Some Properties of Constant of Trapezoidal Fuzzy Number Matrices

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Abstract- *The fuzzy set theory has been applied in many fields such as management, engineering, theory of matrices and so on. In this paper, some elementary operations on proposed trapezoidal fuzzy numbers (TrFNs) are defined. We also have been defined some operations on trapezoidal fuzzy matrices(TrFMs). The notion of Constant of trapezoidal fuzzy matrices (constant TrFMs) is proposed and some properties of Constant trapezoidal fuzzy matrix (constant TrFM) are verified. Some of their relevant examples are also included to justify the proposed notions.*

Keywords- Fuzzy Arithmetic, Fuzzy number, Trapezoidal fuzzy number (TrFN), Trapezoidal fuzzy matrix(TrFM), Constant of Trapezoidal fuzzy matrix(Constant TrFM).

I. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[13] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [2] in 1951, by means of Zadeh's extension principle [12,14], the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubosis and Prade [1] has defined any of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers.

Trapezoidal fuzzy number's (TrFNs) are frequently used in application. It is well known that the matrix formulation of a mathematical formula gives extra facility to study the problem. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology. A presented new ranking function and arithmetic operations on type-2 generalized type-2 trapezoidal fuzzy numbers by Stephen Dinagar and Anbalagan [9].

We introduce trapezoidal fuzzy matrices (TrFMs). To the best of our knowledge, no work is available on TrFMs, through a lot of work on fuzzy matrices is available in literature. A brief review on fuzzy matrices is given below. Fuzzy matrices were introduced for the first time by Thomason [11] who discussed the convergence of power of fuzzy matrix. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in [6,7,8].

Ragab et.al [5] presented some properties of the min-max composition of fuzzy matrices. Kim [4] presented some important results on determinant of square fuzzy matrices. Stephen Dinagar et.al [10] presented some important properties on constant type-2 Triangular fuzzy matrices. Jaisankar and Mani [3] proposed the Hessenberg of Trapezoidal fuzzy number matrices.

The paper organized as follows, Firstly in section 2, we recall the definition of Trapezoidal fuzzy number and some operations on trapezoidal fuzzy numbers (TrFNs). In section 3, we have reviewed the definition of trapezoidal fuzzy matrix (TrFM) and some operations on Trapezoidal fuzzy matrices (TrFMs). In section 4, we defined the notion of Constant of trapezoidal fuzzy matrix (Constant TrFM). In section 5, we have presented some properties of Constant of trapezoidal fuzzy matrix (Constant TrFM). Finally in section 6, conclusion is included.

II. PRELIMINARIES

In this section, we recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1: Fuzzy set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A=\{(x,\mu_A(x))\ ;\ x\in X\}$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x\in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2: Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(x) = 1$.

Definition 2.3: Convex fuzzy set

A fuzzy set $A = \{(x, \mu_A(x))\} \subseteq X$ is called Convex fuzzy set if all A_α are Convex set (i.e.,) for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0,1]$, $\lambda x_1 + (1-\lambda) x_2 \in A_\alpha$ for all $\lambda \in [0,1]$ otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.4: Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\tilde{A}$ is normal
ii. $\tilde{A}$ is convex
iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 2.5: Trapezoidal fuzzy number

A fuzzy number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}^{TzL}}(x) = \begin{cases} 0 & ; \quad x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & ; \quad a_1 \le x \le a_2 \\ 1 & ; \quad a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & ; \quad a_3 \le x \le a_4 \\ 0 & ; \quad x > a_4 \end{cases}$$

