A Comparative Study on Regular And Edge Regular Fuzzy Graph

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Abstract- *In this paper, regular fuzzy graph and edge regular fuzzy graph are discussed on the various aspects. The regular fuzzy graph and edge regular fuzzy graph are compared through various examples are provided. Many procedures have been established during the past few decades yet there is need of procedure to compare the regular and edge regular fuzzy graph.*

Keywords- degree of a vertex, edge, totally regular,totally edge regular.

I. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Now, fuzzy graphs have been witnessing a tremendous growth and finds application in many branches of engineering and technology. NagoorGani and Ratha introduced the concept of regular fuzzy graphs in 2008. Ratha and Kumaravel introduced the concept of edge degree, total edge degree and discussed about the degree of an edge in some fuzzy graphs.In this paper,regular and edge regular fuzzy graph are studied.We make a comparative study between regular and edge regular fuzzy graph.

II. PRELIMINARIES

Some important definitions are given below.

DEFINITION: 2.1

Let V be a finite non empty set. The triplet G = (V, σ, μ) is called a **fuzzy graph** on V, where σ and μ are fuzzy sets on V and $E(V \times V)$ respectively such that $\mu(a,b) \leq \min\{\sigma(a),\sigma(b)\}$ for all $a,b \in$ V.

EXAMPLE a(0.5)

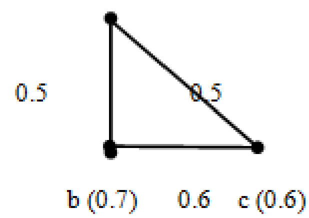

Figure 2.1: G = (V, σ, μ)

DEFINITION: 2.2

A fuzzy graph G is **complete** if $\mu(ab)$ = $\sigma(a) \wedge \sigma(b)$ for all a,b$\in$ V, where ab denotes the edge between a and b.

DEFINITION: 2.3

Let G: (σ ,μ) be a fuzzy graph on G^* : (V,E) .The **degree of a vertex** a is $d_G(a) = \sum_{a \neq b} \mu(ab)$. The minimum degree of G is $\delta(G) = \wedge\{ d_G(v), \forall v \in V\}$ and the maximum degree of G is $\Delta(G) = \vee\{d_G(v), \forall v \in V\}$.

DEFINITION: 2.4

Let G: (σ,μ) be a fuzzy graph on G^*:(V,E).The **total degree of a vertex** a$\in$ V is defined by $td_G(a) = \sum_{a \neq b} \mu(ab) + \sigma(a)$.

DEFINITION: 2.5

Let G : (σ,μ) be a fuzzy graph on G^* : (V,E). The **degree of an edge** ab is

$$d_G(ab) = d_G(a) + d_G(b) - 2\mu(ab)$$

This is equivalent to $(ab) = \sum_{\substack{ac \in E \\ c \neq b}} \mu(ac) + \sum_{\substack{ac \in E \\ c \neq a}} \mu(cb)$

$$= \sum_{ac \in E} \mu(ac) + \sum_{cb \in E} \mu(cb) - 2\mu(ab).$$

The minimum edge degree of G is $\delta_E(G) = \wedge\{ d_G(ab), \forall ab \in E \}$

The maximum edge degree of G is $\Delta_E(G) = \vee\{ d_G(ab), \forall ab \in E \}$.

DEFINITION: 2.6

Let G: (σ,μ) be a graph on G^*: (V,E) .The **total degree of an edge** ab $\in$ E is defined by

$$td_G(ab) = d_G(a) + d_G(b) - \mu(ab).$$

This is equivalent to

$$td_G(ab) = \sum_{\substack{ac\in E\\ c\neq b}} \mu(ac) + \sum_{\substack{cb\in E\\ c\neq a}} \mu(cb) + \mu(ab) = d_G(ab) + \mu(ab)$$

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DEFINITION: 2.7

Let G: (σ,μ) be a fuzzy graph on G^* : (V,E) . If each vertex in G has same degree.Then G is said to be a **regular fuzzy graph** or n – regular fuzzy graph.

DEFINITION: 2.8

LetG: (σ,μ) be a fuzzy graph on G^* : (V,E) . If each vertexin G has same total degree n. Then G is said to be a **totally regular fuzzy graph** or n – totally regular fuzzy graph

Example: 2.8

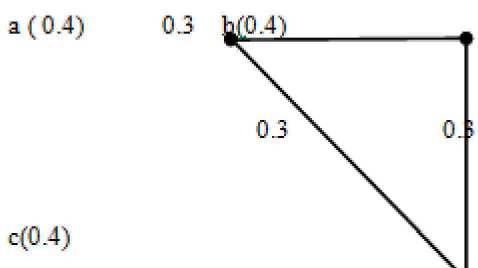


Figure 2.2: G^* : (V,E)

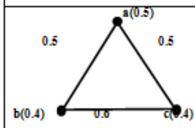
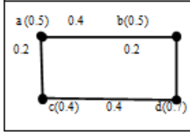
$d_G(a) = 0.6$, $d_G(b) = 0.6$, $d_G(c) = 0.6$ $td_G(a) = 1.0$, $td_G(b) = 1.0$, $td_G(c) = 1.0$

Here, G is a regular fuzzy graph. Also G is a totally regular fuzzy graph.

