Vibration Analysis of Internal Combustion Petrol Engine By Mathematical Model

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Abstract- *Any motion that repeats itself after an interval of time is vibration. In general, vibration on vehicle engine occurs due to over load, reciprocating unbalanced mass, rough ride, improper maintenance, broken mounts, over speed, faulty timing belt, etc. This results in loosening of chain sprocket, requires frequent maintenance, affects ride behavior of the vehicle and overall performance as well as, it produces noise and transmits vibration to the rider. To investigate this vibration in economical bike engine, a mathematical model of reciprocating unbalanced mass and rough road condition in harmonic manner has been considered. In this method, the equation of motion for the coupled system of piston, connecting rod, crank and suspension system has been derived with two degrees of freedom. Material properties of engine components has been selected for standard bike HERO SPLENDOR PRO. The vibratory response of the engine for various speeds and road conditions have been determined with Matlab Software. From this, response of the system with respect to vibration has been analyzed using different material properties and the vibration parameters are plotted. The better vibration withstanding material combination for the selected bike was found.*

Keywords- Economical bike, Engine,Matlab, Vibration.

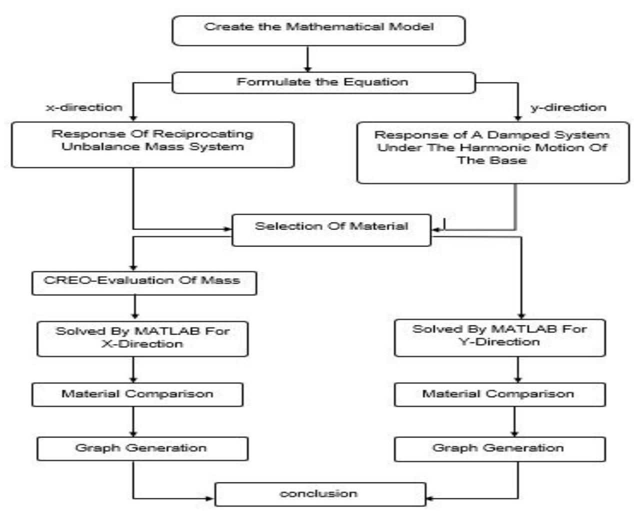
I. INTRODUCTION

A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper). This system involves the transfer of its potential energy to kinetic energy and of kinetic energy to potential energy, alternately. It has been classified based on force acting on the system, energy dissipation on the system, excitation with respect to time, etc.Some of them are free and forced vibration, damped and undamped vibration, deterministic and random vibration. In this proposed work forced and damped vibrations are considered.

It usually depends on the bike you're riding to some extent, but here are the major reasons-the engine is not sitting properly, servicing time has been long overdue, engine oil might be less; engine not running smoothly, the suspensions are not tuned correctly, the handlebar might have gotten loose due to rough usage, the air pressure might be low.

Periyasamy and Alwarsamy , 2013 have studied the engine block displacement with respect to vibration of an internal combustion engine in the radial direction due to combustion force and inertia forces.Dr. Nasir H. Abdul Hussain and Muthanna L. Abdullah in, 1999 presented an analytical model of the pressure force and vibratory response of the cylinder induced by the piston movement of compression ignition engine. In this method, the equation of motion for the coupled system of piston and cylinder is derived, taking account of three – degree of freedom system of the piston to simulate accurately time of the pressure force and vibratory response. In this proposed work , a mathematical model of reciprocating unbalanced mass and rough road condition in harmonic manner has been considered to measure the vibration amplitude for an IC petrol engine.

II. METHODOLOGY



III. MATHEMATICAL FORMULATION

Figure 1 shows the vibration model for reciprocating unbalance system and rough road condition of four stroke petrol engine.