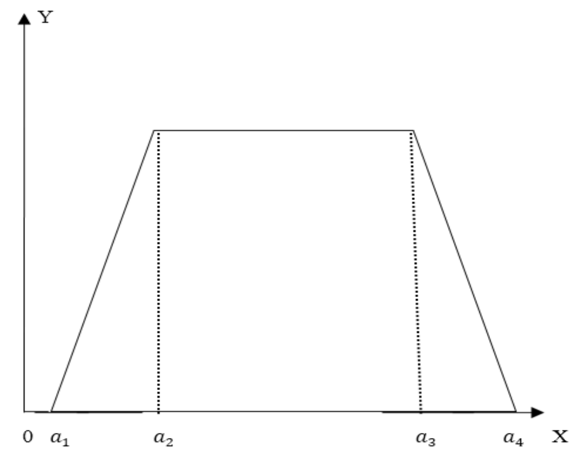

Figure 1. Trapezoidal fuzzy number

Definition 2.6: Ranking function

We defined a ranking function $\Re$: $F(R) \to R$ which maps each fuzzy numbers to real line F(R) represent the set of all trapezoidal fuzzy number. If R be any linear ranking function

$$\Re(\tilde{A}^{TzL}) = \left(\frac{a_1 + a_2 + a_3 + a_4}{4}\right)$$

Also we defined orders on F(R) by

$$\Re(\tilde{A}^{TzL}) \ge \Re(\tilde{B}^{TzL}) \text{ if and only if } \tilde{A}^{TzL} \ge_R \tilde{B}^{TzL}$$
$$\Re(\tilde{A}^{TzL}) \le \Re(\tilde{B}^{TzL}) \text{ if and only if } \tilde{A}^{TzL} \le_R \tilde{B}^{TzL}$$
$$\Re(\tilde{A}^{TzL}) = \Re(\tilde{B}^{TzL}) \text{ if and only if } \tilde{A}^{TzL} =_R \tilde{B}^{TzL}$$

Definition 2.7: Arithmetic operations on trapezoidal fuzzy numbers (TrFNs)

Let $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers (TrFNs) then we defined,

Addition

$$\tilde{A}^{TzL} + \tilde{B}^{TzL} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

Subtraction

$$\tilde{A}^{TzL} - \tilde{B}^{TzL} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

Multiplication

$$\tilde{A}^{TzL} \times \tilde{B}^{TzL} = (a_1 \Re(B), a_2 \Re(B), a_3 \Re(B), a_4 \Re(B))$$

where $\Re(\tilde{B}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$ or

$\Re(\tilde{b}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$

Division

$$\tilde{A}^{TzL} / \tilde{B}^{TzL} = \left(\frac{a_1}{\Re(\tilde{B}^{TzL})}, \frac{a_2}{\Re(\tilde{B}^{TzL})}, \frac{a_3}{\Re(\tilde{B}^{TzL})}, \frac{a_4}{\Re(\tilde{B}^{Tzl})}\right)$$

Where $\Re(\tilde{B}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$ or

$\Re(\tilde{b}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$

Scalar multiplication

$$K\,\tilde{A}^{TzL} = \begin{cases} (ka_1, ka_2, ka_3, ka_4) & if\ K \geq 0 \\ (ka_4, ka_3, ka_2, ka_1) & if k < 0 \end{cases}$$

Definition 2.8: Zero trapezoidal fuzzy number

If $\tilde{A}^{TzL}$ = (0,0,0,0) then $\tilde{A}^{TzL}$ is said to be zero trapezoidal fuzzy number. It is defined by $\tilde{0}^{TzL}$.

Definition 2.9: Zero equivalent trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{A}^{TzL}$ is said to be a zero equivalent trapezoidal fuzzy number if $\Re(\tilde{A}^{TzL})=0$. It is defined by $\tilde{0}^{TzL}$.

Definition 2.10: Unit trapezoidal fuzzy number

If $\tilde{A}^{TzL}$ = (1,1,1,1) then $\tilde{A}^{TzL}$ is said to be a unit trapezoidal fuzzy number. It is denoted by $\tilde{1}^{TzL}$.

Definition 2.11: Unit equivalent trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{A}^{TzL}$ is said to be unit equivalent triangular fuzzy number. If $\Re(\tilde{A}^{TzL}) = 1$. It is denoted by $\tilde{1}^{TzL}$.

