On Changing Behavior of Edges of Some Graphs I

S. Shenbaga Devi [1], A. Nagarajan[2]
Department of Mathematics
[1]Assistant Professor, Aditanar College of Arts and Science, Tiruchendur 628 215
[2]Associate Professor, V.O.Chidambaram College, Thoothukudi 628 008

Abstract- *Let G be a (p,q)graph and $f: V(G) \rightarrow \{1,2,\ldots,p+q-1,p+q+2\}$ be an injection. For each edge $e=uv$, the induced edge labeling f^* is defined as follows:*

$$f^*(e)=\begin{cases}\dfrac{|f(u)-f(v)|}{2} & if\ |f(u)-f(v)|\ is\ even\\ \dfrac{|f(u)-f(v)|+1}{2} & if\ |f(u)-f(v)|\ is\ odd\end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and are from $\{1,2,3,\ldots.q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem diffence mean graph. In this paper, a new parameter E^+ is introduced and applied for the graphs $(K_4 * S_n)$, $J(m,n)$, C_n and $K_{1,n}$.

Keywords- Star, Jelly fish, Near Skolem Difference Mean labeling, Near Skolem difference mean graphs.

I. INTRODUCTION

All graphs in this paper are finite, undirected and simple. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. For standard terminology and notations, we follow Harary (1) and for graph labeling, we refer to Gallian (2).

In this paper, a Near skolem difference mean graph G is investigated and a new parameter E^+ is introduced to find the minimum number of edges that should be added to G to convert the Near skolem difference mean graph G into a non-Near skolem difference mean graph G^*.

Definition 1.1: $(K_4 * S_n)$ is a graph obtained from K_4 by appending a star S_n with n vertices to its four vertices and has a vertex set $V(G)=\{u_i, v_{ij}/1\leq i\leq 4, 1\leq j\leq 4\}$ and edge set $E(G)=\{u_i u_{i+1}/1\leq i\leq 3\}\cup\{u_2 u_{i+2}\}\cup\{u_3 u_4\}\cup\{u_i v_{ij}/1\leq i\leq 4, 1\leq j\leq n\}$.

Definition 1.2: For integers *m*, *n*≥ 0, a Jelly fish is a graph J(m,n) with vertex set and edge set as

$V\ (J(m,n))\ =\ \{u,v,x,y\}\cup\{x_1,x_2,\ldots,x_m\}\cup\{y_1,y_2,\ldots,y_n\}$

$E\ (J(m,n=\ \{(u,x),(u,v),(u,y),(v,x),(v,y)\}\cup\{(x_i,x): 1\ \leq\ i\ \leq\ n\}\cup\{(y_i,y): 1\ \leq\ i\ \leq\ n\}$.

II. MAIN RESULT

Definition 2.1 A graph $G=(V,E)$ with p vertices and q edges is said to have Nearly skolem difference mean labeling if it is possible to label the vertices x∈V with distinct elements $f(x)$ from $\{1,2,\ldots\ldots,p+q-1,p+q+2\}$ in such a way that each edge e = uv , is labeled as $f^*(\text{e})=\frac{|f(u)-f(v)|}{2}$ if $|\ f(u)-f(v)|$ is even and $f^*(\text{e})=\frac{|f(u)-f(v)|+1}{2}$ if $|\ f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and are from $\{1,2,\ldots\ldots,q\}$. A graph that admits a Near skolem difference mean labeling is called a Near skolem difference mean graph.

Definition 2.2: Let G be a Near skolem difference mean graph. Then the parameter $E^+$of a graph G is defined as the minimum number of edges to be added to G, so that the resulting graph is non-Near skolem difference mean.

Theorem 2.3:$E^+(K_4 * S_n)=1, n\geq 1$.

Proof: Let G be the graph $K_4 * S_n$.

