On Strong Interval-Valued Bipolar Fuzzy Graph

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Abstract- *In this paper, the strong interval-valued bipolar fuzzy graphs are introduced. Cartesian product, composition and join of two strong interval valued bipolar fuzzy graphs are defined. Some propositions involving strong interval-valued bipolar fuzzy graphs are stated and proved.*

Keywords- Bipolar fuzzy graph, Interval valued bipolar fuzzy graph, Strong interval valued bipolar fuzzy graph.

I. INTRODUCTION

In 1965, Zadeh [10] introduced the notion of a fuzzy subset of a set. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, signal processing, multiagent systems, pattern recognition, robotics, computer networks, expert systems, decision making and automata theory. In 1994, Zhang [11] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. A bipolar fuzzy set is an extension of Zadeh's fuzzy set theory whose membership degree range is [-1, 1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. In 1975, Rosenfeld [8] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [6] in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analougs of several graph theoretical concepts. Bhattacharya [5] gave some remarks on fuzzy graphs. Interval-Valued Fuzzy Graphs (IVFG) are defined by Akram and Dudec [4] in 2011. The complement of a fuzzy graph was defined and discussed by Mordeson and Nair [7], M.Sunitha and A.Vijayakumar[9]. Recently, the bipolar fuzzy graphs have been discussed by M. Akram [1,2,3]. In this paper, we introduce the notion of Strong Interval-Valued Bipolar Fuzzy Graphs (SIVBFG).

II. PRELIMINARIES

In this section, we first review some definitions of undirected graphs that are necessary for this paper.

Definition 2.1[12] Recall that a graph is an ordered pair $G^* = (V,E)$, where V is the set of vertices of G^* and E is the set of edges of G^*. Two vertices x and y in an undirected graph G^* are said to be adjacent in G^* if $\{x,y\}$ is an edge of G^*. A simple graph is an undirected graph that has no loops and no more than one edge between any two different vertices.

Definition 2.2[12] A subgraph of a graph $G^* = (V,E)$ is a graph $H=(W,F)$ where $W \subseteq V$ and $F \subseteq E$.

Definition 2.3[12] The complementary graph $\bar{G}^*$ of a simple graph has the same vertices as G^*. Two vertices are adjacent in $\bar{G}^*$ if and only if they are not adjacent in G^*.

Definition 2.4[10] A fuzzy subset μ on a set X is a map $\mu : X \to [0,1]$. A map $\mu : X \times X \to [0,1]$ is called a fuzzy relation on X if $\mu(x,y) \le \mu(x) \wedge \mu(y)$ for all $x,y \in X$. A fuzzy relation μ is symmetric if $\mu(x,y) = \mu(y,x)$ for all $x,y \in X$.

Definition 2.5[13] Let X be a non empty set. A bipolar fuzzy set B in X is an object having the form $B = \{(x, \mu^P(x), \mu^N(x)) | x \in X\}$ where $\mu^P : X \to [0,1]$ and $\mu^N : X \to [-1,0]$ are mappings.

Definition 2.6[1] A *bipolar fuzzy graph* with underlying set V is defined to be a pair $G = (A,B)$ where the functions $\mu_A : V \to [0,1]$ and $\gamma_A : V \to [-1,0]$ denote the degree of positive membership and negative membership of the element $x \in V$ respectively, the functions $\mu_B : E \subseteq V \times V \to [0,1]$and $\gamma_B : E \subseteq V \times V \to [-1,0]$ are defined by $\mu_B(x,y) \le min(\mu_A(x), \mu_A(x))$ and $\gamma_B(x,y) \ge max(\gamma(x), \gamma_A(x)), \forall (x,y) \in E$.

Definition 2.7[1] A bipolar fuzzy graph $G = (A,B)$ is called *strong bipolar fuzzy graph* if
$\mu_B(x,y) = min(\mu_A(x), \mu_A(x))$ and
$\gamma_B(x,y) = max(\gamma(x), \gamma_A(x)), \forall (x,y) \in E$.

