A New Ranking Approach on Solving Fuzzy Assignment Problem Using Symmetric Hexagonal Fuzzy Number

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Abstract- *Assignment problem is a special kind of Linear Programming Problem. In this paper, the cost values of the Fuzzy Assignment Problem are considered as Symmetric Hexagonal Fuzzy Numbers. First, the Symmetric Hexagonal Fuzzy Numbers are converted into crisp values using Hexagonal ranking method. Then the optimum assignment schedule of the Fuzzy Assignment Problem is obtained by usual Hungarian Method. The proposed approach is illustrated by a numerical example.*

Keywords- Fuzzy Number, Fuzzy Set, Hexagonal Fuzzy Number, Symmetric Hexagonal Fuzzy Number, Fuzzy Assignment Problem, Hexagonal Ranking Function.

I. INTRODUCTION

A fuzzy set is a class of objects with a continuum of grades of membership such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one was introduced by Zadeh [11] in 1965.

Jatinder Pal Singh and Neha Thakur[3] discussed a various methods to solve assignment problem in which parameters are represented by triangular or trapezoidal fuzzy numbers. To compare the assignment cost calculated by existing method with the assignment cost which has been found out.

Nagoor Gani and V.N.Mohamed[5] discussed a assignment problem is a well known topic and is used very often is solving problems of engineering and management science. The fuzzy assignment problem has been transformed into a crisp assignment problem in the Linear Programming Problem form.

K.Sangeetha et.al [9] proposed to solve the fuzzy salesman problem using Hungarian method. An unbalanced assignment problem is solved using Ranking of Triangular Fuzzy Number. Bellman and Zadeh[1]proposed the concept of decision making problems involving uncertainty and imprecision.

P.K.De, BhartiYadev[2]discussed a general approach for solving assignment problems involving with fuzzy cost coefficients. Kadhirvel.K and Balamururgan.K [4] discussed method for solving Hungarian assignment problems using triangular and trapezoidal fuzzy number. S. Kumara Ghuru[6] discussed the solution of solving fuzzy transportation problem using symmetric triangular fuzzy number.

G.Nirmala, R.Anju[7] discussed about cost minimization assignment problem using fuzzy quantifier. K.Ruth Isabels, G.Uthra[8] proposed an application of linguistic variables in as- signment problem with fuzzy costs. K.R. Shoba[10] proposed profit maximization of fuzzy assignment problem.

In this paper a Fuzzy Assignment problem is considered. The cost values of the Fuzzy Assignment Problem are taken as Symmetric Hexagonal Fuzzy Numbers. The Symmetric Hexagonal Fuzzy Numbers are converted into crisp values using Hexagonal Ranking Procedure. The problem is then solved by the usual Hungarian method.

The rest of this paper organized as follows. In section 2, some basic definitions and Section 3 Hexagonal Ranking Procedure of Symmetric Hexagonal Fuzzy Number are given. Section 4, presents introduction of Fuzzy Assignment Problem. In section 5, procedure and numerical example for the proposed method are given followed by result. In section 6, conclusion is discussed.

II. DEFINITIONS

2.1 Fuzzy Set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs $A = \{(x, \mu_A(x)) ;\ x \in X\}$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A (x) is called the membership value of $x \in X$ on the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Fuzzy Number

A fuzzy set $\hat{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\hat{A}$ is normal

ii. $\hat{A}$ is convex

iii. The support of $\hat{A}$ is closed and bounded then $\hat{A}$ is called fuzzy number.

