Stability Analysis of E-Bike

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Abstract- *Bicycles are convenient, environmental friendly, and efficient transportation devices. But, its dynamics is not that easy. As it is a 2-wheel vehicle, inherently it is not stable system. Ever since the development of bicycle, there has been research by Engineers, Physicist to understand and to improve the dynamics of the bicycle. The complexity in the dynamics also comes through gyroscopic effect when wheel is spinning and steering of the handlebar is done. Also, newer bicycles are having more complex dynamics due to the presence of shock-absorbers at front fork as well as part of the rest of the structures too. Electric bicycles are bringing in more complexity due to presence of additional components and forces/torque applied. The additional e-bike components from dynamics perspective are motor and battery, which are sufficiently heavy. The focus of the work is to understand the stability of the bicycle. Lateral dynamics of the bicycle will be modeled. The bicycle will be modeled as lumped parameter system, which is simpler to carry out further stability studies. The degrees of freedom modeled in system will be having speed, steering angle, lean etc. Also, studies will be carried out to see effect of several parameters like frame mass-distribution, handlebar mass-distribution, speed, front fork angle etc. on stability. The codes for the present work have been programmed using MATLAB software and results are obtained in the form of Eigen values and suitable graphs are plotted and results are analyzed.*

I. INTRODUCTION

An electric cycle which is also called as electric-bike or booster bike. E-bike are bicycles which integrates electric motor, which helps for propulsions. Many varieties of E-bikes now are available worldwide. E-bikes will be having small compact motor to assist the power given by the rider pedal is called "PEDELECS". Rechargeable batteries are used in e-bikes. Lighter variety motors can be used to travel up to 24 to 34 km/h, which mainly depends on the rules and regulations of that particular countries law, were the products are marketed. Where has motors having high powered variants will travel up to 45 km/h. E-bikes are powered by electric motor and it's like version of motorcycles, which are present from late 19th century. Electric bikes are the only transportation that are managed in last 20 years to take part significant where in bicycle market. E-Bike's simple design which are closely mimics that of traditional bicycles and with efficient and small electric motor it's easy to control. Its growth rapidly increasing from 1998. Many aspects favors the use of e-bikes in various situation. These may incorporate lower vitality cost per distances travelled by e-bikes. 1% to 2% of passing via auto while passing by e-bike for a solitary rider. Also, reserve funds should be possible in different costs, for example, insurance, license, registrations and also parking. Even it improves the traffics flow, environment friendly and also the medical and health benefits for e-bike riders.

II. SURVEY WORK

The bicycle, though made of fewer components than a motorcycle or automobile, is a complex system to model accurately. There have been numerous efforts through the 19th to 21st century where researchers have attempted to accurately model all the dynamic characteristics of a bicycle. These research efforts have intertwined with the investigation of motorcycle dynamics since motorcycles and bicycles share many common characteristics, including governing equations of motion. Typically, the objective of this modeling has been to develop a baseline that can be used to comprehend the dynamic characteristics in terms of design parameters. These design parameters could then be tuned so as to enhance the handling and maneuvering characteristics of a bike. A purely objective quantification of handling capability can be challenging, however, use of an eigenvalue analysis can provide a means of comparing some of the critical dynamic characteristics. When all the eigenvalues of the system matrix, formulated from the equations of motion, are found to be simultaneously negative, the bicycle system is considered to be self-stable. This implies that at some determined forward speed, the bicycle will remain in an upright position without any external inputs. Not being self-stable does not preclude a bicycle from being ride-worthy, nor does self-stability make the bicycle unresponsive. The mechanism of self-stability is best explained through an analogy of balancing an inverted pendulum. As the pendulum tends to tip over, quickly moving the base in the direction of the tip prevents the pendulum from falling. In much the same way, when a bicycle tends to tip in a direction, the wheel also turns in the same direction. This causes the base of the bicycle to accelerate in the direction of the tip and right itself during certain self-stable speeds. Many different authors have examined specific aspects of bicycle stability, including elaborate tire modeling and models that incorporate rider control. However, the 6 literature discussed in this chapter is limited to rider positioning, frame architecture, and the influence of frame stiffness or compliance. The Whipple model is commonly recognized as a baseline model with a full set of linearized equations of motion for an uncontrolled bicycle. This model consists of a rigid front frame, rear frame with rider, and two knife-edge wheels. This model has been used and revised by researchers including Carvallo, Döhring, Weir, Sharp, and Hand to name a few. These revisions, coupled with additional hand derivations by Papadopoulos, form the basis of the model commonly referred to as the benchmark bicycle. Addition of parameters to the governing equations of motion of the benchmark bicycle to develop a more accurate two-wheeled vehicle model forms a majority of the simulations that have been performed in the literature.

