Annamalai Computing Method for Formation of Geometric Series using in Science and Technology

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Abstract- *This paper presents a technique or method for forming and computing the generalized geometric series in a new way. It will be very interesting and informative for current students and researchers.*

Keywords- ACM-Geometric series, Annamalai geometric series method

I. INTRODUCTION

Annamalai computing method (ACM) provides a novel approach for computation of geometric series in a different manner. ACM can be used in the research field of science and technology such as computational biology [1, 2], medicine [3, 4], networking and security [5, 6].

We commonly know that a geometric series is any series that can be written in the following form: $a + ar + ar^2 + ar^3 + \dots\dots$, where the common ration is r.

This series can also be written in the notation $\sum_{i=0}^{\infty} ar^i$

For Examples,

$(i)\ 5 + 5\frac{1}{2} + 5\frac{1}{2^2} + 5\frac{1}{2^3} + \dots\dots$, where $a = 5$ and $r = \frac{1}{2}$.

$(ii)\ 1 + 2 + 2^2 + 2^3 + \dots\dots$, where a = 1 and r = 2.

It is eventually understood by the presently existing books of mathematics the summation of finite geometric series is:

$$\sum_{i-0}^{n-1} ar^i = \frac{a(r^n - 1)}{(r-1)}$$

II. NOVEL COMPUTATION OF GEOMETRIC SERIES

This new ideas can help students to form the geometric series and computing it explicitly.

$$(i)\ 2 = 2 \Rightarrow 2 = 1 + 1 \Rightarrow 2 = 1 + \frac{1}{2} + \frac{1}{2} \Rightarrow 2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$$

i.e, $2 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots.. + \frac{1}{2^{n-1}} + \frac{1}{2^{n-1}} \Rightarrow \sum_{i=0}^{n-1} \frac{1}{2^i} = \frac{1-2^n}{2^{n-1}}$ [1, 2]

$$(ii)\ 2^n = 2^n \Rightarrow 2^n = 2^{n-1} + 2^{n-1} \Rightarrow 2^n = 2^{n-1} + 2^{n-2} + 2^{n-3} + \dots. + 2^k + 2^k \Rightarrow \sum_{i=k}^{n-1} 2^i = 2^n - 2^k$$ [2, 6]

If the integer ‘k’ reaches to the integer ‘0’, then

$$2^n = 2^{n-1} + 2^{n-2} + 2^{n-3} + 2^{n-4} + \dots.. + 1 + 1 \Rightarrow \sum_{i=0}^{n-1} 2^i = 2^n - 1$$ [2, 6]

(iii) Let us choose an integer ‘13’ for forming and computing the geometric series in a new way.

$$13^n = 13^n \Rightarrow 13^n = (13-1)13^{n-1} + 13^{n-1} \Rightarrow 13^n = (13-1)13^{n-1} + (13-1)13^{n-2} + 13^{n-2}$$

We can further expand the terms as follows:

$$13^n = 13^n \Rightarrow 13^n = (13-1)13^{n-1} + (13-1)13^{n-2} + (13-1)13^{n-3} + \dots.. + (13-1)13^k + 13^k$$

By simplifying we get

$$13^n = 13^n \Rightarrow \sum_{i=k}^{n-1} 13^i = \frac{13^n - 13^k}{13-1} = \frac{13^n - 13^k}{12}$$

If the integer 'k'reachesto the integer '0', then $13^n = 13^n \Leftrightarrow \sum_{i=0}^{n-1} 13^i = \frac{13^n - 1}{12}$ (for this proof we can refer to the following theorem**).**

Theorem (New Formation of Generalized Geometric Series and its Computation):

$$ar^n = ar^n \Leftrightarrow \sum_{i=k}^{n-1} ar^i = \frac{a\left(r^n - r^k\right)}{r-1}, \ (r \neq 1)$$

Proof

LHS $\Rightarrow ar^n = ar^n$

$\Rightarrow ar^n = a(r-1)r^{n-1} + ar^{n-1}$

$\Rightarrow ar^n = a(r-1)r^{n-1} + a(r-1)r^{n-2} + ... + a(r-1)r^i + ... + a(r-1)r^k + ar^k$

$$ar^n = ar^n \Rightarrow \sum_{i=k}^{n-1} ar^i = \frac{a\left(r^n - r^k\right)}{r-1} \quad \text{---- (1)}$$

RHS $\Rightarrow \sum_{i=k}^{n-1} ar^i = \frac{a\left(r^n - r^k\right)}{r-1}$

$\Rightarrow ar^n = a(r-1)r^{n-1} + a(r-1)r^{n-2} + ... + a(r-1)r^i + ... + a(r-1)r^k + ar^k$

$\Rightarrow ar^n = ar^n$

$$\sum_{i=k}^{n-1} ar^i = \frac{a\left(r^n - r^k\right)}{r-1} \Rightarrow ar^n = ar^n \quad \text{---- (2)}$$

From (1) and (2) we get:

$$ar^n = ar^n \Leftrightarrow \sum_{i=k}^{n-1} ar^i = \frac{a\left(r^n - r^k\right)}{r-1}, \ (r \neq 1)$$

We can also write this expression as

$$ar^n = ar^n \Leftrightarrow \sum_{i=0}^{n-1} ar^i = \frac{a\left(r^n - 1\right)}{r-1}, \ (r \neq 1)$$

Corollaries:

(i) $ar^n = ar^n \Leftrightarrow ar^n = a(r-1)r^{n-1} + ... + a(r-1)r + a(r-1) + a(r-1)r^{-1} + ... + a(r-1)r^{-k} + ar^{-k}$

$$ar^n = ar^n \Leftrightarrow \sum_{i=-k}^{n-1} ar^i = \frac{a\left(r^n - r^{-k}\right)}{r-1},\ (r \neq 1)$$

$$(ii)\ ar = ar \Leftrightarrow ar = a(r-1)1 + a1 \Leftrightarrow ar = (r-1)\frac{a}{r^0} + (r-1)\frac{a}{r} + (r-1)\frac{a}{r^2} + ... + (r-1)\frac{a}{r^n} + \frac{a}{r^n}$$

$$ar = ar \Leftrightarrow \sum_{1=0}^{n} \frac{a}{r^i} = \frac{a\left(1 - r^{n+1}\right)}{(1-r)r^n}$$

$$(iii)\ a\sum_{i=k}^{n-1} r^i = \frac{a\left(r^n - 1 + 1 - r^k\right)}{r-1} = \frac{a\left(r^n - 1\right)}{r-1} - \frac{a\left(r^k - 1\right)}{r-1} = a\left(\sum_{i=0}^{n-1} r^i - \sum_{i=0}^{k-1} r^i\right)$$

IV. CONCLUSION

In the research study, a novel technique has been introduced to form the generalized geometric series and computing it.

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Author's Profile

Prof. C. Annamalai has experience for more than 20 years in research and teaching of computer science and information technology and admin. He has published several papers in diverse fields of science and technology. Also, he has reviewed recent IT and Computer books published by William Stallings (USA) and many other articles published by IEEE journals and top journals in the fields of computing sciences and its related subjects. Presently he is working at Indian Institute of Technology Kharagpur.

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