Let $V(G)=\{u_i, v_{ij}/1\leq i\leq 4, 1\leq j\leq n\}$ and

$E(G)=\{u_i u_j/1\leq i<j\leq 4\}\cup\{u_i v_{ij}/1\leq j\leq n\}$

Then $|V(G)|=4n+4$ and $|E(G)|=4n+6$

Define $f: V(G)\rightarrow\{1,2,\ldots\ldots\ldots\ldots\ldots,8n+9,8n+12\}$ as follows:

$f(u_1)=8n+12$
$f(u_2)=3$
$f(u_3)=8n+9$
$f(u_4)=1$
$f(v_{1j})=4+2j,\quad 1\leq j\leq n$
$f(v_{2j})=2n+4+2j,\quad 1\leq j\leq n$
$f(v_{3j})=7+2j,\quad 1\leq j\leq n$
$f(v_{4j})=2n+7+2j,\quad 1\leq j\leq n$

Let f^* be the induced edge labeling of f. Then,

$f^*(u_1u_2)=4n+4$
$f^*(u_2u_3)=4n+3$
$f^*(u_3u_4)=4n+5$
$f^*(u_1u_4)=4n+6$
$f^*(u_1u_3)=1$
$f^*(u_1v_{1j})=2+i,\quad 1\leq i\leq n$
$f^*(u_2v_{2j})=2n+2+i,\quad 1\leq i\leq n$

$f^*(u_3v_{3j}) = n + 2 + i, \quad 1 \le i \le n$

$f^*(u_4v_{4j}) = 3n + 2 + i, \quad 1 \le i \le n$

Clearly the induced edge labels $f^*(E(G)) = \{1,2, \dots\dots\dots .4n + 6\}$ are all distinct.

Hence, the graph $K_4 * S_n$ admits Near skolem difference mean labeling.

Let G^* be the graph $K_4 * S_n \cup \{uv\}$ where u *and* v are any non-adjacent vertices of $K_4 * S_n$.

u_i are the vertices of $K_4 (1 \le i \le 4)$ and

v_{ij} are the vertices of the star S_n ($1 \le j \le n$ *and* $1 \le i \le 4$).

Then $V(G^*) = \{u_i, v_{ij} / 1 \le i \le 4, 1 \le j \le n\}$ and

$E(G^*) = \{u_i u_j / 1 \le i < j \le 4\} \cup \{u_i v_{ij} / 1 \le j \le n\} \cup \{(uv)\}$.

Then $|V(G^*)| = 4n + 4$ and $|E(G^*)| = 4n + 7$

let $f: V(G^*) \to \{1,2, \dots ,8n + 10, 8n + 13\}$

Let $e = xy \in E(G^*)$ with $1 \le f(x) < f(y) \le 8n + 13$.

Now two cases arise:

Case (i) $\frac{|f(y)-f(x)|}{2} = 4n + 7$

This implies $f(y) = 8n + 14 + f(x)$

$\ge 8n + 14 + 1$

$= 8n + 15$

Case (ii) $\frac{|f(y)-f(x)|+1}{2} = 4n + 7$

This implies $f(y) - f(x) = 8n + 14 - 1$

This implies $f(y) = 8n + 13 + f(x)$

$\ge 8n + 13 + 1$

$= 8n + 14$

Thus, in both cases, we get a contradiction since by definition, $f(v) \le 8n + 13$

Hence, G^* is not a Near Skolem Difference Mean graph.

Hence, $E^+(K_4 * S_4) = 1$.

Theorem 2.4: *$E^+(J(m,n)) = 2$ where $J(m,n)$ is a Jelly fish graph.*

Proof: It has already been proved that the graph $J(m,n)$ is Near Skolem Difference Mean. [4]

Let G be a graph obtained from $J(m,n)$ by adding an edge.

Let $V(G) = \{u, v, x, y, u_i, v_j / 1 \le i \le m, 1 \le j \le n\}$ and

$E(G) = \{xu, xv, yu, yv, xy\} \cup \{uu_i / 1 \le i \le m\} \cup \{vv_j / 1 \le j \le n\}\} \cup \{(uv)\}$.