III. STRONG INTERVAL-VALUED BIPOLAR FUZZY GRAPH

Let $D_1[0,1]$ and $D_2[-1,0]$ be the set of all closed subintervals of the intervals $[0,1]$ and $[-1,0]$ respectively and elements of these sets are denoted by uppercase letters. If

$P \in [0,1]$ and N$\in [-1,0]$ then these can be represented as $P = [P_L, P_U]$ and $N = [N_L, N_U]$ where the suffices L and U are the lower and upper limits of the respective intervals.

Definition 3.1 An *interval valued bipolar fuzzy graph* with underlying set V is defined to be a pair $G = (A, B)$ where

1) the functions $P_A: V \to D_1[0,1]$ and $N_A: V \to D_2[-1,0]$ denote the degree of positive membership and negative membership of the element $x \in V$ respectively.

2) the functions $P_B: E \subseteq V \times V \to D_1[0,1]$ and $N_B: E \subseteq V \times V \to D_2[-1,0]$ are defined by
$P_{BL}(x, y) \le \min(P_{AL}(x), P_{AL}(y))$ and
$N_{BL}(x, y) \ge \max(N_{AL}(x), N_{AL}(y))$
$P_{BU}(x, y) \le \min(P_{AU}(x), P_{AU}(y))$ and
$N_{BU}(x, y) \ge \max(N_{AU}(x), N_{AU}(y))\ \forall (x, y) \in E.$

Here after, we use the notation xy for (x, y) an element of E.

Definition 3.2 An interval valued bipolar fuzzy graph $G = (A, B)$ is called *strong interval valued bipolar fuzzy graph* if
$P_{BL}(x, y) = \min(P_{AL}(x), P_{AL}(y))$ and
$N_{BL}(x, y) = \max(N_{AL}(x), N_{AL}(y))$
$P_{BU}(x, y) = \min(P_{AU}(x), P_{AU}(y))$ and
$N_{BU}(x, y) = \max(N_{AU}(x), N_{AU}(y))\ \forall xy \in E.$

Example 3.1 Figure 1 is an example for IVBFG, $G = (A, B)$ defined on a graph $G^* = (V, E)$ such that $V = \{x, y, z\}$, $E = \{xy, yz, zx\}$ A is an interval valued bipolar fuzzy set of V and let B is an interval-valued ipolar fuzzy set of $E \subseteq V \times V$. Here

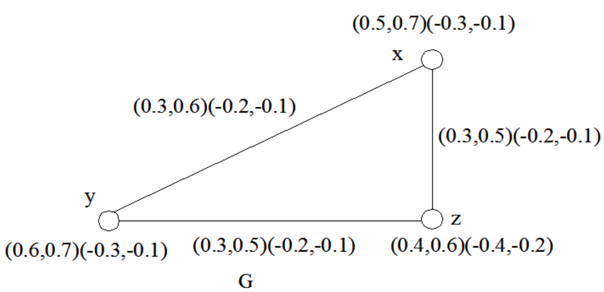

Figure 1

$$A = \left\{ \begin{pmatrix} x, [0.5, 0.7] \\ [-0.3, -0.1] \end{pmatrix}, \begin{pmatrix} y, [0.6, 0.7], \\ [-0.3, -0.1] \end{pmatrix}, \begin{pmatrix} z, [0.4, 0.6], \\ [-0.4, -0.2] \end{pmatrix} \right\}$$

$$B = \left\{ \begin{pmatrix} xy, [0.3, 0.6], \\ [-0.2, -0.1] \end{pmatrix}, \begin{pmatrix} yz, [0.3, 0.5], \\ [-0.2, -0.1] \end{pmatrix}, \begin{pmatrix} xz, [0.3, 0.5], \\ [-0.2, -0.1] \end{pmatrix} \right\}$$