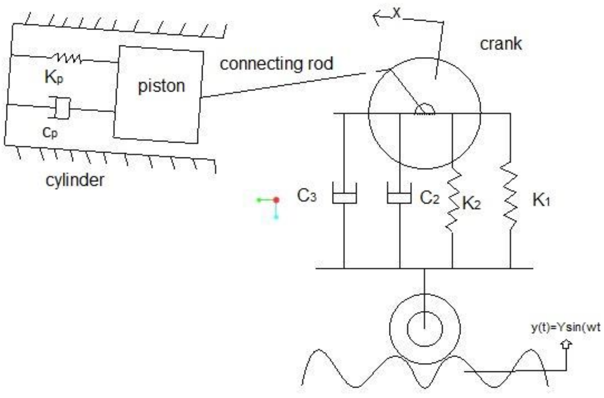


Figure 1Mathematical Model of IC Engine

The general form of forced vibration under damping is,

$m\ddot{x}+c\dot{x}+kx=F(t)$

Where,

m =mass of the system

c =damping ratio,

K =Stiffness of the system,

$\ddot{x}$= acceleration

$\dot{x}$ =velocity

x=displacement

This analysis had two response system. They are,

1.RESPONSE OF A RECIPROCATING UNBALANCE SYSTEM IN X-DIRECTION[19]

The governing equation hormonic response of the system is

$$m\ddot{x}+c\dot{x}+kx=F*\sin\omega t$$

Considering a reciprocating engine as shown in FIGURE let the equivalent mass of the reciprocating part be m_0 and the total mass of the engine including reciprocating part is m_eq. The crank length and the connecting rod length are e and l respectively.

Inertia force due to reciprocating mass m_0 is approximately equal to

$$F = m_0 e\omega^2[\sin\omega t + \left(\frac{e}{l}\right)\sin 2\omega t]$$

If (e/l) is small, second harmonic of the above equation part can be neglected and the exciting force becomes equal to $m_0 e\omega^2 * \sin\omega t$which is the same as that for rotating unbalance.

The require equation for our study

$$m_{eq}\ddot{x} + c_p\dot{x} + k_p x = m_0 e\omega^2 * \sin\omega t$$

,The solution of this system,

$$x(t)=x_c + x_p$$

where,

$x_c = A_1 e^{-\zeta\omega_n t}(\cos\omega_d t + \phi_1)$

$\omega_d = \sqrt{1-\zeta^2}\omega_n$

x_p= A sin ($\omega t - \phi$)

$A = \frac{F}{\sqrt{(K_p - m_{eq}\omega^2)^2 + (c_p\omega)^2}}$;

$\phi = \tan^{-1}\left[\frac{2\zeta r}{1-r^2}\right]$

2.RESPONSEOF A DAMPED SYSTEM UNDER THE HARMONIC MOTION OF THE BASE IN Y-DIRECTION[18]

During ride on rough road the wheel of a spring-mass-damper system undergoes harmonic motion, as shown in FIGURE. Let y(t) denote the displacement of the wheel and x(t) the displacement of the vehicle from its static equilibrium position at time t.

The governing equation for this response is

$m\ddot{x}+c\dot{x}+kx= A\sin(\omega t - \alpha)$

The required equation for our study is

$M\ddot{y} + C_{eq}\dot{y} + k_{sus}y = A\sin(\omega t - \alpha)$

Here,

A=Y$\sqrt{K_{sus}^2 + (c\omega)^2}$ $\alpha = \tan^{-1}(-\frac{C_{eq}\omega}{K_{sus}})$

$K_{sus} = K_1 + K_2$ $C_{eq} = C_1 + C_2$

The solution of this equation is,

x(t)=X sin($\omega t - \phi$)

Where,

X=Y * $[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}]^{1\backslash 2}$; $\zeta = \frac{C_{eq}}{C_c}$

$\phi = \tan^{-1}\frac{2\zeta r^3}{1+(4\zeta^2-1)r^2}$; $r = \frac{\omega}{\omega_n}$

The ratio of the amplitude of the response x(t), to that of the base motion y(t), is called the displacement transmissibility Td =X/Y.

IV. RESULT AND DISCUSSION

1.INERTIA DUE TO RECIPROCATING UNBALANCE

1.1.CALCULATION

1.Natural frequency $\omega_n=(2*\pi*N_p)/60$;
Where, $N_p=8000$ rpm, $\omega_n=837.75$ rad/s

2 .$\omega=(2*\pi*N)/60$
Where,
N={Speed of kmph,the tyre rpm }*Prim Red*Final Red*Top gear ratio*v
N=11.6*3.72*3.07*0.95*v

The values various angular velocity for different speeds are shown in Table 1

Table 1- Values of ω for different vehicle speed

s.no	v(km/hr)	N(rpm)	ω (rad/sec)
1	20	2517.05	263.55
2	40	5034.11	527.168
3	60	7551.16	790.74
4	80	10068.22	1059.3

3. $\zeta=\zeta_p+\zeta_{cd}+\zeta_{cr}$
The damping factor values were refered. [17]

1.2. EVALUATION OF MASSES

Using CREO software the model of piston connecting rod and crank is drawn, then mass was found out using mass properties option from given density.