Then $|V(G)| = m + n + 4$ and $|E(G)| = m + n + 6$

Define: $f: V(G) \to \{1,2,3, \dots ,2m + 2n + 9, 2m + 2n + 12\}$ as follows:

$f(u) = 2m + 2n + 12$

$f(v) = 2m + 2n + 9$

$f(x) = 2m + 1$

$f(y) = 2m + 3$

$f(u_i) = 2i - 1, \quad 1 \le i \le m$

$f(v_j) = 2m + 3 + 2j, \quad 1 \le j \le n.$

Let f^* be the induced edge label of f. Then,

$f^*(uu_i) = m + n + 7 - i, \quad 1 \le i \le m$

$f^*(vv_j) = n + 3 - j, \quad 1 \le j \le n$

$f^*(xu) = n + 6$

$f^*(xv) = n + 4$

$f^*(yu) = n + 5$

$f^*(yv) = n + 3$

$f^*(xy) = 1$

$f^*(uv) = 2$

The induced edge labels are all distinct and are $\{1,2, \dots , m + n + 6\}$.

Hence, from the above labeling pattern, the graph $G = J(m,n) \cup \{(uv)\}$ admits Near Skolem Difference Mean labeling.

Hence, G is a Near Skolem Difference Mean graph.

Now, consider $G^* = J(m,n) \cup \{u_1u_2, u_2u_3\}$ where

Let $V(G^*) = \{u, v, x, y, u_i, v_j, \ |1 \le i \le m, 1 \le j \le n\}$ and

$E(G^*) = \{xu, xv, yu, yv, xy\} \cup \{uu_i / 1 \le i \le m\} \cup \{vv_j / 1 \le j \le n\} \cup \{u_1u_2, u_2u_3\}$.

Then $|V(G^*)| = m + n + 4$ and $|E(G^*)| = m + n + 7$

Let $f: V(G^*) \to \{1,2, \dots ,2m + 2n + 10, 2m + 2n + 13\}$.

Let $e = w_1w_2 \in E(G^*)$ with

Now, $1 \le f(w_1) < f(w_2) \le 2m + 2n + 13$

There are two cases:

Case(i): Suppose $\frac{|f(w_2)-f(w_1)|}{2} = m + n + 7$

$f(w_2) - f(w_1) = 2m + 2n + 14.$

$f(w_2) = 2m + 2n + 14 + f(w_1)$

$\ge 2m + 2n + 15.$

Case(ii): Suppose $\frac{|f(w_2)-f(w_1)|+1}{2} = m + n + 7$

This implies $f(w_2) - f(w_1) = 2m + 2n + 14 - 1$

$f(w_2) = 2m + 2n + 13 + f(w_1)$

$\ge 2m + 2n + 14.$

From both the cases, we have $f(w_2) \ge 2m + 2n + 13$.

This is a contradiction since, by definition, $f(w_2) \le 2m + 2n + 13$

Hence, G^* is not Near Skolem Difference Mean graph.

Hence $E^+(J(m,n)) = 2$.

Theorem 2.5: $E^+(C_n) = 3$.

Proof: It has already been proved that the cycle graph C_n is Near Skolem Difference Mean for $n \ge 3$.[3]

To prove the theorem, we consider the following two cases:

Case(i): When $E^+(C_n) = 1$

Subcase (i): When $n = 2k + 1$

Without loss of generality, let G be the graph $C_{2k+1} \cup \{u_k v_{k-1}\}$.

Let $V(G) = \{u_i, v_j, 1 \le i \le k + 1, 1 \le j \le k\}$ and

$E(G) = \{u_1v_1, u_{k+1}v_k, u_iu_{i+1}, v_jv_{j+1}, 1 \le i \le k, 1 \le j \le k - 1\} \cup \{u_kv_{k-1}\}$.

Then $|V(G)| = 2k + 1$ and $|E(G)| = 2k + 2$.

Let $f: V(G) \to \{1,2,\ldots,4k+2,4k+5\}$ be defined as follows:

When k is odd:

$f(u_{2i+1}) = 4i+1, \quad 0 \le i \le \frac{k-1}{2}.$

$f(u_{2i}) = 4k+6-4i, \quad 1 \le i \le \frac{k+1}{2}.$

$f(v_1) = 4k+5.$

$f(v_{2i+1}) = 4k+5-4i, \quad 1 \le i \le \frac{k-1}{2}.$

$f(v_{2i}) = 4i+2, \quad 1 \le i \le \frac{k-1}{2}.$

When k is even:

$f(u_{2i+1}) = 4i+1, \quad 0 \le i \le \frac{k}{2}.$

$f(u_{2i}) = 4k+6-4i, \quad 1 \le i \le \frac{k}{2}.$

$f(v_1) = 4k+5$

$f(v_{2i+1}) = 4k+5-4i, \quad 1 \le i \le \frac{k-2}{2}.$

$f(v_{2i}) = 4i+2, \quad 1 \le i \le \frac{k}{2}.$

Let f^* be the induced edge labeling for f.