Example 3.2 Figure 2 is an SIVBFG $G = (A, B)$, where

$$A = \left\{ \begin{pmatrix} x, [0.5, 0.7], \\ [-0.3, -0.1] \end{pmatrix}, \begin{pmatrix} y, [0.6, 0.7], \\ [-0.3, -0.1] \end{pmatrix}, \begin{pmatrix} z, [0.4, 0.6], \\ [-0.4, -0.2] \end{pmatrix} \right\}$$

$$B = \left\{ \begin{pmatrix} xy, [0.5, 0.7], \\ [-0.3, -0.1] \end{pmatrix}, \begin{pmatrix} yz, [0.4, 0.6], \\ [-0.3, -0.1] \end{pmatrix}, \begin{pmatrix} xz, [0.4, 0.6], \\ [-0.3, -0.1] \end{pmatrix} \right\}$$

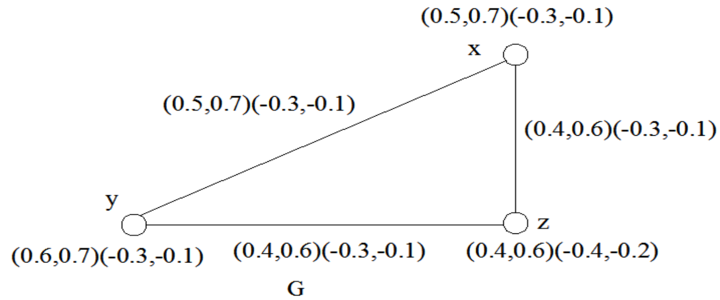

Figure 2

Definition 3.3 Let A_1 and A_2 be interval-valued bipolar fuzzy subsets of V_1 and V_2 respectively. Let B_1 and B_2 interval-valued bipolar fuzzy subsets of E_1 and E_2 respectively. The ***Cartesian product*** of two SIVBFGs G_1 and G_2 is denoted by $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ and is defined as follows:

1) $(P_{A_1L} \times P_{A_2L})(x_1, x_2) = min\left(P_{A_1L}(x_1), P_{A_2L}(x_2)\right)$
$(P_{A_1U} \times P_{A_2U})(x_1, x_2) = min\left(P_{A_1U}(x_1), P_{A_2U}(x_2)\right)$
$(N_{A_1L} \times N_{A_2L})(x_1, x_2) = max\left(N_{A_1L}(x_1), N_{A_2L}(x_2)\right)$
$(N_{A_1U} \times N_{A_2U})(x_1, x_2) = max\left(N_{A_1U}(x_1), N_{A_2U}(x_2)\right)$
$\forall x_1 \in V_1, x_2 \in V_2$

2)
$(P_{B_1L} \times P_{B_2L})((x, x_2), (x, y_2)) =$
$min\left(P_{A_1L}(x), P_{B_2L}(x_2, y_2)\right)$

$(P_{B_1U} \times P_{B_2U})((x, x_2), (x, y_2)) =$
$min\left(P_{A_1U}(x), P_{B_2U}(x_2, y_2)\right)$

$(N_{B_1L} \times N_{B_2L})((x, x_2), (x, y_2)) =$
$max\left(N_{A_1L}(x), N_{B_2L}(x_2, y_2)\right)$

$(N_{B_1U} \times N_{B_2U})((x, x_2), (x, y_2)) =$
$max\left(N_{A_1U}(x), N_{B_2U}(x_2, y_2)\right)$
$\forall x \in V_1, x_2 y_2 \in E_2$