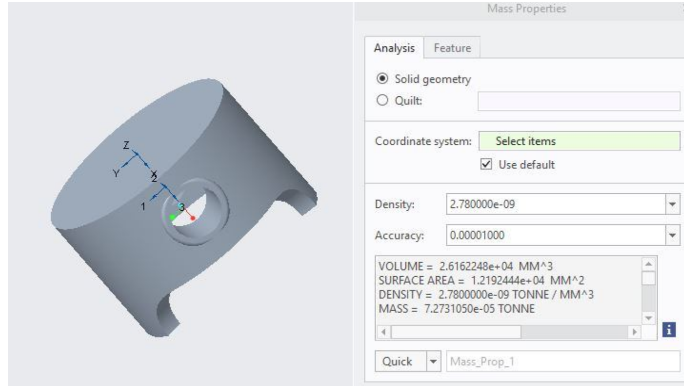


Figure 2 CREO model of piston

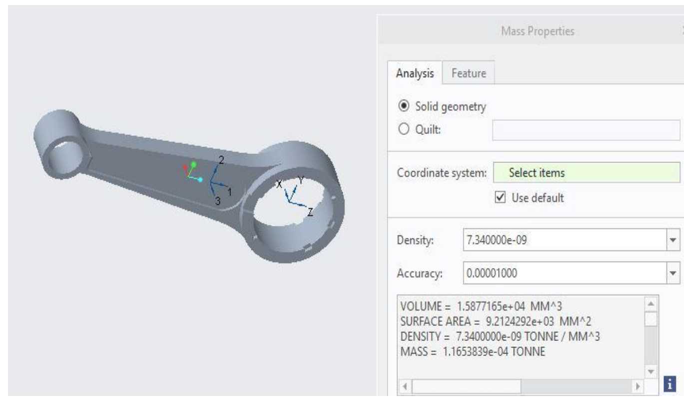


Figure 3 CREO model of connecting rod

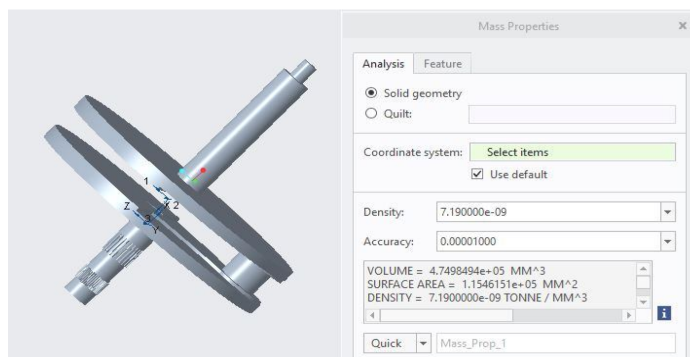


Figure 4 CREO model of crank

Table 2 Mass of the existing material

Component	Piston	Connecting Rod	Crank Shaft
Material	Al-2024-T6	Grey Cast Iron	Alloy Steel 41cr4
Density(kg/m^3)	2780	7340	7190
Mass(gram)	72.73	116.53	3415

Table 3 Mass of the proposed material 1

Component	Piston	Connecting Rod	Crank Shaft
Material	Grey Cast Iron	Stainless steel grade 304	Titanium alloy Ti-6Al-4V
Density(kg/m^3)	7340	8060	4420
Mass(g)	188	123.8	2099

Table 4 Mass of the proposed material 2

Component	Piston	Connecting Rod	Crank Shaft
Material	SILUMIN(Al-Si-Mg)	Al 360 alloy	Ni-200
Density(kg/m^3)	2659	2680	8890
Mass(g)	69.56	42.869	3390

Table 5 Mass and Damping coefficient of each combination

parameters	Existing Material	Preferred Material 1	Preferred Material 2
m_o	0.1843	0.3118	0.11236
ζ	0.0031	0.0076	0.0108