2.3 Triangular fuzzy number

A fuzzy number $\hat{A}$ = (a_1, a_2, a_3) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \le a_1 \\ \frac{x-a_1}{a_2-a_1} & ; a_1 \le x \le a_2 \\ 1 & ; x = a_2 \\ \frac{a_3-x}{a_3-a_2} & ; a_2 \le x \le a_3 \\ 0 & ; x > a_3 \end{cases}$$

2.4 Trapezoidal Fuzzy Number

A fuzzy number $\hat{A}$ = (a_1, a_2, a_3, a_4) is said to be a trapezoidal fuzzy number if its membership function is given by where $a_1 \le a_2 \le a_3 \le a_4$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

III. HEXAGONAL FUZZY NUMBERS

3.1 Hexagonal Fuzzy Number

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H(a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\hat{A}}(x)$ is given below.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)} & for\ a_2 \le x \le a_3 \\ 1 & for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\frac{(x-a_4)}{(a_5-a_4)} & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\frac{(a_6-x)}{(a_6-a_5)} & for\ a_5 \le x \le a_6 \\ 0 & for\ x > a_6 \end{cases}$$

3.2 Symmetric Hexagonal Fuzzy Number

A Symmetric Hexagonal Fuzzy Number $\tilde{A}_H = (a_L - s - t, a_L - s, a_L, a_U, a_U + s + t)$

Where $a_L, a_U, s\ and\ t$ are real numbers and its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-(a_L-s-t)}{t}\right) & for\ a_L - s - t \le x \le a_L - s \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-(a_L-s)}{s}\right) & for\ a_2 - s \le x \le a_L \\ 1 & for\ a_L \le x \le a_U \\ 1 - \frac{1}{2}\left(\frac{x-a_u}{s}\right) & for\ a_U \le x \le a_U + s \\ \frac{1}{2}\left(\frac{(a_u+s+t)-x}{t}\right) & for\ a_U + s \le x \le a_u + s + t \\ 0 & otherwise \end{cases}$$

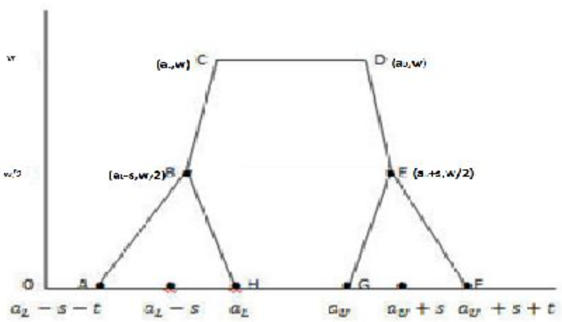

Fig-1 Symmetric Hexagonal Fuzzy Number

3.3 New Ranking of Hexagonal Fuzzy Number

If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is a fuzzy number then the Hexagonal ranking is defined by

$$R(\tilde{A}) = \left(\frac{3}{2}(a_L + a_u), \frac{5W}{6}\right)$$

3.4 Arithmetic Operation on Symmetric Hexagonal Fuzzy Number

Addition:

$$\tilde{A}_H + \tilde{B}_H = [(a_L - b_L) - t, (a_L - b_L) - S, (a_L + b_L), (a_U + b_U), (a_U + b_U + S), (a_U + b_U + t)]$$

Subtraction:

$$\tilde{A}_H - \tilde{B}_H = [(a_L - b_L) - t, (a_L - b_U) - S, (a_L + b_U), (a_U + b_L), (a_U - b_L) + S, (a_U - b_L) + t)]$$

Where $S = (S_1 + S_2), t = (S_1 + S_2 + t_1 + t_2)$

IV. FUZZY ASSIGNMENT PROBLEM

Consider the situation of assigning n machines to n jobs and each machine is capable of doing each job at different costs. Let C^*_{ij} be an fuzzy cost of assigning the i^{th} machine to the j^{th} job.

Let x_{ij} be the decision variable denoting the assignment of the i^{th} machine to the j^{th} job. The objective is to minimize the total cost.

The mathematical model of the Fuzzy Assignment Problem is given by

$$Minimize$$

$$subject\ to\ \ z^* = \sum_{j=1}^{n} C^*_{ij}\, x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad for\ j = 1,2, \ldots\ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad for\ i = 1,2, \ldots\ldots, n$$

$$where\ \ x_{ij} = \begin{cases} 1, if\ the\ i^{th}\ \text{machine is assigned to}\ j^{th}\ \text{job} \\ 0, if\ the\ i^{th}\ \text{machine is not assigned to}\ j^{th}\ \text{job} \end{cases}$$

$$C^*_{ij} = C^1_{ij}, C^2_{ij}, C^3_{ij}$$

V. PROCEDURE

Step 1: First convert the cost values of the fuzzy assignment problem which are all in symmetric hexagonal fuzzy numbers into crisp values by using Hexagonal Ranking.

