A New Approach For Solving Triangular Intuitionistic Fuzzy Transportation Problem

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Abstract- *In this paper a new method is proposed for finding an optimal solution for Intuitionistic fuzzy transportation problem,directly. Here the supply and demand are triangular intuitionistic fuzzy numbers.The procedure is illustrated by numerical example.*

Keywords- Intuitionistic fuzzy number,Triangular Intuitionistic Fuzzy Number (TrIFN) Intuitionistic Fuzzy Transportation Problem ,Transportation Problem (TP)

I. INTRODUCTION

The concept of intuitionistic fuzzy set was presented by Atanassov[2] which was established to be extremely useful to deal with vagueness .The intuitionistic fuzzy set separates the degree of non membership of an element in the set.

Many authors have used fuzzy and intuitionistic fuzzy set theory for solving real life optimization problems such as planning,scheduling,transportation, manufacturing etc. (Xu [12] ; Cascetta et al.[4];Ganesan and Veeramani [8]; Asuncion et al.[1]; Kaur and Kumar[9];De and sana [5]).In real world transportation problems many situations occur where transportation cost can be uncertain due to several factors (Dempe and Starostina [6]).transportation problem.Nagoorani and Razak [10] presented a two stage cost minimizing fuzzy transportation problem in which supplies and demand are triangular problem (FTP) by taking triangular fuzzy numbers.Pandian and Natarajan [11] did a comparative study on transportation problem in fuzzy environment. Basirzadeh [3] developed an approach for solving fuzzy transportation problem. Kaur and Kumar [9] solved fuzzy transportation problem taking generalized triangular fuzzy numbers. Transportation model deals with transportation of a product available at several origin(sources) to a number of different destinations (jobs).

This model can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling and many others. The total movement from each origin and the total to each destination are given and it is desired to find how the associations can be made subject to the limitation on total In such problems,sources can be divided among the jobs may be done with a combination of sources. The objective is to minimize the cost of transportation while meeting the requirement at the destination.

This paper is organized as follows, section 2 deals with some basic concepts of Intuitionistic Fuzzy Set theory. Section 3, discusses the algorithm to solve the Triangular Fuzzy of Intuitionistic Fuzzy Transportation Problem. In section 4, a numerical example is given. Finally, we concluded in section 5.

II. PRELIMINARIES

2.1 A fuzzy set: Ã is defined by Ã={(X,(x)):X€A,(x)€[0,1].In the pair(x,(x),the first element x belong to the classical set A,the second element (x), belong to the interval [0,1], called membership function

2.2 Fuzzy Number

The notion of fuzzy numbers was introduced by Dubois.D and Prade.H (1980). A fuzzy subset A of the real line R with membership function $\mu_A : R \to [0,1]$ is called a fuzzy number if

i. A is normal, i.e., there exists an element $x_0 \in A$ such that $\mu_A(x_0) = 1$

ii. A is fuzzy convex,

$$i.e., \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2) \forall (x_1, x_2) \in R \& \forall \lambda \in [0,1]$$

iii. μ_A is upper semi continuous

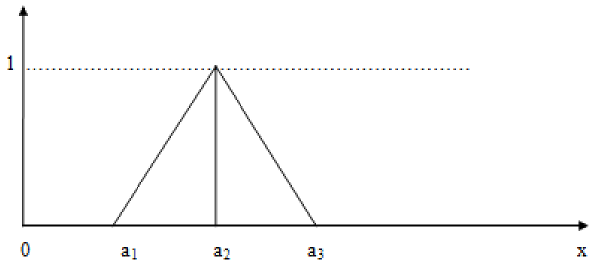
iv. Supp A is bounded where Supp $A = \{x \in R : \mu_A(x) > 0\}$

2.3 Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}$ is a fuzzy number fully specified by 3-tuples (a_1, a_2, a_3) such that $a_1 \leq a_2 \leq a_3$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, if a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, if a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

This is represented diagrammatically as



Triangular fuzzy number

2.4 Fuzzy Arithmetic Operations of Triangular Fuzzy Numbers

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then the fuzzy arithmetic operations under function principle introduced by Chen, S.H (1985) are furnished below.