3)
$$\left(P_{B_1L} \times P_{B_2L}\right)\left((x_1,z),(y_1,z)\right) = min\left(P_{B_1L}(x_1,y_1), P_{A_2L}(z)\right)$$

$$\left(P_{B_1U} \times P_{B_2U}\right)\left((x_1,z),(y_1,z)\right) = min\left(P_{B_1U}(x_1,y_1), P_{A_2U}(z)\right)$$

$$\left(N_{B_1L} \times N_{B_2L}\right)\left((x_1,z),(y_1,z)\right) = max\left(N_{B_1L}(x_1,y_1), N_{A_2L}(z)\right)$$

$$\left(N_{B_1U} \times N_{B_2U}\right)\left((x_1,z),(y_1,z)\right) = max\left(N_{B_1U}(x_1,y_1), N_{A_2U}(z)\right)$$
$\forall z \in V_2, x_1y_1 \in E_1$

Proposition 3.1 *If G_1 and G_2 are the strong interval valued bipolar fuzzy graphs, then the cartesian product $G_1 \times G_2$ is a strong interval valued bipolar fuzzy graph.*

Proof:

Let G_1 and G_2 are SIVBFGs, there exist $x_i, y_i \in E_i, i = 1,2$ such that

$$P_{B_iL}(x_i,y_i) = min\left(P_{A_iL}(x_i), P_{A_iL}(y_i)\right), i = 1,2.$$
$$P_{B_iU}(x_i,y_i) = min\left(P_{A_iU}(x_i), P_{A_iU}(y_i)\right), i = 1,2.$$
$$N_{B_iL}(x_i,y_i) = max\left(N_{A_iL}(x_i), N_{A_iL}(y_i)\right), i = 1,2.$$
$$N_{B_iU}(x_i,y_i) = max\left(N_{A_iU}(x_i), N_{A_iU}(y_i)\right), i = 1,2.$$

Let

$E = \{(x,x_2),(x,y_2) / \forall x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1,z),(y_1,z) / \forall z \in V_2, x_1y_1 \in E_1\}$

Consider, $(x,x_2),(x,y_2) \in E$, we have

$$\left(P_{B_1L} \times P_{B_2L}\right)\left((x,x_2),(x,y_2)\right) = min\left(P_{A_1L}(x), P_{B_2L}(x_2,y_2)\right)$$
$$= min\left(P_{A_1L}(x), P_{A_2L}(x_2), P_{A_2L}(y_2)\right)$$

Similarly,

$$\left(P_{B_1U} \times P_{B_2U}\right)\left((x,x_2),(x,y_2)\right) = min\left(P_{A_1U}(x), P_{B_2U}(x_2,y_2)\right) = min\left(P_{A_1U}(x), P_{A_2U}(x_2), P_{A_2U}(y_2)\right)$$

$$\left(P_{A_1L} \times P_{A_2L}\right)\left((x_1,x_2)\right) = min\left(P_{A_1L}(x_1), P_{B_2L}(x_2)\right)$$
$$\left(P_{A_1U} \times P_{A_2U}\right)\left((x_1,x_2)\right) = min\left(P_{A_1U}(x_1), P_{B_2U}(x_2)\right)$$
$$\left(P_{A_1L} \times P_{A_2L}\right)\left((x_1,y_2)\right) = min\left(P_{A_1L}(x_1), P_{B_2L}(y_2)\right)$$
$$\left(P_{A_1U} \times P_{A_2U}\right)\left((x_1,y_2)\right) = min\left(P_{A_1U}(x_1), P_{B_2U}(y_2)\right)$$

$$min\left(\left(P_{A_1U} \times P_{A_2U}\right)(x,x_2), \left(P_{A_1U} \times P_{A_2U}\right)(x,y_2)\right) =$$
$$min\left(min\left(P_{A_1U}(x), P_{A_2U}(x_2)\right), min\left(P_{A_1U}(x), P_{A_2U}(y_2)\right)\right)$$
$$= min\left(P_{A_1U}(x), P_{A_2U}(x_2), P_{A_2U}(y_2)\right)$$

