Cube Difference Labeling of Some Snakes

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Abstract- *Let G be a (p,q) graph. G is said to have a cube difference labeling if there exists a injection $f{:}V(G) \to \{0,1,2,\dots,p-1\}$ such that the edge set of G has assigned a weight defined by the absolute cube difference of its end-vertices, the resulting weights are distinct. A graph which admits cube difference labeling is called cube difference graph. In this paper, a discussion is made on cube difference labeling of some snakes like quadrilateral snake Q_n, alternate quadrilateral snake $A(Q_n)$, triangular snake T_n and alternate triangular snake $A(T_n)$.*

Keywords- Cube difference labeling, cube difference graph, path, snakes, quadrilateral snake Q_n, alternate quadrilateral snake $A(Q_n)$, triangular snake T_n, alternate triangular snake $A(T_n)$.

I. INTRODUCTION

Mathematics is an important branch of science. Graph theory is an important branch in mathematics, plays a vital role in solving real life problems. Labeling concept in mathematics are used in computer networks, circuit design, communication networks, data base management, astronomy etc…,. There are many types of labeling are discussed by various researchers. Cube difference labeling was introduced by J. Shiama in 2013[6]. S.S. Sandhya, E. Ebin Raja Merly and B. Shiny discussed super geometric mean labeling of double quadrilateral snake graphs in 2015[5]. I.I. Jadav and G.V. Ghodasara investigated alternate quadrilateral snake graphs are strongly*-graphs in 2016[2]. Triangular snake and alternate triangular snake satisfies mean cordiality which was proved by R.Ponraj and S.Sathish Narayanan in 2015[4]. In this paper, a study is made on cube difference labeling of quadrilateral snake Q_n, alternate quadrilateral snake $A(Q_n)$, triangular snake T_n and alternate triangular snake $A(T_n)$.

II. PRELIMINARIES

Cube difference labeling: Let $G = (V(G), E(G))$ be a graph. G is said to be cube difference labeling if there exist a injection $f{:}V(G) \to \{0,1,2,\dots,p-1\}$ such that the induced function $f^*{:}E(G) \to N$ given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ is injective.

Path: A walk is called a path if all its points are distinct. A path of length n is denoted by P_n.

Quadrilateral snake: The quadrilateral snake Q_n is obtained from a path $u_1, u_2, u_3, \dots, u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and then joining v_i and w_i. That is every edge of path is replaced by a cycle C_4.

Alternate quadrilateral snake: An alternate quadrilateral snake $A(Q_n)$ is obtained from a path P_n, each alternate edge of P_n is replaced by a cycle C_4.

Triangular snake: The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Alternate triangular snake: An alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, u_3, \dots, u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by cycle C_3.

III. CUBE DIFFERENCE LABELING OF SOME SNAKES

Theorem 1: Quadrilateral snake Q_n satisfies cube difference labeling for all values of n.

Proof: Let G be quadrilateral snake Q_n of order $p = 3n + 1$ and size $q = 4n$ .The vertex set $V(G) = \{u_0, u_1, u_2, \dots, u_{n-1}, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$. The function $f{:}V(G) \to \{0,1,2,\dots,p-1\}$ is defined as follows:
$f(u_i) = 3i$, for $0 \le i \le n$;
$f(v_i) = 3i - 1$, for $1 \le i \le n$;
$f(w_i) = 3i - 2$, for $1 \le i \le n$. Then the induced function $f^*{:}E(G) \to N$ is defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct. Hence quadrilateral snake Q_n is a cube difference graph.

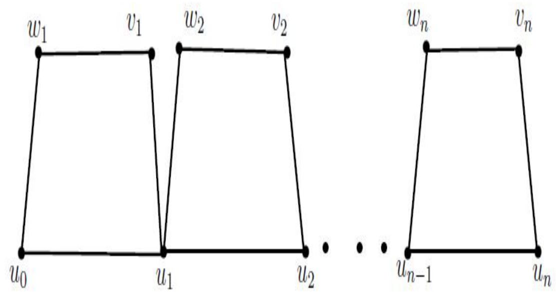

Figure 1. Quadrilateral snake Q_n

Example 1:

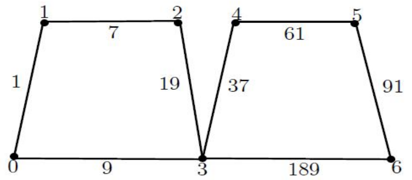

Figure 2. Quadrilateral snake Q_2

Theorem 2: Alternate quadrilateral snake $A(Q_n)$ is a cube difference graph.

Proof: Let G be alternate quadrilateral snake $A(Q_n)$. Then $p = 4n$, $q = 4n + (n-1)$ are order and size of $G = A(Q_n)$. The vertex set $V(G) = \{u_0, u_1, u_2, \dots, u_n, u_{n+1}, u_{n+2}, \dots, u_{n+(n-1)}, w_1, w_2, \dots, w_n, v_1, v_2, \dots, v_n\}$. Now define the function $f: V(G) \to \{0,1,2,\dots,p-1\}$ as follows:

$f(u_i) = i$, for $0 \le i \le n-1$;

$f(u_{n+i}) = n + i$, for $0 \le i \le n-1$;

$f(w_{i+1}) = 2n + 2i$, for $0 \le i \le n-1$;

$f(v_{i+1}) = 2n + 2i + 1$, for $0 \le i \le n-1$. Then the induced function $f^*: E(G) \to N$ admits cube difference labeling. Hence $A(Q_n)$ is cube difference graph.