Addition

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,

Where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

Subtraction

$\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$,

Where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

2.5 Intuitionistic Fuzzy Set (IFS)

Let X be the universe of discourse, then an intuitionistic fuzzy set A in X is given by $A = \{x, \mu_A(x), \gamma_A(x) / x \in X\}$ where $\mu_A(x)$: X $\rightarrow$[0 , 1] and $\gamma_A(x)$: X $\rightarrow$[0 , 1] determine the degree of membership and non membership of the element x $\in$X, respectively and for every $x \in X, 0 \le \mu_A(x) + \gamma_A(x) \le 1.$

2.6 Intuitionistic Fuzzy Graph

Let X be the universe, containing fixed graph vertices and let V $\subset$X be a fixed set.

Construct the IFS $V = \{x, \mu_v(x), \gamma_v(x) / x \in X\}$ where the functions $\mu_v(x)$: X $\rightarrow$[0 , 1] and $\gamma_v(x)$: X $\rightarrow$[0 , 1] determine the degree of membership and non membership to set V of the element(vertex) x $\in$X, respectively and for every $x \in X, 0 \le \mu_v(x) + \gamma_v(x) \le 1.$

2.7 Intuitionistic Fuzzy Number (IFN)

Let $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$ be an IFS, then we call the pair $(\mu_A(x), \gamma_A(x))$ an intuitionistic fuzzy number. We denote an intuitionistic fuzzy number by $(\langle a, b, c\rangle, \langle l, m, n\rangle)$, where $\langle a, b, c\rangle \in F(I), \langle l, m, n\rangle \in F(I), I = [0,1], 0 \le c + n \le 1.$

2.8 Triangular Intuitionistic Fuzzy Number (TIFN) and its arithmetic

A TIFN 'A' is given by $A = \{(\mu_A, \gamma_A) / x \in R\}$, where μ_A and γ_A are triangular fuzzy numbers with $\gamma_A \le \mu_A{}^C$. So a triangular intuitionistic fuzzy number A is given by A = $(\langle a, b, c\rangle, \langle e, f, g\rangle)$ with $\langle e, f, g\rangle \le \langle a, b, c\rangle^c$ i.e., either e $\ge$ band f$\ge$ corf$\le$ aand g $\le$ b are membership and non-membership fuzzy numbers of A.

An intuitionistic fuzzy number $(\langle a, b, c\rangle, \langle e, f, g\rangle)$ with e $\ge$ band f$\ge$ c is shown in the following figure:

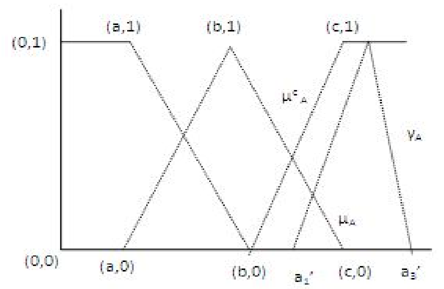


Figure 1 Triangular Intuitionstic fuzzy number

The additions of two triangular intuitionistic fuzzy numbers are as follows:

Addition

For two triangular Intuitionistic Fuzzy Numbers A = $\left(\langle a_1, b_1, c_1\rangle : \mu_A, \langle e_1, f_1, g_1\rangle : \gamma_A\right)$ and B = $\left(\langle a_2, b_2, c_2\rangle : \mu_B, \langle e_2, f_2, g_2\rangle : \gamma_B\right)$ with $\mu_A \neq \mu_B$ and $\gamma_A \neq \gamma_B$, define

A+B = $\left(\langle a_1+a_2, b_1+b_2, c_1+c_2\rangle : Min(\mu_A, \mu_B), \langle e_1+e_2, f_1+f_2, g_1+g_2\rangle : Max(\gamma_A, \gamma_B)\right)$