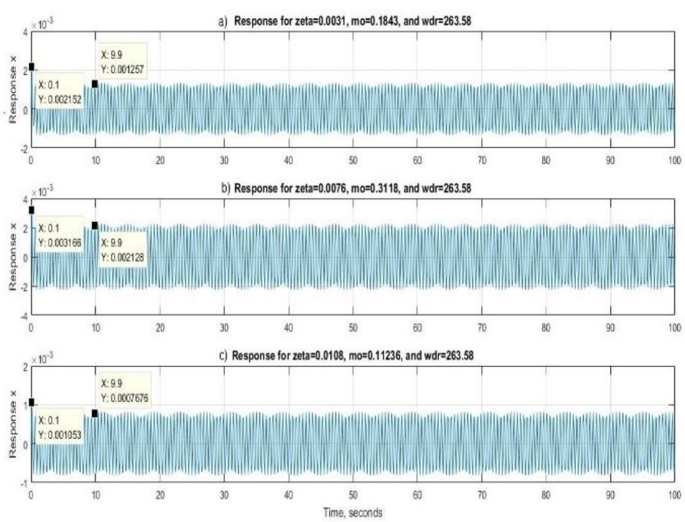


Figure 5 a) Existing Material; d) Proposed Material 1; c) Proposed Material 2

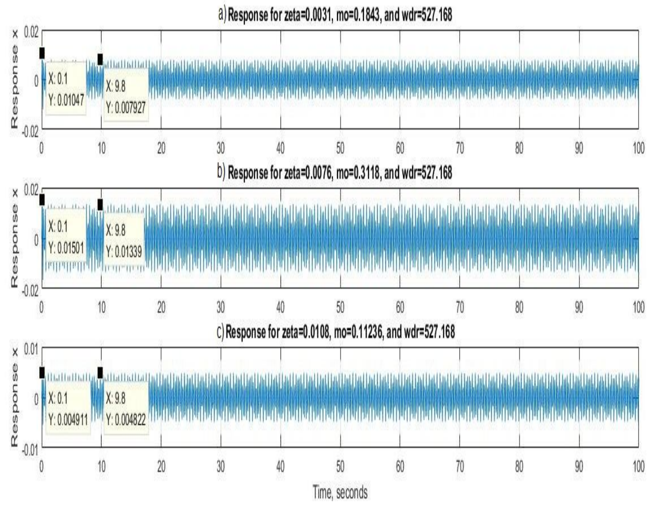


Figure 6 a)Existing Material; d) Proposed Material 1; c) Proposed Material 2

a) SPEED OF THE VEHICLE=20 km/hr

Figure 5 shows that the response of the system with respect to time for different ζ value respect to masses at 20 km/hr

b) SPEED OF THE VEHICLE=40 km/hr

Figure 6 shows that the response of the system with respect to time for different ζ value respect to masses at 40 km/hr

c) SPEED OF THE VEHICLE=60 km/hr

Figure 7 shows that the response of the system with respect to time for different ζ value respect to masses at 60 km/hr

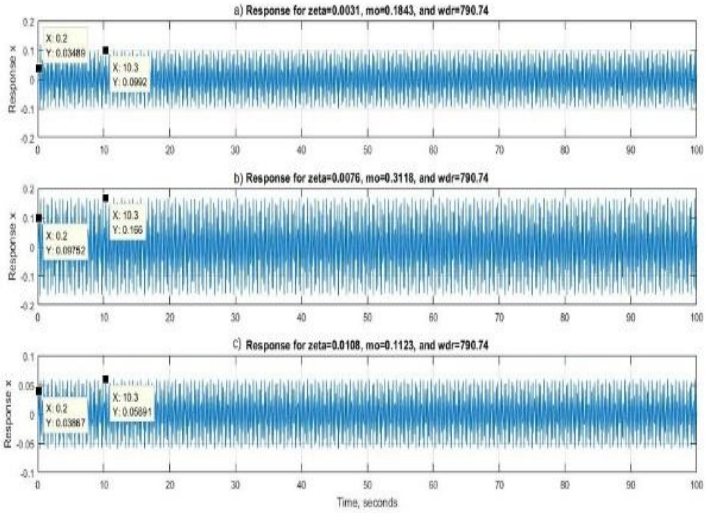


Figure 7a)Existing Material; d) Proposed Material 1; c) Proposed Material 2

1.3. RESULT

- Table 6 shows that the Maximum amplitude response for different material combinations with respect to vehicle speed.
- Table 7 shows that the % of amplitude reduction for Proposed Material 2 with respect to speed of the vehicle and 20 km/hr speed has highest % of amplitude reduction.
- Table 8 shows that the % of amplitude reduction for Proposed Material 2 with respect to speed of the vehicle and 40 km/hr speed has highest % of amplitude reduction.