Hence,

$$\left(P_{B_1L} \times P_{B_2L}\right)\left((x,x_2),(x,y_2)\right)$$
$$= min\left(\left(P_{A_1L} \times P_{A_2L}\right)(x,x_2), \left(P_{A_1L} \times P_{A_2L}\right)(x,y_2)\right)$$
$$\left(P_{B_1U} \times P_{B_2U}\right)\left((x,x_2),(x,y_2)\right)$$
$$= min\left(\left(P_{A_1U} \times P_{A_2U}\right)(x,x_2), \left(P_{A_1U} \times P_{A_2U}\right)(x,y_2)\right)$$

Similarly we can show that

$$\left(N_{B_1L} \times N_{B_2L}\right)\left((x,x_2),(x,y_2)\right)$$
$$= max\left(\left(N_{A_1L} \times N_{A_2L}\right)(x,x_2), \left(N_{A_1L} \times N_{A_2L}\right)(x,y_2)\right)$$
$$\left(N_{B_1U} \times N_{B_2U}\right)\left((x,x_2),(x,y_2)\right)$$
$$= max\left(\left(N_{A_1U} \times N_{A_2U}\right)(x,x_2), \left(N_{A_1U} \times N_{A_2U}\right)(x,y_2)\right)$$

Hence $G_1 \times G_2$ is a strong interval valued bipolar fuzzy graph. This completes the proof.

Proposition 3.2 *If $G_1 \times G_2$ is strong interval valued bipolar fuzzy graph then at least G_1 or G_2 must be strong.*

Proof: Suppose that G_1 and G_2 are not strong interval valued bipolar fuzzy graphs, there

exist Let G_1 and G_2 are SIVBFGs, there exist $x_i, y_i \in E_i, i = 1,2$ such that

$$P_{B_iL}(x_i,y_i) < min\left(P_{A_iL}(x_i), P_{A_iL}(y_i)\right), i = 1,2.$$
$$P_{B_iU}(x_i,y_i) < min\left(P_{A_iU}(x_i), P_{A_iU}(y_i)\right), i = 1,2.$$
$$N_{B_iL}(x_i,y_i) > max\left(N_{A_iL}(x_i), N_{A_iL}(y_i)\right), i = 1,2.$$
$$N_{B_iU}(x_i,y_i) > max\left(N_{A_iU}(x_i), N_{A_iU}(y_i)\right), i = 1,2.$$

Let

$E = \{(x,x_2),(x,y_2) / \forall x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1,z),(y_1,z) / \forall z \in V_2, x_1y_1 \in E_1\}$

Consider, $(x,x_2),(x,y_2) \in E$, we have

$$\left(P_{B_1L} \times P_{B_2L}\right)\left((x,x_2),(x,y_2)\right) = min\left(P_{A_1L}(x), P_{B_2L}(x_2,y_2)\right)$$
$$< min\left(P_{A_1L}(x), P_{A_2L}(x_2), P_{A_2L}(y_2)\right)$$

Similarly,

$$\left(P_{B_1U} \times P_{B_2U}\right)\left((x,x_2),(x,y_2)\right) = min\left(P_{A_1U}(x), P_{B_2U}(x_2,y_2)\right)$$

$$< min\left(P_{A_1U}(x), P_{A_2U}(x_2), P_{A_2U}(y_2)\right)$$

$$(P_{A_1L} \times P_{A_2L})((x_1, x_2)) = min\,(P_{A_1L}(x_1), P_{B_2L}(x_2))$$
$$(P_{A_1U} \times P_{A_2U})((x_1, x_2)) = min\,(P_{A_1U}(x_1), P_{B_2U}(x_2))$$
$$(P_{A_1L} \times P_{A_2L})((x_1, y_2)) = min\,(P_{A_1L}(x_1), P_{B_2L}(y_2))$$
$$(P_{A_1U} \times P_{A_2U})((x_1, y_2)) = min\,(P_{A_1U}(x_1), P_{B_2U}(y_2))$$