III. MODELING OF SYSTEM

I used Whipple bike model consisting of four rigid bodies. They are, a bicycle rear frame called as 'B', where the bicycle rider's body is well rigidly attached to the system. A bicycle rear wheel called 'R', a front frame 'H', consisting of fork's assembly and the handlebar of bicycle, and a bicycle front wheel tire set 'F' of bicycle. Which are shown schematically in the below figure. Inside the constraints of the complete entire (right – left) and also wheels circularly symmetric, with the mass distribution and the shape are generally with single careat. The model which regards these symmetries permits the non-planer thick wheels. Here, we take into account such thickness parameter of wheels while defining inertial properties at the same time, like Whipple, it limits the consideration to the knife edge rolling contact point.

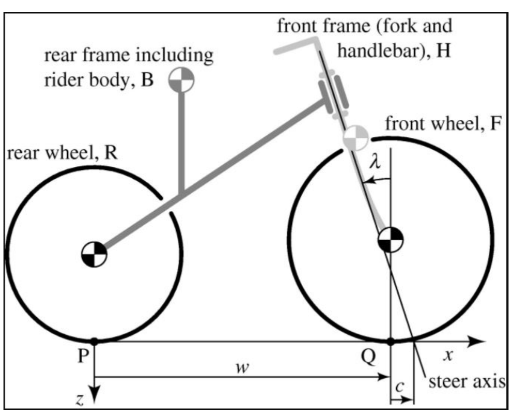


Fig 1: Whipple bike model

Benchmark parameters: A benchmark bike is defined with all the parameters esteems given in the below tables. The parameters esteems were chosen to limit the likelihood of cancelation that could happen if utilized as a part of an inaccurate model. In this benchmark bike the 2 wheel are of distinctive in all the properties & having no 2 values of the angles, mass or distance same.

The bike model demonstrated here is characterized totally by 25 various design parameters, which are shown in above figure. This is not a negligible depiction for dynamics analysis of the model. For instance, the inertial property of back wheel(R) with exception of the polar moments of inertias i.e., (mR. & IRxx.) however no with IRyy. This will be joined with various inertias properties of rear frame structure (B) lessening the quantity of the parameter by 2. A similar blend will be utilized for the front frame assembly, lessening the quantity of parameters to (25) - (2) - (2) = 21. The polar inertias of the each bicycle wheel will be supplanted with the gyrostatic constants which gives its spin angular-momentums of the wheel in term of straight forward velocities. It does not diminish the quantity of the parameter in nonlinear modellings. Yet, in the linear modellings the radii of all wheels of bicycle model are unimportant for the steer geometry and the lean and also their impact on angular momentums is exemplified in gyrostatic constants. Wiping out wheels radii lessens the quantity of the parameter by two to (21) - (2) = 19.

