

Various Graph Terminologies

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Abstract- A Graph is basically a collection of vertices (also called nodes) and edges that connect these vertices. A vertex can be connected to any number of vertices means instead of one to one relationship there exist many to many relationship.

Keywords- Directed Graph, Weighted graph, undirected graph

I. INTRODUCTION

A graph G is defined as an ordered set (V, E) , where $V(G)$ represents the set of vertices and $E(G)$ represents the edges that connect these vertices. Figure. A graph can be directed or undirected. In an undirected graph, edges do not have any direction associated with them. That is, if an edge is drawn between nodes A and B , then the nodes can be traversed from A to B as well as from B to A . In a directed graph, edges form an ordered pair. If there is an edge from A to B , then there is a path from A to B but not from B to A . The edge (A, B) is said to initiate from node A (also known as initial node) and terminate at node B (terminal node)

II. GRAPH TERMINOLOGY

Adjacent nodes or neighbours: For every edge, $e = (u, v)$ that connects nodes u and v , the nodes u and v are the end-points and are said to be the adjacent nodes or neighbours.
Degree of a node Degree of a node u , $\text{deg}(u)$, is the total number of edges containing the node u . If $\text{deg}(u) = 0$, it means that u does not belong to any edge and such a node is known as an isolated node.

Regular graph It is a graph where each vertex has the same number of neighbours. That is, every node has the same degree. A regular graph with vertices of degree k is called a k -regular graph or a regular graph of degree k .

Path A path P written as $P = \{v_0, v_1, v_2, \dots, v_n\}$, of length n from a node u to v is defined as a sequence of $(n+1)$ nodes.

Closed path A path P is known as a closed path if the edge has the same end-points.

Simple path A path P is known as a simple path if all the nodes in the path are distinct with an exception that v_0 may be equal to v_n . If $v_0 = v_n$, then the path is called a closed simple path.

Cycle A path in which the first and the last vertices are same. A simple cycle has no repeated edges or vertices (except the first and last vertices).

Connected graph A graph is said to be connected if for any two vertices (u, v) in V there is a path from u to v . That is to say that there are no isolated nodes in a connected graph. A connected graph that does not have any cycle is called a tree.

Complete graph A graph G is said to be complete if all its nodes are fully connected. That is, there is a path from one node to every other node in the graph. A complete graph has $n(n-1)/2$ edges, where n is the number of nodes in G .

Labelled graph or weighted graph A graph is said to be labelled if every edge in the graph is assigned some data. In a weighted graph, the edges of the graph are assigned some weight or length. The weight of an edge denoted by $w(e)$ is a positive value which indicates the cost of traversing the edge. Figure 13.4(c) shows a weighted graph.

Multiple edges Distinct edges which connect the same end-points are called multiple edges. That is, $e = (u, v)$ and $e' = (u, v)$ are known as multiple edges of G .

Loop An edge that has identical end-points is called a loop. That is, $e = (u, u)$.

Multi-graph A graph with multiple edges and/or loops is called a multi-graph. shows a multi-graph.

Size of a graph The size of a graph is the total number of edges in it.

III. TERMINOLOGY OF A DIRECTED GRAPH

Out-degree of a node The out-degree of a node u , written as $\text{outdeg}(u)$, is the number of edges that originate at u .

In-degree of a node The in-degree of a node u , written as $\text{indeg}(u)$, is the number of edges that terminate at u .

Degree of a node The degree of a node, written as $\text{deg}(u)$, is equal to the sum of in-degree and out-degree of that node.

Therefore, $\text{deg}(u) = \text{indeg}(u) + \text{outdeg}(u)$.

Isolated vertex A vertex with degree zero. Such a vertex is not an end-point of any edge.

Pendant vertex (also known as leaf vertex) A vertex with degree one.

Cut vertex A vertex which when deleted would disconnect the remaining graph.

Source A node u is known as a source if it has a positive out-degree but a zero in-degree.

Sink A node u is known as a sink if it has a positive in-degree but a zero out-degree.

Reachability A node v is said to be reachable from node u , if and only if there exists a (directed) path from node u to node v .

you will observe that node D is reachable from node A . **Strongly connected directed graph** A digraph is said to be strongly connected if and only if there exists a path between every pair of nodes in G . That is, if there is a path from node u to v , then there must be a path from node v to u .

Unilaterally connected graph A digraph is said to be unilaterally connected if there exists a path between any pair of nodes u, v in G such that there is a path from u to v or a path from v to u , but not both.

Weakly connected digraph A directed graph is said to be weakly connected if it is connected by ignoring the direction of edges. That is, in such a graph, it is possible to reach any node from any other node by traversing edges in any direction (may not be in the direction they point). The nodes in a weakly connected directed graph must have either out-degree or in-degree of at least 1.

Parallel/Multiple edges Distinct edges which connect the same end-points are called multiple edges. That is, $e = (u, v)$ and $e' = (u, v)$ are known as multiple edges of G .

Simple directed graph A directed graph G is said to be a simple directed graph if and only if it has no parallel edges. However, a simple directed graph may contain cycles

with an exception that it cannot have more than one loop at a given node.

IV. CONCLUSION

This paper gives brief introduction about various terms used in graph. These terms are useful to get the detail of any graph whether it is directed or undirected graph and concept of graph is used in various real life applications.

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