Fuzzy Regular generalized Super Open Sets In Fuzzy Topological Space

M.K. Mishra[1], D. Anandhi[2], M.Prabhavathy[3], R.Vishalatchi[4]
Director R&D,Asst. Prof, E.G.S.
[1, 2, 3, 4] Pillay Arts and Science College Nagapattinam

Abstract- *In this paper we introduce and study fuzzy regular generalized super open (fuzzy rg- super open) sets in topological fuzzy topological space and obtain some of their properties. Also, studythe fuzzyrg-neighborhood (fuzzy rg-nbhd)and explorefuzzy rg-super closed sets and introduce fuzzy rg-super closure and discuss some its basic properties in fuzzy topological fuzzy spaces .*

Keywords- Fuzzy super closure, fuzzy super interior, fuzzy super open set, fuzzy super closed set, fuzzy rg- super open sets,, fuzzy rg-nbhd, fuzzy rg-closure.

I. INTRODUCTION

Throughout this paper (X,τ) represents a fuzzy topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a fuzzy topological space X, cl(A) and int(A) denote the super closure of A and the super interior of A respectively. X\A or A^c denotes the complement of A in X. We recall the following definitions and results.

1.1. Definition

A subset A of a fuzzy topological space X is called

1. Fuzzy pre super open set if A ≤ int(cl (A)) and a presuper closed set if cl(int (A))≤ A.
2. Fuzzy semi super open set if A ≤clint (A) and a semisuper closed set if int(cl (A))≤A.
3. Fuzzy regular super open set if A = int(cl (A)) and a regular super closed set if A = cl(int (A)).
4. Fuzzyπ- super open set if A is a finite union of regular super open sets.
5. Fuzzy regular semi super openif there is a regular super open U such thatU ≤A ≤cl(U).

1.2. Definition: A subset A of (X,τ) is called

1. Fuzzy generalized super closed set (fuzzy g-super closed) if cl (A)≤ U whenever A≤U and U is super open in X.
2. Fuzzy regular generalized super closed set (fuzzy rg-super closed)if cl (A)≤U whenever A≤U and U is fuzzy regular super open in X.
3. Fuzzy generalized preregular super closed set (fuzzy gpr-super closed) if pcl (A)≤U whenever A ≤ U and U is fuzzy regular super open in X.
4. Fuzzy weakly generalized super closed set (fuzzy wg-super closed) if clint (A) ≤U whenever A≤ U and U is fuzzy super open in X .
5. Fuzzyπ-generalized super closed set (fuzzy πg-super closed) if cl(A)≤U whenever A≤U and U is fuzzy π-super open in X.
6. Fuzzy weakly super closed set (fuzzy w-super closed)if cl(A) ≤U whenever A≤U and U is fuzzy semi super open in X.
7. Fuzzy regular weakly generalized super closed set (fuzzy rwg-super closed) if clint(A)≤U whenever A ≤U and U is fuzzy regular super open in X.
8. Fuzzyrw-super closed ifcl(A)≤U whenever A≤U and U is fuzzy regular semi super open.
9. Fuzzy*g-super closed ifcl(A)≤U whenever A≤U and U is fuzzy w- super open.
10. Fuzzyrg-super closed ifcl(A)≤U whenever A≤U and U is fuzzyrw- super open.

II. FUZZYREGULAR GENERALIZED NEIGHBOURHOODS

2.1. Definition: A subset A of a fuzzy space X is called fuzzyregular generalized super open (briefly fuzzyrg- super open) set if its complement is fuzzyrg-super closed. The family of all fuzzyrg– super open sets in X is denoted by FRGSO(X).

2.1. Remark:cl(X\A) = X\int(A).

2.1. Theorem:A subset A of X is fuzzyrg- super open if and only if F≤int(A) whenever F is fuzzy rw-super closed and F≤A.

Proof:(*Necessity*):- Let A be fuzzyrg- super open . Let F be fuzzyrw-super closed and F≤A then X\A≤X\F, whenever X\F is fuzzyrw- super open. Since X\A is fuzzyrg-super closed,cl(X\A)≤X\F. By Remark 2.2, X\int(A)≤ X\F. That is F≤int(A).

(***Sufficiency***)**:-** Suppose F is fuzzyrw-super closed and F≤A implies F≤int(A). Let X\A≤U where U is fuzzyrw- super open. Then X\U≤A, where X\U is fuzzyrw-super closed. By hypothesis X\U≤int(A). That is X\int(A)≤U ,so cl(X\A)≤U, implies, X\A is fuzzyrg-super closed and A is fuzzyrg- super open.