At last, linearized conditions of motions that the polar moment of inertias of the 2 frames are immaterial and decreasing to essential no. of design parameters to (19) – (2) = 17. In their most lessened shape, the linear conditions of motions having eleven arbitrary independent matrix entries. Every section is the combination of the 17 parameters, which are quite recently portrayed. Still, advance decreases can be gotten by assessment of fourth-order characteristics equations. For simple and easy comparison we utilize all 25 various design parameters. This particular framework have non-holonomic kinematic constraints. The non holonomic non slip rolling and holonomic hinges and ground contact constraints limits these 4 linked 3-dimension objects in the space. Beginning with 24 DOF for 4-rigid bogies each with 3 rotational & translational DOF in the physical spaces. i.e., $4\times(3+3) = 24$. Now, subtract five DOF for all of the 3 hinges and 1 more for the each wheels to the ground surface plane contact $24-3\times5-2=7$. In this manner, before we decide the non slipping wheels contacts point's requirements, the available design spaces is 7-dimensional. The 4 non holonomic moving constraint. i.e., 2 for each wheel-to-ground contacts, don't further limit these available design spaces. Kinematically reasonable rolling's motions will interpret and direct the bike on the planes arbitrary ways and furthermore will rotates the wheels with respect to outline with no amount of net of change of general bike positions or even orientations. In this way, the available configurations of space for this particular models is 7-dimensions.

The particular 7-dimension configurations space will be parameterized as shown in below figure. The point of bike rear wheel contacting with the ground surface are (xP., yP.) with respect to the global fixed coordinate-systems with the origin (O). The initial orientations of bike rear frames structure is with the relative to the global references frame structure (O-xyz) is defined by the sequences of the angular-rotations. All various rotations are clearly shown in the below figure with fictitious hinges. Which is placed at the back hub. i.e., yaw-rotations (ψ) about the z-axis, lean-rotations (ϕ) about the x-axis, and the pitch-rotations (θB) about the y-axis of the plane. Steering-angle (δ) is the rotations of the bike front structure handlebar assembly frame with relation to the rear frame assembly about the bicycle steering axis. Bicycle right turns of straight forwarding moving cycle have δ > 0.

At last, the rotations of the bicycle front wheel (F) & the bicycle rear wheel (R) with relation to their respective frame structures H & B are given by θF & θR. In outline, the configurations spaces is parametrized with the xp, yp, θF, θR, ϕ, ψ, δ. various other quantities like wheel center coordinates & also rear frame pitch are also can be controlled by above these variables.

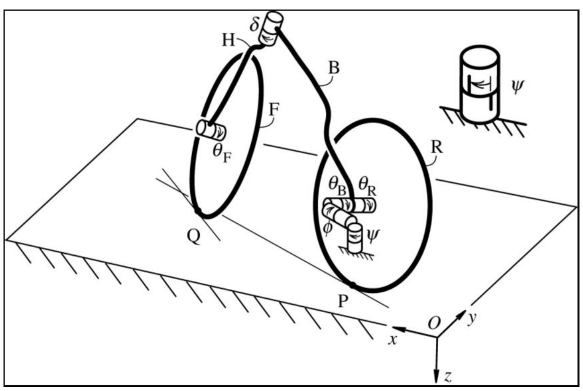


Fig 2: Configuration Space model

IV. SIMULATION

The stabilities of the uprights straights ahead running solutions at steady forwarding speed v will be got by exploring the Eigen values of the systems. However, the transient reactions of the systems, without any of forcing is given by the linearized combinations of their corresponding Eigen modes. These Eigen modes all together with the eigenvalues are found by accepting an exponentials solutions of the forms q = qo (λt) for the homogeneous equations. This prompts to a characteristics polynomials which are quartic in λ. The coefficients in this polynomials are complex equations of the 25 designs parameters, gravity, and speed v. The solutions λ of the characteristics polynomials for a range of forwarding speeds are shown in below figure. Eigenvalues with a positive real parts relates to the unstable motions though the eigenvalues with a negative real parts relates to the asymptotic steady motions for the corresponding modes. Imaginary Eigen values relates to the oscillatory motions.

On the basic level there are up to 4 Eigen modes, whereas oscillatory Eigen modes come up sets. 2 are huge and are called as Capsize modes & Weave modes. The capsize mode relates to the real Eigen value with Eigen vectors dominated by the lean. When the unstable bicycles just falls over like the capsizing ship. The weave mode is an oscillatory motions in which the bicycles sways about the headed directions. The 3rd remaining eigen mode is the caster mode which corresponding to the larger negative real eigen value with eigen vector dominated by steering's.