Definition 2.12: Inverse of trapezoidal fuzzy number

If $\tilde{a}^{TzL}$ is trapezoidal fuzzy number and $\tilde{a}^{TzL} \neq \tilde{0}^{TzL}$ then we define

$$\tilde{a}^{Tzl^{-1}} = \frac{\tilde{1}^{Tzl}}{\tilde{a}^{TzL}}$$

Definition 2.13: Equal and Equivalent Trapezoidal fuzzy number

Let $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers (TrFNs). Then $\tilde{A}^{TzL}$ and $\tilde{B}^{TzL}$ are said to be equal if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4$. It is denoted by $\tilde{A}^{TzL} = \tilde{B}^{TzL}$.

Suppose if $\Re(\tilde{A}^{TzL}) = \Re(\tilde{B}^{TzL})$ then $\tilde{A}^{TzL}$ and $\tilde{B}^{TzL}$ are said to be equivalent fuzzy numbers. It is denoted by $\tilde{A}^{TzL} \approx \tilde{B}^{TzL}$.

Remark:

Note that all equal trapezoidal fuzzy numbers are also equivalent trapezoidal fuzzy numbers. But the converse need not be true.

III. Trapezoidal fuzzy matrices (TrFMs)

In this section, we introduced the trapezoidal fuzzy matrix and the operations of the matrices some examples provided using the operations.

Definition 3.1: Trapezoidal fuzzy matrix (TrFM)

A trapezoidal fuzzy matrix of order m×n is defined as A= $(\tilde{a}_{ij}^{TzL})_{m\times n}$, where $a_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is the ij^{th} element of A.

Definition 3.2: Operations on Trapezoidal Fuzzy Matrices (TrFMs)

As for classical matrices. We define the following operations on trapezoidal fuzzy matrices. Let A = $(\tilde{a}_{ij}^{TzL})$ and B =$(\tilde{b}_{ij}^{TzL})$ be two trapezoidal fuzzy matrices (TrFMs) of same order. Then, we have the following

i) Addition

A+B = $\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right)$

ii) Subtraction

A-B = $\left(\tilde{a}_{ij}^{TzL} - \tilde{b}_{ij}^{TzL}\right)$

iii) For A = $\left(\tilde{a}_{ij}^{TzL}\right)_{m\times n}$ and B = $\left(\tilde{b}_{ij}^{TzL}\right)_{n\times k}$ then AB = $\left(\tilde{c}_{ij}^{TzL}\right)_{m\times k}$ where $\tilde{c}_{ij}^{TzL} = \sum_{p=1}^{n} \tilde{a}_{ip}^{TzL} \cdot \tilde{b}_{pj}^{TzL}$, i=1,2,…,m and j=1,2,…,k.

iv) A^T or $A^1 = \left(\tilde{a}_{ji}^{TzL}\right)$

v) $KA = \left(K\tilde{a}_{ij}^{TzL}\right)$ where K is scalar.

Examples

1) If $A = \begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$ and

B= $\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

Then A+B=$\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right)$

$A+B = \begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix} + \begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

$A+B = \begin{bmatrix}(1,5,7,11) & (5,9,13,17)\\ (4,8,10,18) & (3,6,13,18)\end{bmatrix}$

2) If A=$\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$ and

B= $\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

Then A-B=$\left(\tilde{a}_{ij}^{TzL} - \tilde{b}_{ij}^{TzL}\right)$

$A-B = \begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix} - \begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

$A-B = \begin{bmatrix}(-8,-2,0,2) & (-7,-3,1,5)\\ (-11,-1,1,3) & (-4,1,8,11)\end{bmatrix}$

If A= $\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$ and

B= $\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

Then A.B= ($\tilde{a}_{ij}^{TzL}\tilde{b}_{ij}^{TzL}$)

$A.B = \begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix} . \begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

$A.B = \begin{bmatrix}(-1,2,3,4)(4) + (2,4,6,8)(6) & (-1,2,3,4)(6) + (2,4,6,3)(3)\\ (1,4,5,6)(4) + (4,5,9,10)(6) & (1,4,5,6)(6) + (4,5,9,10)(3)\end{bmatrix}$

$A.B = \begin{bmatrix}(8,32,48,64) & (0,24,36,48)\\ (28,46,74,84) & (18,39,57,66)\end{bmatrix}$IV.