Table 6 Maximum amplitude response for various speed

Speed (km/hr)	Maximum amplitude Response(m)		
	Existing Material	Proposed Material 1	Proposed Material 2
20	$0.002152*10^{-3}$	$0.003166*10^{-3}$	$0.001053*10^{-3}$
40	0.01047	0.01503	0.004911
60	0.0992	0.166	0.0589

Table 7 % of amplitude reduction for Proposed Material 1

Speed (km/hr)	Maximum amplitude response (m)		% of amplitude reduction
	Existing Material	Proposed Material 1	
20	$0.002152*10^{-3}$	$0.003166*10^{-3}$	-0.32
40	0.01047	0.01503	-0.43
60	0.0992	0.166	-0.67

Note: -ve sign indicate the increase of amplitude.

Table 8 % of amplitude reduction for Proposed Material 2

Speed (km/hr)	Maximum Response (m)		% in amplitude reduction
	Existing Material	Proposed Material 2	
20	$0.002152*10^{-3}$	$0.001053*10^{-3}$	0.5
40	0.01047	0.004911	0.53
60	0.0992	0.0589	0.406

1.4. DISCUSSION

From Table 8 it is shown that Proposed Material 2 has higher % of amplitude reduction for reciprocating unbalance in harmonic motion. Because of its higher stiffness/weight ratio and higher structural damping coefficient. It helps to minimize the valve overlapping, chain sprocket loosening and mileage drop.

2.RESPONSE OF A DAMPED SYSTEM UNDER THE HARMONIC MOTION OF THE BASE

Table 9 shows that the materials used in vehicle suspension

Table 9 Material selection

components	Existing material	Material combination 1	Material combination 2
Suspension coil spring	Carbon Steels C1074	Chrome Vanadium 50CrV4	Spring steel ASTM A227
Suspension oil	Oil 15w -40 (viscous damping coefficient 0.874)		

2.1 CALCULATION

1. $\omega_n = \sqrt{\frac{K_{sus}}{M}}$;
2. $K_{sus} = 2 * K * \cos\theta$; $K = \frac{G*d}{8c^3 n}$; $c = \frac{D}{d}$;
3. $\zeta_{eq} = 2 * \zeta_{sus} + \zeta_{oil}$;
4. $\omega = 2 * \pi \left(\frac{v*1000}{3600}\right)\left(\frac{1}{\lambda}\right)$

Table 10 shows that the property of selected Material from journals.

Table 10 Property of selected Material

Property	Spring steel ASTM A227	Chrome Vanadium 50CrV4	Carbon Steels C1074
Structural damping coefficient(ζ)	0.0016	0.00015	0.0018
Density	7700	8030	7900
Torsional rigiditymodulus(G)(N/mm^2)	78600	82677.16	74210

Table 11 shows that the value of ω_n, ω ,$K_{sus}$and ζ_{eq} from calculation.

Table 11Values from calculation

parameters	Existing Material	Preferred Material 1	Preferred Material 2
ω_n(rad/s) (for 2 person)	15.016	15.85	15.45
ω_n(rad/s) (for 1 person)	17.599	18.58	18.11
K_{sus}(N/m)	53893.488	60075.95	57111.53
ζ_{eq}	0.8776	0.8743	0.8772

2.2. VEHICLE MASS CONSIDER WITH SINGLE PERSON

a) FOR EXSISTING MATERIAL

Figure 8shows that the displacement transmissibility of different speed and maximum T_d is occurred at 52 km/hr.

b) FOR PROPOSED MATERIAL 1

Figure 9 shows that the displacement transmissibility of different speed and maximum T_d is occurred at 65 km/hr.

c) FOR PROPOSED MATERIAL 2

Figure 10 shows that the displacement transmissibility of different speed and maximum T_d is occurred at 61 km/hr.