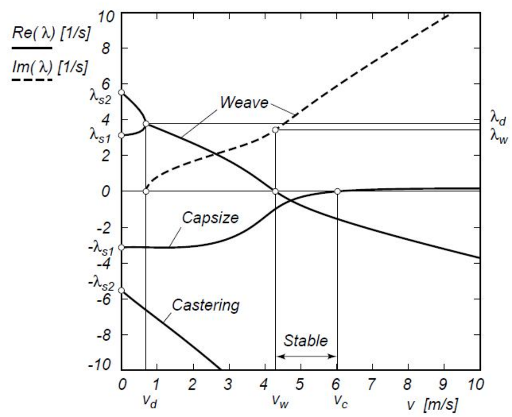


Fig.3: Eigen values λ from the value of linearized stability analysis

Battery locations: The below figure shows the various battery locations mounted in bicycle, in order to analyze the bicycle stability with different battery positions and also with different battery weights on the bicycle model.

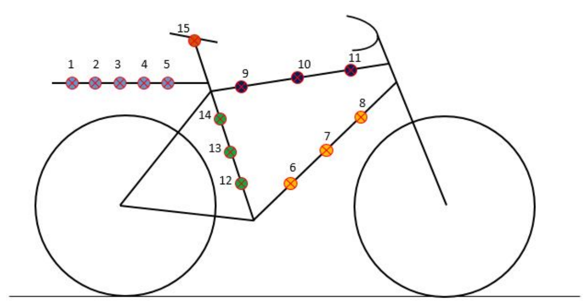
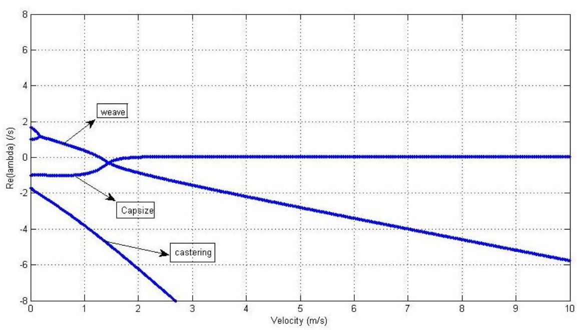


Fig 4: Various battery locations lumped mass points

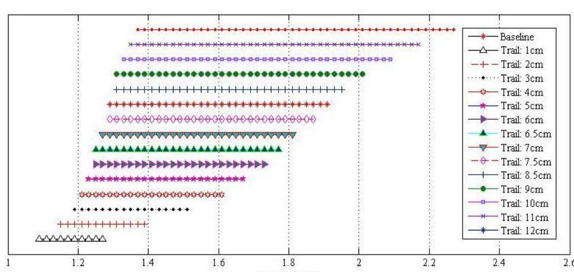
V. RESULTS & DISCUSSIONS

At close zero speeds, almost 0 < v < 0.4 m/s there are 2 sets of real Eigen values. Both the pairs comprises of a positives & also a negatives Eigen values & relates to an inverted pendulums like fallings of the bicycles. The positive roots in each set relates to fallings, though the negative roots corresponding to the time reversals of this fallings. i.e., risings. For this benchmarks, when speed is increased to the 0.7 m/s then 2 real eigen values coalesce and then it splits to form a complex conjugates set. Here, the oscillatory weave motions emerges. Initialing the motions is unstable but at 1.2 m/s, the weave speed these Eigen values crosses the imaginary axis in a Hopf bifurcations, this modes become the stables. At higher speed, the capsize Eigen values crosses the origin line in a pitchfork bifurcations at 2m/s the capsize speeds and the bike becomes to mildly unstable. The speed ranges for which the uncontrolled bike shows asymptotically stable behavior with the all Eigen values having negative real parts.



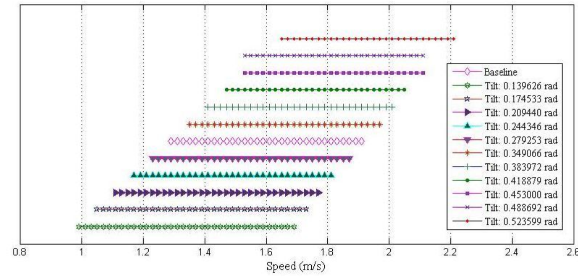
Graph 1: Eigenvalue plot for Parameter

Effect of Trail: From the below graph it indicates that as the trail increases, the stability range of bicycle increases. Out of various trails, 12cm trail provides good stability and 1cm trails shows the minimum stable region. Optimum trail is chosen based on frame size and head angle.