$f^*(u_1v_1) = 2k+2$

$f^*(u_{k+1}v_k) = \begin{cases} 1, when\ k\ is\ even \\ 2, when\ k\ is\ odd \end{cases}$

$f^*(u_iu_{i+1}) = 2k+3-2i, \quad 1 \le i \le k$

$f^*(v_jv_{j+1}) = 2k+2-2i, \quad 1 \le j \le k-1.$

$f^*(u_kv_{k-1}) = \begin{cases} 2, when\ k\ is\ even \\ 1, when\ k\ is\ odd \end{cases}$

The edge labels are all distinct and are $f^*(E(G)) = \{1,2,\ldots,2k+2\}$.

Subcase (ii): When $n = 2k$

Without loss of generality, let G be the graph $C_{2k} \cup \{u_kv_{k-1}\}$.

Let $V(G) = \{u_i, v_j / 1 \le i, j \le k\}$ and

$E(G) = \{u_1v_1, u_kv_k, u_iu_{i+1}, v_jv_{j+1} / 1 \le i, j \le k-1\} \cup \{u_kv_{k-1}\}$.

Then $|V(G)| = 2k$ and $|E(G)| = 2k+1$

Let $f: V(G) \to \{1,2,\ldots,4k,4k+3\}$ be defined as follows.

When k is odd:

$f(u_{2i+1}) = 1+4i, \quad 0 \le i \le \frac{k-3}{2}.$

$f(u_k) = 2k.$

$f(u_{2i}) = 4k+4-4i, \quad 1 \le i \le \frac{k-3}{2}.$

$f(u_{k-1}) = 2k+7$

$f(v_{2i+1}) = 4k+3-4i, \quad 0 \le i \le \frac{k-3}{2}.$

$f(v_k) = 2k+4$

$f(v_{2i}) = 2+4i, \quad 1 \le i \le \frac{k-3}{2}.$

$f(v_{k-1}) = 2k-1.$

When k is even:

$f(u_{2i+1}) = 4i+1, \quad 0 \le i \le \frac{k-2}{2}.$

$f(u_{2i}) = 4k+4-4i, \quad 1 \le i \le \frac{k}{2}.$

$f(v_{2i+1}) = 4k+3-4i, \quad 0 \le i \le \frac{k-2}{2}.$

$f(v_{2i}) = 4i+2, \quad 1 \le i \le \frac{k}{2}.$

Let f^* be the induced edge labeling of f. Then,

$f^*(u_1v_1) = 2k+1$

$f^*(u_kv_k) = \begin{cases} 1, when\ k\ is\ even \\ 2, when\ k\ is\ odd \end{cases}$

$f^*(u_iu_{i+1}) = 2k+2-2i, \quad 1 \le i \le k-1.$

$f^*(v_jv_{j+1}) = 2k+1-2j, \quad 1 \le j \le k-1$

$f^*(u_kv_{k-1}) = \begin{cases} 2, when\ k\ is\ even \\ 1, when\ k\ is\ odd \end{cases}$

Then the induced edge labels are distinct and are

$\{1,2,\ldots,2k+1\}$

Hence the graph G admits Near skolem difference mean labeling even after adding the edge $\{u_kv_{k-1}\}$.

Case (ii): When $E^+(C_n) = 2$

Subcase(i): When $n = 2k+1$

Without loss of generality, let G be the graph obtained by adding 2 edges to C_{2k+1}.

Let $V(G) = \{u_i, v_j, 1 \le i \le k+1, 1 \le j \le k\}$ and

$E(G) = \{u_1v_1, u_{k+1}v_k, u_iu_{i+1}, v_jv_{j+1}, 1 \le i \le k, 1 \le j \le k-1\} \cup \{u_2v_1, u_4v_1(when\ k\ is\ odd)\}$ or $\cup\{u_kv_{k-1}, u_{k+1}v_{k-1}(when\ k\ is\ even)\}$.