IV. CONSTANT OF TRAPEZOIDAL FUZZY MATRIX (CONSTANT TRFM)

In this section, we introduce the new matrix namely Constant matrix in the fuzzy nature.

Definition 4.1: Constant of trapezoidal fuzzy matrix (Constant TrFM)

A trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ of order $n \times n$ is said to be Constant TrFM if it is either R-constant TrFM or C-constant TrFM.

Definition 4.2: R-constant trapezoidal fuzzy matrix (R constant TrFM)

A trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ of order $n \times n$ is said to be R-constant TrFM if all its rows are equal to each other.

i.e., $\tilde{a}_{ij}^{TzL} = \tilde{a}_{kj}^{TzL}$ for all $i, j, k = 1,2, \dots, n.$

Definition 4.3: C-constant trapezoidal fuzzy matrix (C-constant TrFM)

A trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ of order $n \times n$ is said to be C-constant TrFM if all its columns are equal to each other.

i.e., $\tilde{a}_{ij}^{TzL} = \tilde{a}_{ik}^{TzL}$ for all $i, j, k = 1,2, \dots, n.$

Definition 4.4: Constant– equivalent of trapezoidal fuzzy matrix (Constant - equivalent TrFM)

A trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ of order $n \times n$ is said to be Constant - equivalent TrFM if it is either R-constant - equivalent TrFM or C-constant - equivalent TrFM.

Definition 4.5: R-constant - equivalent trapezoidal fuzzy matrix (R-constant-equivalent TrFM)

A trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ of order $n \times n$ is said to be R-constant - equivalent TrFM if all its rows are equivalent to each other.

i.e., $\tilde{a}_{ij}^{TzL} \approx \tilde{a}_{kj}^{TzL}$ for all $i, j, k = 1,2, \dots, n$.

Definition 4.6: C-constant- equivalent trapezoidal fuzzy matrix (C-constant-equivalent TrFM)

A trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ of order $n \times n$ is said to be C-constant - equivalent TrFM if all its columns are equivalent to each other.

i.e., $\tilde{a}_{ij}^{TzL} \approx \tilde{a}_{ik}^{TzL}$ for all $i, j, k = 1,2, \dots, n$.

4.7: Numerical example:

4.7.1: R-constant trapezoidal fuzzy matrix (R-constant TrFM)

$$A = \begin{bmatrix} (2,4,6,8) & (3,5,9,11) & (4,8,12,24) \\ (2,4,6,8) & (3,5,9,11) & (4,8,12,24) \\ (2,4,6,8) & (3,5,9,11) & (4,8,12,24) \end{bmatrix}$$

is a R-constant TrFM.

4.7.2: R-constant - equivalent trapezoidal fuzzy matrix (R-constant - equivalent TrFM)

$$B = \begin{bmatrix} (2,4,6,8) & (1,6,9,12) & (0,8,16,24) \\ (1,4,6,9) & (4,5,9,10) & (3,8,15,22) \\ (0,4,6,10) & (3,5,9,11) & (6,8,16,18) \end{bmatrix}$$

is a R-constant - equivalent TrFM.

4.7.3: C-constant trapezoidal fuzzy matrix (C-constant TrFM)

$$C = \begin{bmatrix} (2,4,6,8) & (2,4,6,8) & (2,4,6,8) \\ (3,5,9,11) & (3,5,9,11) & (3,5,9,11) \\ (4,8,12,24) & (4,8,12,24) & (4,8,12,24) \end{bmatrix}$$

is a C-constant TrFM.