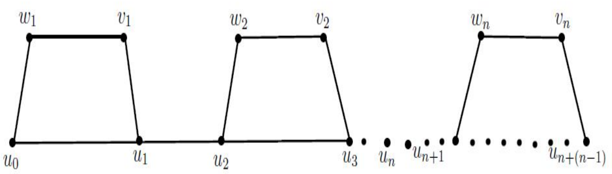

Figure 3. Alternate quadrilateral snake $A(Q_n)$

Example 2:

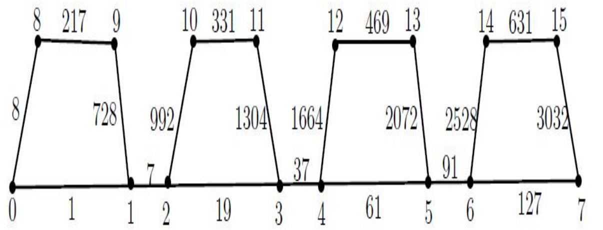

Figure 4. Alternate quadrilateral snake $A(Q_4)$

Theorem 3: Triangular snake T_n admits cube difference labeling for all values of n.

Proof: Let G be triangular snake T_n. By the definition of T_n, the order is $p = 2n + 1$ and the size is $q = 3n$. The vertex set $V(G) = \{u_0, u_1, u_2, \dots, u_{n-1}, u_n, v_0, v_1, v_2, \dots, v_{n-1}\}$. Now define the function $f: V(G) \to \{0,1,2,\dots,p-1\}$ as follows:

$f(u_i) = 2i$, for $0 \le i \le n$;

$f(v_i) = 2i + 1$, for $0 \le i \le n-1$. Then the induced function $f^*: E(G) \to N$ is defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct. Hence triangular snake T_n is a cube difference graph.

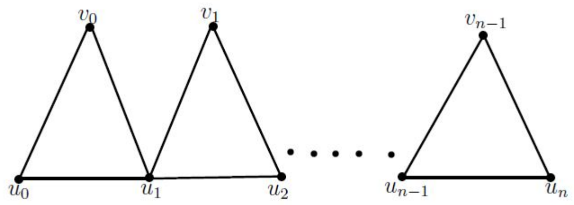

Figure 5. Triangular snake T_n

Example 3:

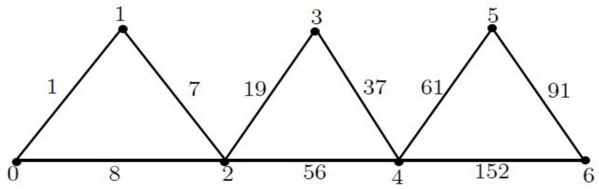

Figure 6. Triangular snake T_3

Theorem 4: Alternate triangular snake $A(T_n)$ is a cube difference graph.

Proof: Let G be alternate triangular snake $A(T_n)$ with order $p = 3n$ and size $q = 3n + (n-1)$. The vertex set $V(G) = \{u_0, u_1, u_2, \dots, u_n, u_{n+1}, u_{n+2}, \dots, u_{n+(n-1)}, v_0, v_1, v_2, \dots, v_{n-1}\}$ as shown in figure 7.

Now define the function $f: V(G) \to \{0,1,2,\dots,p-1\}$ as follows:

$f(u_i) = i$, for $0 \le i \le n-1$;

$f(u_{n+i}) = n + i$, for $0 \le i \le n-1$;

$f(v_i) = 2n + i$, for $0 \le i \le n-1$. Then the induced function $f^*: E(G) \to N$ is defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct. Hence alternate triangular snake $A(T_n)$ is a cube difference graph.

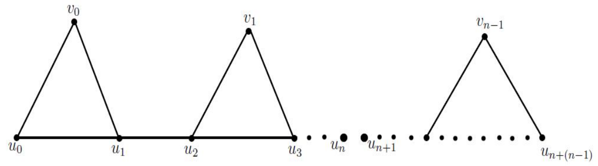

Figure 7. Alternate triangular snake $A(T_n)$

Example 4:

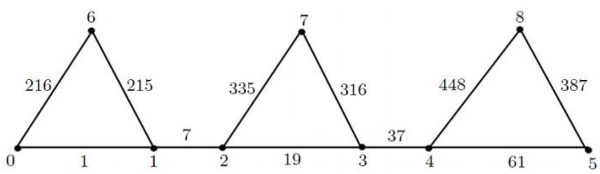

Figure 8. Alternate triangular snake $A(T_3)$

IV. CONCLUSION

In this paper, a study on cube difference labeling of some snakes are successfully discussed. This might be a small area of research but may be a spark for other labeling concepts.

REFERENCES

[1] J.A. Bondy. and U.S.R. Murthy, Graph Theory with applications, North Holland Elsevier, New York, 1976.
[2] I.I. Jadev & G.V. Ghodasara, Snakes related strongly * - graphs, Vol-3, Issue-9, Sep 2016.
[3] Joseph A. Gallian, A Dynamic survey of Graph Labeling, 2015.
[4] R. Ponraj and S. Sathish Narayanan, Mean cordiality of Some Snake graphs, Palestine Journal of Mathematics, Vol.4 (2), 439-445, 2015.
[5] S.S. Sandhya, E. Ebin Raja Merly and B. Shiny, Super Geometric Mean Labeling on double Quadrilateral Snake graphs, Asia pacific Journal of Research, Vol:1.Issue XXI, January 2015.
[6] J. Shiama, Cube difference labeling of Some Graphs, International Journal of Engineering Science and Innovative Technology (IJESIT) , Volume 2, Issue 6, November 2013.
[7] R. Uma, S. Divya, Cube Difference Labeling of Star Related Graphs, International Journal of Scientific Research in Computer Science, Engineering and Information Technology , Volume 2, Issue 5, ISSN: 2456-3307 , October 2017.