Then $|V(G)| = 2k+1$ and $|E(G)| = 2k+3$.

Let $f: V(G) \to \{1,2,\ldots,4k+3,4k+6\}$ be defined as follows:

When k is odd:

$f(u_{2i+1}) = 4i+1, \quad 0 \le i \le \frac{k-1}{2}.$

$f(u_{2i}) = 4k+7-4i, \quad 1 \le i \le \frac{k+1}{2}.$

$f(v_{2i+1}) = 4k+6-4i, \quad 0 \le i \le \frac{k-1}{2}.$

$f(v_{2i}) = 4i-1, \quad 1 \le i \le \frac{k-1}{2}.$

When k is even:

$f(u_{2i+1}) = 4i+1, \quad 0 \le i \le \frac{k}{2}.$

$f(u_{2i}) = 4k+7-4i, \quad 1 \le i \le \frac{k}{2}.$

$f(v_{2i+1}) = 4k+6-4i, \quad 0 \le i \le \frac{k-2}{2}.$

$f(v_{2i}) = 4i-1, \quad 1 \le i \le \frac{k}{2}.$

Let f^* be the induced edge labeling for f.

$f^*(u_1v_1) = 2k+3$

$f^*(u_{k+1}v_k) = \begin{cases} 1, when\ k\ is\ even \\ 2, when\ k\ is\ odd \end{cases}$

$f^*(u_iu_{i+1}) = 2k + 3 - 2i, \quad 1 \le i \le k$
$f^*(v_jv_{j+1}) = 2k + 4 - 2j, \quad 1 \le j \le k - 1.$

When k is odd,
$f^*(u_2v_1) = 2$
$f^*(u_4v_1) = 4$

When k is even,
$f^*(u_kv_{k-1}) = 1$
$f^*(u_{k+1}v_{k-1}) = 4$

The edge labels are all distinct and are $f^*(E(G)) = \{1,2,..,2k + 3\}$.

Subcase(i): When $n = 2k$

Without loss of generality, let G be the graph obtained by adding 2 edges to C_{2k}.

Let $V(G) = \{u_i, v_j, 1 \le i \le k + 1, 1 \le j \le k\}$ and
$E(G) = \{u_1v_1, u_{k+1}v_k, u_iu_{i+1}, v_jv_{j+1}, 1 \le i \le k, 1 \le j \le k - 1\} \cup \{u_2v_1, u_3v_2\}$.

Then $|V(G)| = 2k$ and $|E(G)| = 2k + 2$.

Let $f: V(G) \to \{1,2, \dots, 4k + 1, 4k + 4\}$ be defined as follows:

When k is odd:

$f(u_{2i+1}) = 1 + 4i, \quad 0 \le i \le \frac{k-3}{2}.$

$f(u_k) = 2k.$

$f(u_{2i}) = 4k + 5 - 4i, \quad 1 \le i \le \frac{k-1}{2}.$

$f(v_{2i+1}) = 4k + 4 - 4i, \quad 0 \le i \le \frac{k-1}{2}.$

$f(v_{2i}) = 4i - 1, \quad 1 \le i \le \frac{k-1}{2}.$

When k is even:

$f(u_{2i+1}) = 4i + 1, \quad 0 \le i \le \frac{k-2}{2}.$

$f(u_{2i}) = 4k + 5 - 4i, \quad 1 \le i \le \frac{k}{2}.$

$f(v_{2i+1}) = 4k + 4 - 4i, \quad 0 \le i \le \frac{k-2}{2}.$

$f(v_{2i}) = 4i - 1, \quad 1 \le i \le \frac{k}{2}.$

Let f^* be the induced edge labeling for f.
$f^*(u_1v_1) = 2k + 2$
$f^*(u_kv_k) = 3$
$f^*(u_iu_{i+1}) = 2k + 2 - 2i, \quad 1 \le i \le k - 1$
$f^*(v_jv_{j+1}) = 2k + 3 - 2j, \quad 1 \le j \le k - 1.$
$f^*(u_2v_1) = 2$
$f^*(u_3v_2) = 1$

The edge labels are all distinct and are
$f^*(E(G)) = \{1,2,..,2k + 2\}$.