Subtraction

A-B = $\left(\langle a_1-a_2, b_1-b_2, c_1-c_2\rangle : Min(\mu_A, \mu_B), \langle e_1-e_2, f_1-f_2, g_1-g_2\rangle : Max(\gamma_A, \gamma_B)\right)$

Where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

2.9 Ranking of Triangular Intuitionistic Fuzzy Number

Several approach for the ranking of triangular intuitionistic fuzzy number have been proposed in the literature . An efficient approach for computing the fuzzy numbers is by the use of a ranking function based on their graded means [6]. That is , for every

The ranking function R: $F^{\Re} \rightarrow \Re$ by graded mean is defined as

$$R(\tilde{A}) = \frac{(x+l)+(y+m)+(z+n)}{6}$$

III. ALGORITHM

Step 1

Construct a TrIFT table from the given fuzzy Transportation problem . For unbalanced problem , we do not require to balance the triangular fuzzy transportation problem .

Step 2

Find the smallest cost cell in the triangular intuitionistic transportation table to make the first allocation. Allocate min { } in the cell (i , j). In case of ties, select the cell where maximum allocation can be allocated. Again in case of some cost sales and allocation the cell for which sum of fuzzy demand and supply is maximum in the original triangular intuitionistic fuzzy Transportation table . Finally if all these are same, select the cell randomly.

Step 3

Adjust the supply and demand requirement in the respective row and column .Then following case arise:

Case (i)

If the allocation =, row is to be crossed out and is reduced to (Θ).Now complete the allocation along column by making the allocation/allocations in the smallest cost cell/cells continuously. Consider that , column is exhausted for the allocation in the cell (k , j) . Now, follows the same procedure to complete the allocation along row and continue this process until entire row and column are exhausted.

Case (ii)

If the allocation =, column is to be crossed out and is reduced to(Θ).Now by the following the same procedure explained in case 1,complete the allocation along row and column the process untill entire rows and columns are exhausted.

Case (iii)

If the allocation find the next smallest cost cell, (i,k) form the rest of the cost cells along row and column. Assign a zero in the cell (i,k) and cross out row and column. After that complete the allocation along column following the process described in case 1 to complete the allocations.

Case (iv)

For any allocation, other than first allocation made along the row/column satisfies both the row and column. In such case find the smallest cost cell which is along the column/row and ssign zero in that cell and continue the process described in above cases to complete the allocation along the column/row and also to complete entire allocations.

Step 4

Finally calculate the total intuitionistic fuzzy transportation cost which is the sum of the product of cost and corresponding allocated value..

IV. NUMERICAL EXAMPLE

Sources	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	(<3,4,5> <10,12,14>)	(<10,11,15> <16,18,20>)	(<2,3,4> <8,9,10>)	(<10,11,12> <14,15,16>)	(<6,7,8> <9,10,14>)	(<8,9,10> <12,15,18>)	(<42,50,60> <70,80,90>)
S_2	(<2,3,4> <5,6,10>)	(<1,2,3> <3,4,5>)	(<7,8,9> <10,12,14>)	(<6,7,8> <8,9,10>)	(<10,12,14> <14,16,18>)	(<1,2,3> <4,5,6>)	(<20,30,45><55,70,80>)
S_3	(<3,4,5> <5,6,7>)	(<5,6,7> <7,8,9>)	(<3,4,5> <5,6,7>)	(<3,5,7> <8,10,15>)	(<4,6,9> <13,16,18>)	(<2,4,6> <7,8,9>)	(<40,60,90> <96,110,140>)
S_4	(<5,6,7> <7,8,9>)	(<1,4,5> <6,8,12>)	(<5,7,8> <8,9,10>)	(<2,4,5> <6,8,11>)	(<3,7,8> <9,10,11>)	(<5,6,7> <7,10,13>)	(<40,60,90> <110,140,160>)
S_5	(<3,4,6> <8,9,12>)	(<3,4,5> <6,7,11>)	(<3,4,5> <5,6,7>)	(<2,4,5> <6,9,10>)	(<4,5,6> <6,7,8>)	(<3,5,6> <7,9,12>)	(<40,60,90> <110,140,160>)
S_6	(<4,5,6> <8,12,13>)	(<7,8,9> <9,10,11>)	(<7,8,9> <10,12,14>)	(<3,5,8> <8,9,15>)	(<2,4,6> <6,8,10>)	(<4,5,6> <7,8,12>)	(<10,50,60> <70,80,90>)
Demand	(<30,40,45><55,75,180>)	(<10,20,60> <70,80,90>)	(<120,125,130> <132,135,200>)	(<20,30,45> <55,70,80>)	(<10,60,90> <110,140,160>)	(<30,40,45 <55,70,180>)	–