2.2. Theorem: If int(A) ≤ B ≤ A and A is fuzzyrg- super open, then B is fuzzyrg- super open.

Proof : Let A be fuzzyrg- super open set and int(A) ≤B≤A. Now int(A)≤B ≤ A implies X\A ≤ X\B ≤ X\int(A). That is X\A ≤ X\B ≤cl(X\A). Since X\A is fuzzyrg-super closed , X\B is fuzzyrg-super closed and B is fuzzyrg- super open.

2.2. Remark: For any A ≤ X, int(cl(A)\A) = ϕ.

2.3. Theorem: If A≤X is fuzzyrg-super closed thencl(A)\A is fuzzyrg- super open.

Proof: Let A be fuzzyrg-super closed. Let F be rw-super closed set such that F≤cl(A)\A. Then , F=ϕ. So, F ≤int(cl(A)\A). This shows cl(A)\A is fuzzyrg- super open.

2.4. Theorem: Every super open set in X is fuzzyrg- super open but not conversely.

Proof: Let A be a fuzzy super open set in a fuzzy topological space X. Then X\A is super closed set. so, X\A is fuzzyrg-super closed. Therefore A is fuzzyrg- super open set in X. The converse of the theorem need not be true, as seen from the following example.

2.1. Corollary:-Every regular super open set is fuzzyrg – super open but not conversely.

Proof: Easy

2.2. Corollary:-Every π- super open set is fuzzyrg- super open but not conversely.

Proof : Easy

2.5.Theorem: Everyfuzzyrg- super open sets in X is rg- super open set in X, but not conversely.

Proof: Let A be fuzzyrg- super open set in fuzzy topological space X. Then X\A is fuzzyrg-super closed set in X. So X\A is fuzzy rg-super closed set in X. Therefore A is fuzzy rg- super open in X. The converse of the above theorem need not be true as seen from the following example.

2.6. Theorem: Every fuzzyrg- super open set in X is fuzzy g* –super open set in X, but not conversely.

Proof : Let A be fuzzyrg- super open set in fuzzy topological space X. Then X\A is fuzzyrg-super closed set in X. So, X\A is fuzzy g*-super closed set in X. Therefore A is g*- super open in X. The converse of the above theorem need not be true as seen from the following example

2.7. Theorem: Everyfuzzyrg- super open set in X is fuzzy g-super open, but not conversely.

Proof.: Let A be fuzzyrg- super open set in X. Then A^c is fuzzyrg-super closed set in X. and if A^c is fuzzy g-super closed set in X. Hence A is fuzzy g- super open in X. The converse of the above theorem need not be true as seen from the following example.

2.8. Theorem: If a subset A of a fuzzy topological space X is fuzzyrg- super open then it is πg- super open set in X.

Proof: Let A be fuzzyrg- super open set in fuzzy topological space X. Then X\A is fuzzyrg-super closed set in X. so , X\A is fuzzy πg-super closed set in X. Therefore A is πg- super open in X.The converse of the above theorem need not be true as seen from the following example.

2.9. Theorem: If A and B are fuzzyrg- super open set in a fuzzy topological space X. Then A∩B is also fuzzyrg- super open set in X.

Proof: If A and B are fuzzyrg- super open sets in a fuzzy topological space X. Then X\A and X\A are fuzzyrg-super closed sets in a fuzzy topological space X. and (X\A)∩ (X\B) is also fuzzyrg-super closed sets in X. Therefore A∩B is fuzzyrg- super open set in X.

2.10.Theorem: If a subset A of a topological fuzzy topological space X is both fuzzy rw-super closed and fuzzyrg- super open then it is fuzzy super open.

Proof. Let A be fuzzyrw-super closed and fuzzyrg- super open set in X. Now A ≤A. but A≤int(A). Hence A is fuzzy super open.

2.11.Theorem : If a set A is fuzzyrg- super open in X, then G=X, whenever G is fuzzy rw- super open and (int(A)≤(X\A))≤G.

Proof: Suppose that A is fuzzyrg- super open in X. Let G is fuzzy rw- super open and (int(A)≤(X\A))≤G. Thus Gc≤$(int(A)\leq A^c)^c$ =$(int(A))^c$≤A. That is G^c≤$(int(A))^c$\A^c. Since $(int(A))^c$ = cl(A^c), G^c≤ cl(Ac)\ A^c. Now, G^c is fuzzyrw-super closed and A^c is fuzzyrg-super closed , then G^c = ϕ. Hence G=X.

2.2. Definition: Let X be a fuzzy topological space and let x ∈X. A subset N of X is said to be a fuzzyrg-nbhd of x iff there exists a fuzzyrg- super open set U such that x ∈ U≤N.

2.3. Definition :A subset N of fuzzy topological space X, is called a fuzzyrg-nbhd of A ≤X iff there exists a fuzzyrg- super open set U such that A≤U≤ N.