4.7.4: C-constant - equivalent trapezoidal fuzzy matrix (C-constant - equivalent TrFM)

$$D = \begin{bmatrix} (2,4,6,8) & (1,4,6,9) & (0,4,6,10) \\ (1,6,9,12) & (4,5,9,10) & (3,5,9,11) \\ (0,8,16,24) & (3,8,15,22) & (6,8,16,18) \end{bmatrix}$$

is a C-constant - equivalent TrFM.

Both A and C are Constant TrFms & both B and D are Constant – equivalent TrFMs.

V. SOME PROPERTIES OF CONSTANT OF TRAPEZOIDAL FUZZY MATRIX

In this section, we introduced the properties of Constant TrFM.

5.1 Properties of Constant TrFM (Constant of Trapezoidal Fuzzy matrix)

Property 5.1.1:

If $A = (\tilde{a}_{ij}^{TzL})$ is an R-constant TrFM of order $n \times n$ and $B = (\tilde{b}_{ij}^{TzL})$ is a TrFM of the same order then AB is a R-constant TrFM of order $n \times n$.

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a R-constant TrFM of order $n \times n$ and $B = (\tilde{b}_{ij}^{TzL})$ be a TrFM of order $\times n$. where $\tilde{a}_{ij}^{TzL} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ and $\tilde{b}_{ij}^{TzL} = (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4})$.

Since A is a R-constant TrFM, $\tilde{a}_{ij}^{TzL} = \tilde{a}_{kj}^{TzL}$ $\forall\ i, j, k = 1,2, \dots, n$.

Let $C = AB$, i.e., $(\tilde{c}_{ij}^{TzL}) = (\tilde{a}_{ij}^{TzL})(\tilde{b}_{ij}^{TzL})$. Then $\tilde{c}_{ij}^{TzL} = \sum_{m=1}^{n} \tilde{a}_{im}^{TzL} . \tilde{b}_{mj}^{TzL}$ $\forall\ i, j = 1,2, \dots, n$.

Since $\tilde{a}_{im}^{TzL} = \tilde{a}_{km}^{TzL}$ $\forall\ i, k, m = 1,2, \dots, n$, we have $a_{km}\ \sigma b = \sum_{m=1}^{n} \tilde{a}_{km}^{TzL} . \tilde{b}_{mj}^{TzL} = \tilde{c}_{kj}^{TzL}$.

Since $\tilde{c}_{ij}^{TzL} = \tilde{c}_{kj}^{TzL}$, by definition $c = (\tilde{c}_{ij}^{TzL})$ is a R-constant TrFM.

That is AB is a R-constant TrFM.

Property 5.1.1 (Remark 1)

If $A = (\tilde{a}_{ij}^{TzL})$ is a R-constant - equivalent TrFM of order $n \times n$ and $B = (\tilde{b}_{ij}^{TzL})$ is a TrFM of the same order then AB is a R-constant – equivalent TrFM of order $n \times n$.

Property 5.1.1 (Remark 2)

The product of two R-constant matrices is also a R-constant matrix and product of two R-constant – equivalent matrices is also a R-constant – equivalent matrix.

Property 5.1.2:

If $A = (\tilde{a}_{ij}^{TzL})$ is an C-constant TrFM of order $n \times n$ and $B = (\tilde{b}_{ij}^{TzL})$ is a TrFM of the same order then BA is a C-constant TrFM of order $n \times n$.

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a C-constant TrFM of order $n \times n$ and $B = (\tilde{b}_{ij}^{TzL})$ be a TrFM of order $\times n$. where $\tilde{a}_{ij}^{TzL} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ and $\tilde{b}_{ij}^{TzL} = (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4})$.

Since A is a C-constant TrFM, $\tilde{a}_{ij}^{TzL} = \tilde{a}_{ik}^{TzL} \ \forall \ i, j, k = 1, 2, \ldots, n$.

Let $C = BA$, i.e., $(\tilde{c}_{ij}^{TzL}) = (\tilde{b}_{ij}^{TzL}).(\tilde{a}_{ij}^{TzL})$.