Step 2: Check the condition that the fuzzy assignment problem is balanced.

(i) If balanced go to step 4. (Number of rows = Number of Columns)

(ii) If not balanced go to step 3. (Number of rows = Number of Columns)

Step 3: If the given Fuzzy Assignment problem is not balanced then add dummy row (or) dummy Column with cost value as zero to make the fuzzy assignment problem balanced.

Step 4: Obtain the optimum assignment schedule by Hungarian method.

Example : A Fuzzy Assignment Problem with rows representing 4 machines M_1, M_2, M_3, M_4 and columns representing the 4 Jobs J_1, J_2, J_3, J_4 is considered. The cost matrix C^*_{ij} whose elements are Symmetric Hexagonal Fuzzy Numbers is given below.

The problem is to find the minimum cost.

$$\begin{pmatrix} (1,2,4,5,7,7) & (15,16,18,20,22,22) & (4,5,7,9,12,12) & (3,4,6,7,9,9) \\ (33,34,36,37,39,39) & (43,46,48,49,50,50) & (52,53,55,57,58,58) & (4,5,7,9,12,12) \\ (3,4,6,7,9,9) & (33,34,36,37,39,39) & (4,6,8,9,10,10) & (11,12,13,15,18,18) \\ (15,16,18,20,22,22) & (1,2,4,5,7,7) & (4,5,7,9,12,12) & (4,5,7,9,12,12) \end{pmatrix}$$

Using Ranking Method,

$$R(1,2,4,7,7) = 11.25$$
$$R(15,16,18,20,22,22) = 47.5$$
$$R(4,5,7,9,12,12) = 20$$
$$R(3,4,6,7,9,9) = 16.25$$
$$R(33,34,36,37,39,39) = 91.25$$
$$R(43,46,48,49,50,50) = 121.25$$
$$R(52,53,55,57,58,58) = 140$$
$$R(4,6,8,9,10,10) = 21.25$$
$$R(11,12,13,15,18,18) = 35$$

After ranking we get,

$$\begin{pmatrix} 11.25 & 47.5 & 20 & 16.25 \\ 91.25 & 121.25 & 140 & 20 \\ 16.25 & 91.25 & 21.25 & 35 \\ 47.5 & 11.25 & 20 & 20 \end{pmatrix}$$

Row wise subtraction,

$$\begin{pmatrix} 0 & 36.25 & 8.75 & 5 \\ 71.25 & 101.25 & 120 & 0 \\ 0 & 75 & 5 & 18.75 \\ 36.25 & 0 & 8.75 & 8.75 \end{pmatrix}$$

Column wise subtraction,

$$\begin{pmatrix} 0 & 36.25 & 3.75 & 5 \\ 71.25 & 101.25 & 115 & 0 \\ 0 & 75 & 0 & 18.75 \\ 36.25 & 0 & 3.75 & 8.75 \end{pmatrix}$$

Assignment schedule,

$$\begin{pmatrix} (0) & 36.25 & 3.75 & 5 \\ 71.25 & 101.25 & 120 & 0 \\ 0 & 75 & (0) & 18.75 \\ 36.25 & (0) & 3.75 & 8.75 \end{pmatrix}$$

The assignment cost $= 11.25 + 20 + 21.25 + 11.25$

$$= \mathbf{63.75}$$

$\therefore \mathit{The\ Asssignment\ cost\ is\ Rs.63.75}$

VI. CONCLUSION

In this paper, Fuzzy Assignment problem with cost values as Symmetric Hexagonal Fuzzy Numbers is considered. The Symmetric Hexagonal Fuzzy Numbers are converted into crisp values using new Hexagonal ranking. The optimum assignment schedule of the Fuzzy Assignment Problem is then obtained by Hungarian Method. We hope that this approach will be effective in assignment problems involving imprecise data.

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