2.12. Theorem: Every fuzzynbhd N of x ∈X is a fuzzyrg-nbhd of X , but not conversely.

Proof: Let N be a fuzzynbhd of point x ∈X. Then there exists a fuzzy super open set U such that x ∈ U ≤ N. Since every fuzzy super open set is fuzzyrg- super open set, U is a fuzzyrg-super open set such that x ∈U ≤ N. This implies N is fuzzyrg-nbhd of X .The converse of the above theorem need not be true as seen from the following example.

2.13. Theorem: Everyfuzzyrg- super open set is fuzzyrg-nbhd of each of its points, but not conversely.

Proof: Suppose N is fuzzyrg- super open. Let x ∈ N. For N is a fuzzyrg- super open set such that x ∈ N≤N. Since x is an arbitrary point of N, it follows that N is a fuzzyrg-nbhd of each of its points. The converse of the above theorem is not true in general .

2.3. Remark: Thefuzzyrg-nbhd N of x ∈X need not be a fuzzyrg- super open in X. It is seen from the following example.

III. FUZZYRG-SUPERCLOSURE AND ITS PROPERTIES

3.1. Definition: For a subset A of X, fuzzyrg-scl(A)=∩{F :A≤F , F is fuzzyrgsuper closed in X}.

3.2. Definition: Let (X,τ) be a topological fuzzy topological space and τfuzzyrg = {V≤X: fuzzyrg-scl(X\V)=X\V}.

3.3. Definition: For any A≤X, fuzzyrg-int(A) is defined as the union of all fuzzyrg- super open set contained in A.

3.1. Remark: If A≤X is fuzzyrg-super closed then fuzzyrg-scl(A) = A, but the converse is not true.

3.1. Theorem: Supposeτfuzzyrg is a fuzzy topology. If A is fuzzyrg-super closed in (X,τ), then A is fuzzysuper closed in (X,τ-fuzzyrg).

Proof: Since A is fuzzyrg-super closed in (X,τ), fuzzyrg-scl(A)=A. This implies X\A∈τfuzzyrg. That is X\A is super open in (X,τfuzzyrg). Hence A is super closed in (X, τfuzzyrg).

3.2. Remark :(i) fuzzyrg-scl(ϕ) =ϕ and fuzzyrg-scl(X)=X

(ii) A ≤fuzzyrg-scl(A).

3.2. Theorem : For any x∈X, x∈fuzzyrg-scl(A) if and only if V∩A ≠ϕ for every fuzzyrg- super open set V containing x.

Proof :(Necessity).: Suppose there exists a fuzzyrg- super open set V containing x such that V∩A=ϕ. Since A≤ X\V, fuzzyrg-scl(A) ≤X\V implies x∈fuzzyrg-scl(A) a contradiction.

(Sufficiency):. Suppose x∈fuzzyrg-scl(A), then there exists a fuzzyrg-super closed subset F containing A such that x∈F. Then x∈ X\F and X\F is fuzzyrg- super open. Also (X\F)∩A =ϕ, a contradiction.

3.3. Remark: Let A and B be subsets of X, if A≤B then fuzzyrg-scl(A)≤fuzzyrg-scl(B).

3.3. Theorem: Let A and B be subsets of X, then fuzzyrg-scl(A∩B) ≤fuzzyrg-scl(A) ∩fuzzyrg-scl(B).

Proof : Since A∩B≤A and B, so fuzzyrg-scl(A∩B)≤fuzzyrg-scl(A) and,fuzzyrg-scl(A∩B)≤fuzzyrg-scl(B).Thus, fuzzyrg-scl(A∩B)≤fuzzyrg-scl(A) ≤fuzzyrg-scl(B). In general, fuzzyrg-scl(A)∩fuzzyrg-scl(B) ≤fuzzyrg-scl(A∩B).

3.4. Theorem : If A and B are fuzzyrg-super closed sets then fuzzyrg-cl(A∪B) = fuzzyrg-scl(A)∪fuzzyrg-scl(B).

Proof : Let A and B be fuzzyrg-super closed in X. Then A∪B is also fuzzyrg-super closed . Then fuzzyrg-scl(A∪B) = A∪B = fuzzyrg-scl(A)∪fuzzyrg-scl(B).

3.5. Theorem :(X\fuzzyrg-sint(A)) = fuzzyrg-scl(X\A).

Proof : Let x∈ X\fuzzyrg-int(A), then x ∈fuzzyrg-sint(A). Thus every fuzzyrg- super open set B containing x is such that B⊄A. This implies every fuzzyrg- super open set B containing x intersects X\A. This means x∈fuzzyrg-scl(X\A). Hence (X\fuzzyrg-sint(A))≤fuzzyrg-scl(X\A). Conversely, let x∈fuzzyrg-scl(X\A). Then every fuzzyrg- super open set U containing x intersects X\A. That is every fuzzyrg- super open set U containing x is such that U⊄A, implies x∈fuzzyrg-sint(A). Hence fuzzyrg-scl(X\A)≤(X\fuzzyrg-sint(A)).Thus (X\fuzzyrg-sint(A))=fuzzyrg-scl(X\A).