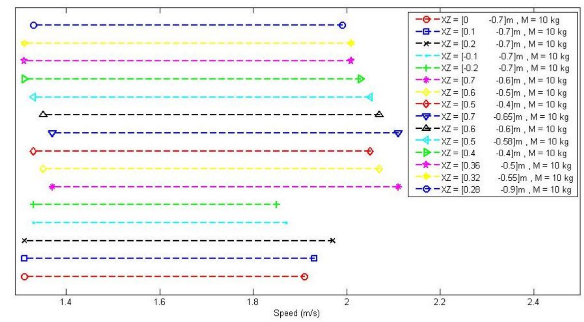
Graph 2: Stability plot for various trails

Effect of steer axis tilt: From the below graph it indicates that as the steer axis tilt increases, the stability range of bicycle increases. Out of various steer axis tilt 0.523599 rad gives good stability and 0.139626 rad trails shows the minimum stable region. Optimum steer axis tilt is chosen based on required wheel base.



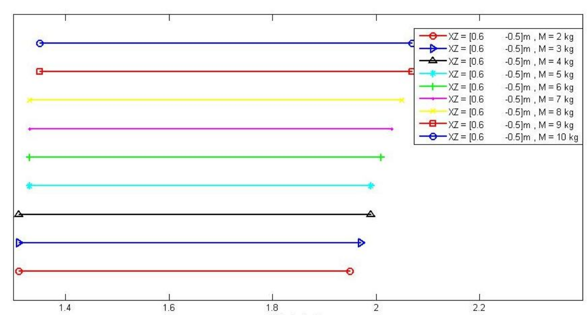
Graph 3: Stability plot for various steer axis tilt

Variation of stability with battery location: Below plot shows for stability region for various locations having battery weight of 10kg. The location XZ= [0.7, -0.6] and XZ= [0.7, -0.65] gives the maximum stability range, whereas region XZ= [-0.2, -0.7] gives minimum stable range for 10kg battery weight.



Graph 4: Stability plot for various battery locations for 10kg

Variation of stability with battery weight: Below plot shows the variation of stable region at the down tube location XZ= [0.6, -0.6] and for the varying battery weight ranging from 2 to 10kg. There is no much variation of stable region for changing in battery weight.



Graph 5: Stability plot for various battery weight at top tube (0.6, -0.6

VI. CONCLUSION

This study focused on dynamics modeling and simulation of electric bicycle. Several studies can be found, which have done detailed modeling of bicycles. But, it is difficult to conduct stability studies on non-linear models. To simplify stability study researchers approached linear studies. In this study the linearized model from Papadopoulos is used as baseline. The degrees of freedom (DOF) used for this model are steering angle and lean angle. The focus of this study is the stability study of the electric bicycle. The additional components present in electric bicycle are motor and battery. Motor and batteries are modeled as lumped masses, which change inertia parameters of bicycle model. The motors considered in this study are hub motors. The hub motors change the mass and inertia of wheels. Whereas battery locations is a critical stability parameter. The batteries considered for this study are light-weight Li-Ion and heavier Lead-Acid batteries. The simulations have been carried out in commercial software, MATLAB. Along with effect of electric components parameters on stability, basic parameters are also studied. The effect of parameters like Trail, Steer axis tilt, wheel radius and its mass on bicycle stability is studied. The effect of hub motor on dynamics is studied increasing wheel mass. The comprehensive studies has been carried out for battery parameters like weight and its location. The different battery locations simulated in are on – top tube, down tube, seat tube, carrier position.

Future Scope: Linearized model works very good for understanding effect of trail, steer angle. But, it doesn't give good idea for effect of battery mass and location. Therefore detailed modeling of bicycles and non-linearity analysis are recommended for studying the electric bicycle.

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