III. COMPARISON BETWEEN REGULAR AND EDGE REGULAR FUZZY GRAPH

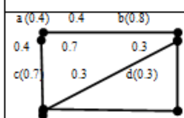
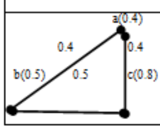
The vertex and edge regular fuzzy graphs are compared and tabulated.Any complete graph is regular. But this result does not carry over to the fuzzy case. A complete fuzzy graph need not be regular.

Table 1: The vertex and edge regular fuzzy graphs are compared

Fuzzy Graphs	Regular (vertex)	Edge regular (edge)	Totally regular (vertex)	Totally edge regular (edge)
	$d_G(a) \neq d_G(b)$ $1 \neq 1.1$ *G is not regular*	$d_G(ab) \neq d_G(bc)$ $1.0 \neq 1.1$ *G is not* Edge *regular*	$td_G(a) = td_G(b)$ $1.5 = 1.5$ ***G is totally regular***	$td_G(ab) = td_G(bc)$ $1.6 = 1.6$ ***G is totally edge regular***
	$d_G(a) = d_G(b)$ 0.6=0.6 ***G is regular***	$d_G(ac) \neq d_G(ab)$ $0.8 \neq 0.4$ *G is not* Edge *regular*	$td_G(a) \neq td_G(c)$ $1.1 \neq 1.0$ *G is not totally regular*	$td_G(ac) \neq td_G(ab)$ $1.0 \neq 0.8$ *G is not totally edge regular*

Any complete graph is regular. But this result does not carry over to the fuzzy case. A complete fuzzy graph need not be an edge regular.

Table 2

Fuzzy Graphs	Regular (vertex)	Edge regular (edge)	Totally regular (vertex)	Totally edge regular(edge)
	$d_G(a) \neq d_G(c)$ $0.8 \neq 1.4$ *G is not regular*	$d_G(ab) = d_G(bc)$ 1.4=1.4 ***G is* Edge *regular***	$td_G(a) \neq td_G(b)$ $1.2 \neq 2.2$ *G is not totally regular*	$td_G(ab) \neq td_G(bd)$ $1.8 \neq 1.7$ *G is not totallyedge regular*
	$d_G(a) \neq d_G(b)$ $0.8 \neq 0.9$ *G is not regular*	$d_G(ab) \neq d_G(bc)$ $0.9 \neq 0.8$ *G is not* Edge *regular*	$td_G(a) \neq td_G(b)$ $1.2 \neq 1.4$ *G is not totally regular*	$td_G(ab) = td_G(bc)$ 1.3=1.3 ***G is totally* edge *regular***

Let G: (σ,μ) be a fuzzy graph on G^*: (V,E). If μ is a constant function if and only if the following are equivalent:

1. G is a regular,
2. G is edge regular,
3. G is a totally edge regular.

Table 3

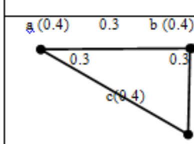
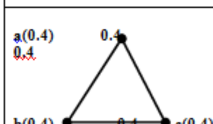
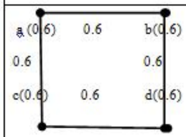
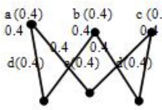
Fuzzy Graphs	Regular (vertex)	Edge regular (edge)	Totally regular (vertex)	Totally edge regular(edge)
	$d_G(v) = 0.6$ *for all* $v \in V$ 0.6	$d_G(uv) = 0.6$ *for all* $uv \in E$ 0.6	$td_G(v) = 1.0$ *for all* $v \in V$ 1.0	$td_G(uv) = 1.0$ *for all* $uv \in E$ 1.0
	$d_G(v) = 0.8$ *for all* $v \in V$ 0.8	$d_G(uv) = 0.8$ *for all* $uv \in E$ 0.8	$td_G(v) = 1.2$ *for all* $v \in V$ 1.2	$td_G(uv) = 1.2$ *for all* $uv \in E$ 1.2

Table 4

Fuzzy Graphs	Regular (vertex)	Edge regular (edge)	Totally regular (vertex)	Totally edge regular (edge)
	$d_c(v) = 1.2$ *for all* $v \in V$ 1.2	$d_c(uv) = 1.2$ *for all* $uv \in E$ 1.2	$td_c(v) = 1.8$ *for all* $v \in V$ 1.8	$td_c(uv) = 1.8$ *for all* $uv \in E$ 1.8
	$d_c(v) = 0.8$ *for all* $v \in V$ 0.8	$d_c(uv) = 0.8$ *for all* $uv \in E$ 0.8	$td_c(v) = 1.2$ *for all* $v \in V$ 1.2	$td_c(uv) = 1.2$ *for all* $uv \in E$ 1.2

IV. CONCLUSION

In this paper, we made a fuzzy graph theory one of the concepts is based on regular fuzzy graph and another side is based on edge regular fuzzy graph. In this regards a diagram is satisfied the both vertex and edges fuzzy graph.It is useful to new research of fuzzy graph theory and also we need to improve the concept of fuzzy graph theory in future.

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