REFERENCES

[1] B. Ghosh, Semi-continuous and semi-closed mappings and semi-connectedness in fuzzy setting,Fuzzy Sets and Systems 35(3) (1990), 345–355.

[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182–190.

[3] C.W. Baker on Preserving g-super closed sets Kyungpook Math. J. 36(1996), 195-199.

[4] G. Balasubramanian and P. Sundaram, On some generalizations of fuzzy continuous functions,Fuzzy Sets and Systems 86(1) (1997), 93–100.

[5] G. Balasubramanian and V. Chandrasekar, Totally fuzzy semi continuous functions, Bull. CalcuttaMath. Soc. 92(4) (2000), 305–312.

[6] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82(1) (1981), 14–32.

[7] K. M. Abd El-Hakeim, Generalized semi-continuous mappings in fuzzy topological spaces, J. Fuzzy Math. 7(3) (1999), 577–589.

[8] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338–353.

[9] Levine. N., Generalized super closed sets in topology, Rend. Circ. Mat. Palermo 19(1970), 89–96.

[10] Maki. H, Devi. R and Balachandran. K., Associated topologies of generalized -super closed sets and -generalized super closed sets, Mem. Sci. Kochi Univ. Ser. A. Math. 15(1994), 51–63.

[11] Levine. N., Semi- super open sets and semi-continuity in topological fuzzy space s, Amer. Math. Monthly,70(1963), 36–41 .

[12] M.K. Mishra et all on "Fuzzy super continuity" International Review in Fuzzy Mathematics July – December2012.

[13] M.K. Mishra M. Shukla M. Fuzzy Regular Generalized Super Closed Set" International Journal of Scientfic and Research December issue July December 2012.

[14] M.K. Mishra,et all on " Fuzzy super closed set" International Journal International Journal of Mathematics and applied Statistics.

[15] Mashhour. A.S., Abd. El-Monsef. M. E. and El-Deeb S.N., On pre continuous mappings and weak pre-continuous mappings, Proc Math, Phys. Soc. Egypt 53(1982), 47–53.

[16] Nagaveni. N., Studies on Generalizations of Homeomorphisms in Topological Fuzzy space s, Ph.D. Thesis, Bharathiar University, Coimbatore, 1999 .

[17] P. M. Pu and Y. M. Liu Fuzzy topology I Neighborhood structure of a fuzzy point and More-Smith Convergence. J. Math. Anal. Appl. 76(1980) ,571-594.

[18] P. M. Pu and Y. M. Liu Fuzzy topology II Product and quotient spaces J.Math. Anal. Appl. 77(1980) 20-37.

[19] P. M. Pu, and Y. M. Liu, Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76(2) (1980), 571–599.

[20] Palaniappan. N., and Rao. K. C., Regular generalized super closed sets, Kyungpook Math. J. 33(1993), 211–219.

[21] Park. J. K. and Park. J.H., mildly generalized super closed sets, almost normal and mildly normal fuzzy space s, Chaos, Solutions and Fractals 20(2004), 1103–1111.

[22] Pushpalatha. A., Studies on Generalizations of Mappings in Topological Fuzzy space s, Ph.D. Thesis, Bharathiar University, Coimbatore, 2000.

[23] R. K. Saraf and M. Khanna, Ongs-closed sets in fuzzy topology, J. Indian Acad. Math. 25(1),(2003), 133–143.

[24] R. K. Saraf, and M. Khanna, Fuzzy generalized semipreclosed sets, J. Tripura Math. Soc.3(2001) 59–68.

[25] R. K. Saraf, and S. Mishra, Fg_-closed sets, J. Tripura Math. Soc. 2 (2000) 27–32.

[26] R. K. Saraf, M. Caldas and S. Mishra, Results via Fg_-closed sets and Fg-closed sets, Pre print.

[27] Stone. M., Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41(1937), 374–481..

[28] Sundaram.P and Sheik John.M., On w-super closed sets in topology, ActaCienciaIndica 4(2000), 389–392.

[29] Syed Ali Fathima. S and Mariasingam. M, On fuzzyregular generalized super closed sets in topological fuzzy space s, International journal of mathematical archive-2(11), 2011, 2497 – 2502.

[30] T. H. Yalva¸c, Semi-interior and semi closure of a fuzzy set, J. Math. Anal. Appl. 132(2) (1988),356–364.

[31] Veera Kumar M.K.R.S., Between g* super closed sets and g-super closed sets , Mem.Fac.Sci.Kochi Univ. Ser .App .Math .,21 (2000),1-19.