Then $\tilde{c}_{ij}^{TzL} = \sum_{m=1}^{n} \tilde{b}_{im}^{TzL} . \tilde{a}_{mj}^{TzL} \ \forall \, i, j = 1, 2, \ldots, n$.

Since $\tilde{a}_{km}^{TzL} = \tilde{a}_{im}^{TzL} \ \forall \, i, k, m = 1, 2, \ldots, n$, we have $b_{km} \ \sigma a = \sum_{m=1}^{n} \tilde{b}_{im}^{TzL} \ . \tilde{a}_{mk}^{TzL} = \tilde{c}_{ik}^{TzL}$.

Since $\tilde{c}_{ij}^{TzL} = \tilde{c}_{ik}^{TzL}$, by definition $c = (\tilde{c}_{ij}^{TzL})$ is a C-constant TrFM.

That is BA is a C-constant TrFM.

Property 5.1.2 (Remark 3)

If $A = (\tilde{a}_{ij}^{TzL})$ is a C-constant - equivalent TrFM of order $n \times n$ and $B = (\tilde{b}_{ij}^{TzL})$ is a TrFM of the same order then BA is a C-constant – equivalent TrFM of order $n \times n$.

Property 5.1.2 (Remark 4)

The product of two C-constant matrices is also a R-constant matrix and product of two C-constant – equivalent matrices is also a C-constant – equivalent matrix.

Property 5.1.3:

The transpose of a R-constant TrFM $A = (\tilde{a}_{ij}^{TzL})$ is a C-constant TrFM and vice versa.

Proof:

Suppose $A = (\tilde{a}_{ij}^{TzL})$ is a R-constant TrFM of order $n \times n$.

Then $\tilde{a}_{ij}^{TzL} = \tilde{a}_{kj}^{TzL} \ \forall \ i, j, k = 1, 2, \ldots, n$. That is, all the rows are equal in A.

Since rows of A become columns of A' all the columns are equal in A'.

That is, $\tilde{a}_{ji}^{TzL} = \tilde{a}_{jk}^{TzL} \ \forall \ i, j, k = 1, 2, \ldots, n$ in A'.

Hence A' is a C-constant TrFM of order $n \times n$.

Property 5.1.3: (Remark 5)

The transpose of an R-constant – equivalent TrFM $A = (\tilde{a}_{ij}^{TzL})$ is a C-constant – equivalent TrFM and vice versa.

Property 5.1.4:

The transpose of a C-constant TrFM $A = (\tilde{a}_{ij}^{TzL})$ is a R-constant TrFM and vice versa.

Proof:

Suppose $A = (\tilde{a}_{ij}^{TzL})$ is a C-constant TrFM of order $n \times n$. Then $\tilde{a}_{ij}^{TzL} = \tilde{a}_{kj}^{TzL} \ \forall \ i, j, k = 1, 2, \ldots, n$. That is, all the columns are equal in A.

Since columns of A become rows of A' all the rows are equal in A'.

That is, $\tilde{a}_{ji}^{TzL} = \tilde{a}_{jk}^{TzL} \ \forall \ i, j, k = 1, 2, \ldots, n$ in A'.

Hence A' is a R-constant TrFM of order $n \times n$.

Property 5.1.4: (Remark 6)

The transpose of an C-constant – equivalent TrFM $A = (\tilde{a}_{ij}^{TzL})$ is a R-constant – equivalent TrFM and vice versa.

Remark 7:

- If $\tilde{A} = (\tilde{a}_{ij}^{TzL})$ is a constant TrFM then $|\tilde{A}| =$, Hence $\tilde{A}$ is singular and also $\tilde{A}$ is not invertible.

VI. CONCLUSION

In this article, we have concentrate the notion of the Constant trapezoidal fuzzy matrices are defined and some relevant properties of their constant fuzzy matrices have also been proved. Few illustrations based on operations of trapezoidal fuzzy matrices have also been justified. In future, the result about TrFMs discussed here may be utilized in further works.

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