Hence, the graph G admits Near Skolem difference mean labeling when $E^+(C_n) = 2$

Case(ii): Let G^* be the graph $C_n \cup \{u_iu_{i+1}, 1 \le i \le 4\}$.

Then $|V(G^*)| = n$ and $|E(G^*)| = n + 3$
Let $f: V(G^*) \to \{1,2, \dots, 2n + 2, 2n + 5\}$.
Let $e = uv \in E(G^*)$ with $1 \le f(u) < f(v) \le 2n + 5$
Now, two subcases arise:

Subcase(i): Suppose, $\frac{|f(v)-f(u)|}{2} = n + 3$.

This implies $f(v) = 2n + 6 + f(u)$
$\ge 2n + 6 + 1.$
$= 2n + 7$

Subcase(ii): Suppose, $\frac{|f(v)-f(u)|+1}{2} = n + 3$

This implies, $f(v) = 2n + 6 + f(u) - 1.$
$\ge 2n + 5 + 1.$
$= 2n + 6$

Thus, in both cases, $f(v) \ge 2n + 6$.
This is a contradiction since, by definition, $f(v) \le 2n + 5$.
Hence, adding 3 edges to the Near skolem difference mean graph C_n makes it a non-Near skolem difference mean graph.
Hence, $E^+(C_n) = 3$.

Theorem 2.6: $E^+(K_{1,n}) = 4$.

Proof: It has already been proved that the graph $K_{1,n}$ is Near skolem difference mean. [6]

To prove the theorem, we consider the following 2 cases:

Case(i): $\boldsymbol{E^+}(K_{1,n}) = 1,2 \text{ or } 3$

Subcase(i): When $E^+(K_{1,n}) = 1$.

Let G be the graph $K_{1,n} \cup \{e = u_1u_2\}$.
Let $V(G) = \{v, u_i / \ 1 \le i \le n\}$ and
$E(G) = \{vu_i / 1 \le i \le n\} \cup \{u_1u_2\}$.
Then $|V(G)| = n + 1$ and $|E(G)| = n + 1$
Let $f: V(G) \to \{1, 2, \dots, 2n + 1, 2n + 4\}$ be defined as follows:
$f(u) = 2n+4$
$f(u_i) = 2i, \quad 1 \le i \le n.$
Let f^* be the induced edge labeling of f. Then,
$f^*(vu_i) = n + 1 - i, \quad 1 \le i \le n.$
$f^*(u_1u_2) = 1$
The induced edge labels are all distinct and are $f^*(E(G)) = \{1, 2, \dots, n + 1\}$.

Subcase(ii): When $E^+(K_{1,n}) = 2$.

Let G be the graph $K_{1,n} \cup \{u_1u_2, u_3u_5\}$.
Let $V(G) = \{v, u_i / 1 \le i \le n\}$ and
$E(G) = \{vu_i / 1 \le i \le n\} \cup \{u_1u_2, u_3u_5\}$.
Then $|V(G)| = n + 1$ and $|E(G)| = n + 2$
Let $f: V(G) \to \{1, 2, \dots, 2n + 2, 2n + 5\}$ be defined as follows:
$f(u) = 2n+5$
$f(u_i) = 2i - 1, \quad 1 \le i \le n.$
Let f^* be the induced edge labeling of f. Then,
$f^*(vu_i) = n + 3 - i, \quad 1 \le i \le n.$
$f^*(u_1u_2) = 1$

$f^*(u_3u_5)$ =2

The induced edge labels are all distinct and are $f^*(E(G)) = \{1, 2, \dots, n+2\}$.

Subcase(iii): If $E^+(G) = 3$

Let G be the graph $K_{1,n} \cup \{u_1u_2, u_3u_5, u_4u_7\}$.

Let $V(G) = \{v, u_i / 1 \le i \le n\}$ and

$E(G) = \{vu_i / 1 \le i \le n\} \cup \{u_1u_2, u_3u_5, u_4u_7\}$.

Then $|V(G)| = n+1$ and $|E(G)| = n+3$.

