effort, or scheduled from various objects such as the large number of date and reports which estimating the effort required to develop for vital importance .This estimation is the process of predicting the effort and cost required to develop effective mathematical model.*

The problem instance has a number of proxies and a number of tasks the agent can be assigned to perform any task, incurring some cost that may vary depends on task assignment is required to perform all tasks. Throughout this paper this process provides accuracy in less time complexity in this study balanced assignment problem whose (number of rows =column) as an example cost metric of order 4 is considered

Keywords- Assignment Problem, Hungarian Method, row min, column min, allocation, optimum cost etc.

I. EXISTING METHOD

Theorem: If a number is added to or subtracted from all of the entries of any one row or column of a cost matrix, then on optimal assignment for the resulting cost matrix are also an optimal assignment for the original cost matrix.

Suppose there are **n** facilitates and **n** jobs it is clear that in this case, there will be **n** assignments. Each facility or say worker can perform each job, one at a time. But there should be certain procedure by which assignment should be made so that the profit is maximized and cost or time is minimized.

Projects	Project Data Set					
	1	2	3	4	j^{th}	n
P_1	Pd_{11}	Pd_{12}	Pd_{13}	Pd_{14}	------	Pd_m
P_2	Pd_{21}	Pd_{22}	Pd_{23}	Pd_{24}	------	Pd_{2n}
P_3	Pd_{31}	Pd_{32}	Pd_{33}	Pd_{34}	------	Pd_{3n}
i^{th}				Pd_{ij}	------	
n^{th}	Pd_{n1}	Pd_{n2}	Pd_{n3}	Pd_{n4}	------	Pd_{nm}

In the table, P_1, P_2, P_3 are the projects and Pd_{ij} is defined project variables of (n x n) matrix. It is a special case of balanced transportation problem when the number of rows is equal to number of columns (R=C).

In the table, Pd_{ij} is defined as the cost when j^{th} job is assigned to i^{th} worker. It maybe noted here that this is a special case of transportation problem when the number of rows is equal to number of columns.

II. MATHEMATICAL FORMULATION:

The basic feasible solution of an Assignment problem may consists (2n – 1) variables out of its (n – 1) variables are zero; n is number of jobs or number of facilities. Due to this high degeneracy, the solution of this problem is complex and time consuming transportation method. Thus a separate technique is derived for it. Before going to the absolute method it is very important to formulate the problem. Suppose x_{ij} is a variable which is defined as1 if the i^{th} job is assigned to j^{th} machine or facility 0 if the i^{th} job is not assigned to j^{th} machine or facility.Now as the problem forms one to one basis or one job is to be assigned to one facility or machine. Therefore $\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1$ The total assignment cost will be given by

$$U = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} c_{ij}$$

The above definition can be developed into mathematical model as follows:

Determine $x_{ij} > 0$ (i, j = 1,2, 3…n) in order to Minimize

$$U = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} c_{ij}$$

Subjected to constraints

$$\sum_{i=1}^{n} x_{ij} = 1$$

When j = 1,2,3 . . . . n.,

$$\sum_{j=1}^{n} x_{ij} = 1$$

When i = 1,2,3 . . . . n., And x_{ij} is either zero or one.

The optimal assignment can be found using the Hungarian algorithm. The Hungarian algorithm has worst case run-time complexity of O(n^3).

Consider cost matrix of order 4:

8	10	17	9
3	8	5	6
10	12	11	9
6	13	9	7

Take min in all rows and Subtract it by each elements (row minima)

0	2	9	1	(-8)
0	5	2	3	(-3)
1	3	2	0	(-9)
0	7	3	1	(-6)

Take min in all column and Subtract it by each elements (column minima)

0	0	7	1
0	3	0	3
1	1	0	0
0	5	1	1
	(-2)	(-2)	

Then Cover all zeros with a minimum number of lines

0	0	7	1	**x**
0	3	0	3	**x**
1	1	0	0	**x**
0	5	1	1	**x**

The optimal assignment

Because there are 4 lines required, allocation i.e . zeros covers an optimal assignment:

0	0	7	1
0	3	0	3
1	1	0	0
0	5	1	1

This corresponds to the following optimal assignment in the original cost matrix:

8	10	17	9
3	8	5	6
10	12	11	9
6	13	9	7

The optimal value equals 30.

Project	Selected features
P_1	10
P_2	5
P_3	9
P_4	6

Solution by Modified Algorithm:

Consider the objective function for minimization type. The algorithm flow for solution of Assignment problem,

1. Check smallest and next smallest element in each row and column then subtracted from each element of that row, column have least one zero will assign in each row, column of new table so least one zero will assign in each row, column.
2. Check rows, column with exactly single (one) zero this is allocation by square □ around it and cross the corresponding column and row and allocate if all allocation is assigned then stop the process else repeat step I crossed rows and column be neutral no operation is required.
3. Draw the minimum number of lines (horizontal and vertical) necessary to cover all zeros in the matrix obtained in step 2,

4. In step 4, if the number of lines drawn are equal to the number of rows, then it is the optimum solution if not, then repeat step-I.
5. Repeat the procedure from step (3) until the number of assignments becomes equal to the number of rows or number of columns.

The optimal assignment can be found has worst case run-time complexity is less than of Hungarian Algorithm run time complexity O(n^3).

This section offers a small example to find out minimal cost of each project by choosing one feature from one row (Project)

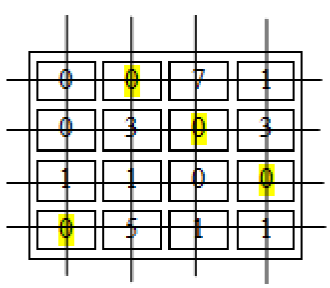
This is the original cost matrix:

8	10	17	9
3	8	5	6
10	12	11	9
6	13	9	7

Step –I

0	0	7	1
0	3	0	3
1	1	0	0
0	5	1	1

Step-II
Cover all zeros with a minimum number of lines
The optimal assignment



Optimal Result: Value is 30

Project	Selected features
P_1	10
P_2	5
P_3	9
P_4	6

III. CONCLUSION

As compared with the existing system time complexity is O(n^3) and proposed method the time complexity is less than time complexity is O(n^3)that is in existing time .This is tested for small entity same concept has to check for unbalanced (row is not equals to column) assignment cost metric.

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