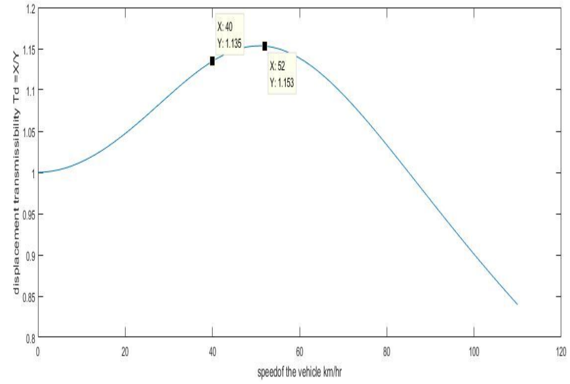


Figure 8 Speed (km/hr) vs displacement transmissibility(X/Y)

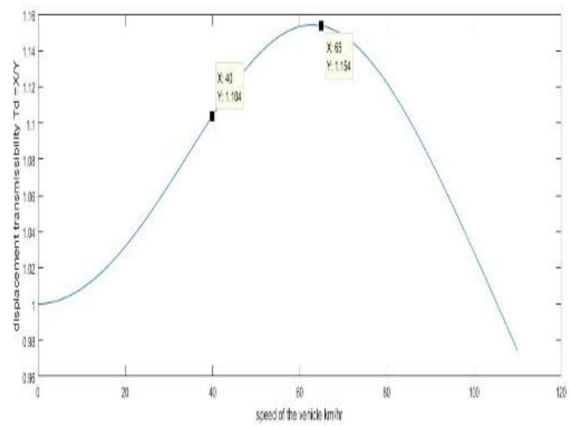


Figure 9Speed (km/hr) vs displacement transmissibility(X/Y)

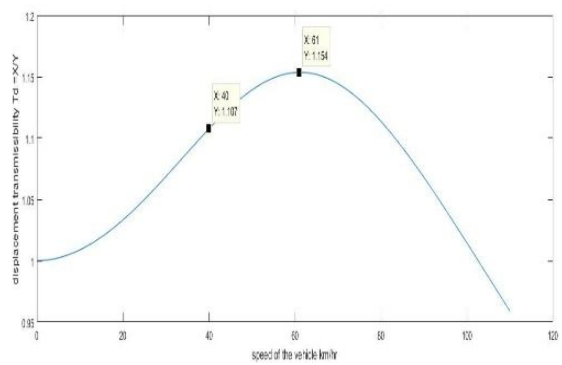


Figure 10 Speed (km/hr) vs displacement transmissibility(X/Y)

2.3. VEHICLE MASS CONSIDER WITH TWO PERSON

a) FOR EXSISTING MATERIAL

Figure 11 shows that the displacement transmissibility of different speed and maximum T_d is occurred at 55 km/hr.

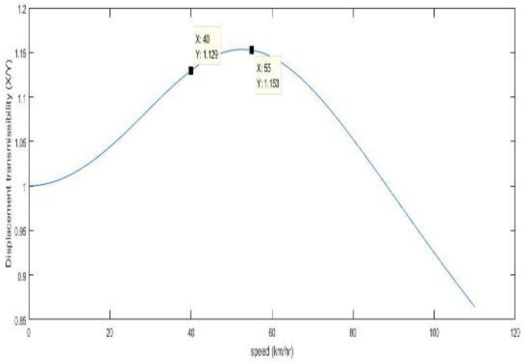


Figure 11 speed (km/hr) vs displacement transmissibility (X/Y)

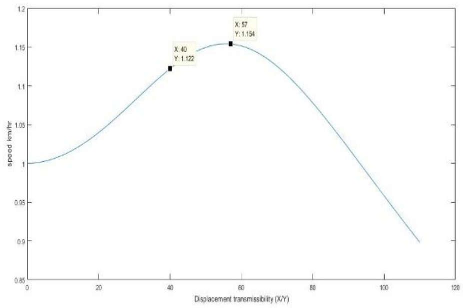


Figure 12 speed (km/hr) vs displacement transmissibility (X/Y)