Let $f: V(G) \to \{1, 2, \dots, 2n+3, 2n+6\}$ be defined as follows:

$f(u) = 2n+6$

$f(u_i) = 2i - 1, \quad 1 \le i \le n.$

Let f^* be the induced edge labeling of f. Then,

$f^*(vu_i) = n + 4 - i, \quad 1 \le i \le n.$

$f^*(u_1u_2)$ =1

$f^*(u_3u_5)$ =2

$f^*(u_4u_7)$ =3

The induced edge labels are all distinct and are

$f^*(E(G)) = \{1, 2, \dots, n+3\}$.

Hence, from all the above three subcases, it can be concluded that adding up to 3 edges to the graph $K_{1,n}$ retains the Near Skolem Difference Mean property of the graph.

Case(ii): Let G^* be the graph $K_{1,n} \cup \{e = u_iu_{i+1}, 1 \le i \le 4\}$.

Then $V^+(G^*) = \{v, u_i, 1 \le i \le n\}$and

$E^+(G^*) = \{vu_i \ 1 \le i \le n\} \cup \{u_ju_{j+1} / 1 \le j \le 4\}$

Then $|V(G^*)| = n + 1$and $|E(G^*)| = n + 4$

Let $f: V(G^*) \to \{1, 2, \dots, 2n+4, 2n+7\}$

Let $e = uv \in E(G^*)$ with$1 \le f(u) < f(v) \le 2n+7$

Now, two subcases arise:

Subcase(i): Suppose, $\frac{|f(v)-f(u)|}{2} = n+4$.

This implies, $|f(v) - f(u)| = 2n + 8$.

This implies $f(v) = 2n + 8 + f(u)$.

$\ge 2n + 8 + 1.$

$= 2n + 9.$

Subcase(ii): Suppose, $\frac{|f(v)-f(u)|+1}{2} = n+4$

This implies, $|f(v) - f(u)| = 2n + 8 - 1$

This implies, $f(v) = 2n + 7 + f(u)$

$\ge 2n + 8.$

Thus, in both subcases, we get a contradiction, since, by definition, $f(v) \le 2n + 7$.

Hence, $G^*$is not a Near Skolem Difference Mean graph.

Hence, $E^+(G) = 4$.

III. CONCLUSION

In this paper we have investigated and concluded that a Near Skolem difference mean graph with $p = q - 2$, $q - 1, q \ and \ q + 1$ becomes non-Near Skolem difference mean graph for $E^+(G) = 1, 2, 3 \ and$ 4 respectively.

REFERENCES

[1] Harary. F, Graph Theory, Narosa Publishing House, New Delhi, (2001).

[2] Gallian. J. A., A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 15(2008), #DS6.

[3] Shenbaga Devi. S, Nagarajan. A, Near Skolem Difference Mean Labeling of cycle related Graphs, International Journal for Science and Advance Research in Technology, December 2017, Volume 3 Issue 12, pages 1037-1042.

[4] Shenbaga Devi. S, Nagarajan. A, Near Skolem Difference Mean labeling of some special types of trees, International Journal of Mathematics Trends and Technology, December 2017, Volume 52 Number 7, pages 474-478.

[5] Shenbaga Devi. S, Nagarajan. A, On Changing behavior of vertices of some graphs, International Journal of Scientific Research in Science, Engineering and Technology, January 2018, Volume 4 Issue 1, pages 400-405.

[6] Shenbaga Devi. S, Nagarajan. A, On Near Skolem Difference Mean Graphs, International Journal of Mathematical Archieve, February-2018, Volume 9, Issue 2, pages 29-36.

[7] Shenbaga Devi. S, Nagarajan. A, Further results on Near Skolem Difference Mean graphs, Journal of Computer and Mathematical Sciences, February 2018, Volume 9, Issue 2.

[8] Shenbaga Devi. S, Nagarajan. A, On Duplication of Near Skolem Difference Mean graph P_n. (Communicated)

[9] Shenbaga Devi. S, Nagarajan. A, Some Results on Duplication of Near Skolem Difference Mean graph C_n. (Communicated)

[10] Shenbaga Devi. S, Nagarajan. A, Near Skolem Difference Mean Labeling of some Subdivided graphs. (Communicated)

[11] Shenbaga Devi. S, Nagarajan. A, On Changing behavior of edges of some special classes of graphs II (Communicated)