Convertion of the given triangular intuitionistic fuzzy transportation problem into a crisp value problem

Table 2

Sources	D_1	D_2	D_3	D_4	D_5	D_5	Supply
S_1	8	15	6	13	9	12	120
S_2	5	3	10	8	14	4	65
S_3	5	7	5	8	11	6	50
S_4	7	6	8	6	8	8	90
S_5	7	6	5	6	6	7	100
S_6	8	9	10	8	6	7	60
Demand	75	55	140	50	95	70	_

Initial basic feasible solution

Table 3

Sources	D_1	D_2	D_3	D_4	D_5	D_5	Supply
S_1	8	15	$\boxed{120}$ 6	13	9	12	120
S_2	5	$\boxed{55}$ 3	10	8	14	$\boxed{10}$ 4	65
S_3	$\boxed{50}$ 5	7	5	8	11	6	50
S_4	$\boxed{25}$ 7	6	8	$\boxed{50}$ 6	8	$\boxed{15}$ 8	90
S_5	7	6	$\boxed{20}$ 5	6	$\boxed{35}$ 6	$\boxed{45}$ 7	100
S_6	8	9	10	8	$\boxed{60}$ 6	7	60
Demand	75	55	140	50	95	70	_

Optimality test of the starting basic triangular intuitionistic fuzzy solution

Table 4

Sources	D_1	D_2	D_3	D_4	D_5	D_5	Supply
S_1	8	15	$\boxed{120}$ 6	13	9	12	120
S_2	5	$\boxed{55}$ 3 $-\theta$	10	8	14	$\boxed{10}$ 4 $+\theta$	65
S_3	$\boxed{50}$ 5	7	5	8	11	6	50
S_4	$\boxed{25}$ 7	6 $+\theta$	8	$\boxed{50}$ 6	8	$\boxed{15}$ 8 $-\theta$	90
S_5	7	6	$\boxed{20}$ 5	6	$\boxed{35}$ 6	$\boxed{45}$ 7	100
S_6	8	9	10	8	$\boxed{60}$ 6	7	60
Demand	75	55	140	50	95	70	_

Table 5

Sources	D_1	D_2	D_3	D_4	D_5	D_5	Supply
S_1	8	15	6	13	9	12	120
S_2	5	$\boxed{40}$ 3	10	8	14	$\boxed{25}$ 4	65
S_3	$\boxed{50}$ 5	7	5	8	11	6	50
S_4	$\boxed{25}$ 7	$\boxed{15}$ 6	8	$\boxed{50}$ 6	8	8	90
S_5	7	6	$\boxed{20}$ 5	6	$\boxed{35}$ 6	$\boxed{45}$ 7	100
S_6	8	9	10	8	$\boxed{60}$ 6	7	60
Demand	75	55	140	50	95	70	_

From the above table,the triangular intuitionistic fuzzy transportation cost

=

(120)(6)+(40)(3)+(25)(4)+(50)(5)+(25)(7)+(15)(6)+(50)(6)+(20)(5)+(35)(6)+(45)(7)+(60)(6)

=720+120+100+250+175+90+300+100+210+315+360

= 2740

=RS.2740

V. CONCLUSION

This new method gives an optimal solution directly for any intuitionistic fuzzy transportation problems. This new algorithm takes very less iterations to obtain optimal solution. So, it will be very helpful for decision makers who are dealing with logistic and supply chain problem.

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