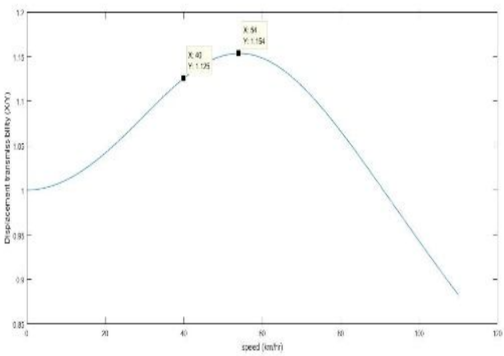


Figure 13 speed (km/hr) vs displacement transmissibility (X/Y)

b) FOR PROPOSED MATERIAL 1

Figure 12 shows that the displacement transmissibility of different speed and maximum T_d is occurred at 57 km/hr.

c) FOR PROPOSED MATERIAL 2

Figure 13 shows that the displacement transmissibility of different speed and maximum T_d is occurred at 54 km/hr.

2.4RESULT

Table 12 shows that the % of Td reduction for different load conditions at 40 km/hr. Proposed Material 1 is higher reduction value for single person load condition

Table12 % of Td reduction for different load conditions

Load condition	Displacement transmissibility (X/Y) at 40 km/hr			% of T_d reduction for Proposed Material 1	% of T_d reduction for Proposed Material 2
	Existing Material	Proposed Material 1	Proposed Material 2		
Single person	1.135	1.104	1.107	0.027	0.024
Double person	1.129	1.122	1.125	0.0062	0.00354

2.5 DISCUSSION

From the Table 12. Proposed Material 1 has higher % of Td reduction at the speed of 40 km/hr for single person load condition. Because of its higher structural damping ratio and stiffness. It helps in ergonomic design consideration for rough road condition.

V. CONCLUSION

The mathematical model developed for reciprocating unbalanced and rough road conditions. The mathematical model was solved by Matlab. The Matlab results and graphs were drawn. The comparative study on material property was considered to show the reduction in damping. The proposed material for the conditions are given below.

Table 13 Inertia Force due to reciprocating unbalance

Component	Existing material	Proposed material	% in amplitude reduction
Piston	Al-2024-T6	SILUMIN(Al-Si-Mg)	0.53 at 40 km/hr
Connecting rod	Grey Cast Iron	Al 360 alloy	
Crank shaft	Alloy Steel 41cr4	Ni-200	

Table 14 Base Excitation due to rough road condition

Component	Existing material	Proposed material	% in amplitude reduction
Suspension Spring	Carbon Steels C1074	Spring steel ASTM A227	0.027 at 40 km/hr, single person load condition

Based on the Table 13 and 14, it is seen that using the proposed material, the vibration is reduced up to 0.577 % at a speed of 40 km/hr by the combined effect inertia force and base excitation.

In future this project would be tested on experimental and dynamic approach

APPENDIX

Nomenclature and values used for numerical analysis

m_o	Reciprocating Mass (mass of piston and comnnecting rod)
$\mathbf{m_{eq}}$	Equilibrium mass of connecting rod, crank, Piston (Kg)
$\mathbf{c_p}$	Damping capacity of piston, connecting road, crank system (Ns/m)
$\mathbf{k_p}$	Structural Stiffness of piston, connecting road, crank system (N/m)
x (t)	Amplitude Response (m)
$\mathbf{x_c}$	Compound factor amplitude response (m)
$\mathbf{x_p}$	Particular Integral (m)
$\boldsymbol{\omega_d}$	Natural frequency of damped system (rad / sec)
ζ	Structural Damping Co-efficient
A	Maximum Amplitude Reduction (m)
Φ	Phase angle difference (Degree)
M	Total mass of the vehicle (Kg)
$\mathbf{C_{eq}}$	Equivalent damping capacity (N s /m)
$\mathbf{k_{sus}}$	Equivalent stiffness of suspension (N/m)
$\mathbf{A_{sus}}$	Maximum Amplitude of suspension (m)
X (t)	Amplitude Response of reciprocating unbalance system (m)
Y (t)	Amplitude Response of base excitation (m)
r	Resonance
K	Stiffness of Suspension (N/m)
θ	Mounting angle between suspension frame and vertical axis (Degree)
G	Torsional Modulus of elasticity (GPa)
d	wire diameter of coil spring (m)
c	Spring Index (D/d)
n	no of coil in spring (13)
$\boldsymbol{\zeta_{oil}}$	Viscous Damping coefficient of 15w 40 oil (0.8743)

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