$$min\,((P_{A_1U} \times P_{A_2U})(x, x_2), (P_{A_1U} \times P_{A_2U})(x, y_2)) =$$

$$min\,(min\,(P_{A_1U}(x), P_{A_2U}(x_2)), min\,(P_{A_1U}(x), P_{A_2U}(y_2)))$$

$$= min\,(P_{A_1U}(x), P_{A_2U}(x_2), P_{A_2U}(y_2))$$

Hence,

$$(P_{B_1L} \times P_{B_2L})((x, x_2), (x, y_2)) < min\,((P_{A_1L} \times P_{A_2L})(x, x_2), (P_{A_1L} \times P_{A_2L})(x, y_2))$$
$$(P_{B_1U} \times P_{B_2U})((x, x_2), (x, y_2)) < min\,((P_{A_1U} \times P_{A_2U})(x, x_2), (P_{A_1U} \times P_{A_2U})(x, y_2))$$

Similarly we can show that

$$(N_{B_1L} \times N_{B_2L})((x, x_2), (x, y_2)) > max\,((N_{A_1L} \times N_{A_2L})(x, x_2), (N_{A_1L} \times N_{A_2L})(x, y_2))$$
$$(N_{B_1U} \times N_{B_2U})((x, x_2), (x, y_2)) > max\,((N_{A_1U} \times N_{A_2U})(x, x_2), (N_{A_1U} \times N_{A_2U})(x, y_2))$$

Hence, $G_1 \times G_2$ is not strong interval valued bipolar fuzzy graph, which is a contradiction. This completes the proof.

Remark: 3.1 *If G_1 is a SIVBFG and G_2 is not a SIVBFG, then $G_1 \times G_2$ is need not be a SIVBFG.*

Example 3.3

Let $G_1 = (A_1, B_1)$ be a SIVBFG, where

$A_1 = \{[a, (0.6, 0.7), (-0.3, -0.1)], [b, (0.6, 0.7), (-0.3, -0.1)]\}$ and $B_1 = \{[ab, (0.6, 0.7), (-0.3, -0.1)]\}$, $G_2 = (A_2, B_2)$ is not a SIVBFG,

where

$A_2 = \{[c, (0.4, 0.6), (-0.4, -0.2)], [d, (0.4, 0.6), (-0.4, -0.2)]\}$ and $B_2 = \{[cd, (0.3, 0.5), (-0.3, -0.1)]\}$,

$G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ is not a SIVBFG.

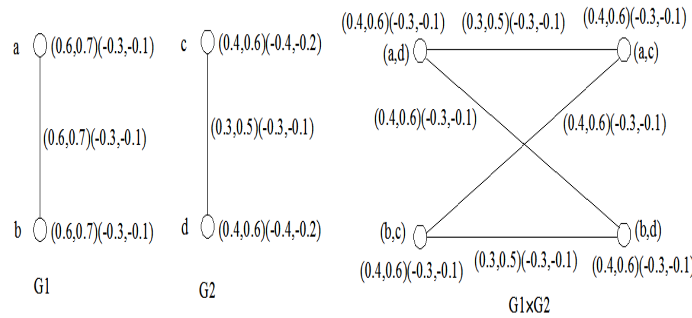

Figure 3

Example 3.4

Let $G_1 = (A_1, B_1)$ be an SIVBFG, where

$A_1 = \{[a, (0.6, 0.7), (-0.3, -0.1)], [b, (0.6, 0.7), (-0.3, -0.1)]\}$ and $B_1 = \{[ab, (0.6, 0.7), (-0.3, -0.1)]\}$, $G_2 = (A_2, B_2)$ is not a SIVBFG, where

$A_2 = \{[c, (0.4, 0.6), (-0.4, -0.2)], [d, (0.4, 0.6), (-0.4, -0.2)]\}$ and $B_2 = \{[cd, (0.3, 0.5), (-0.3, -0.1)]\}$,

$G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ is a SIVBFG.

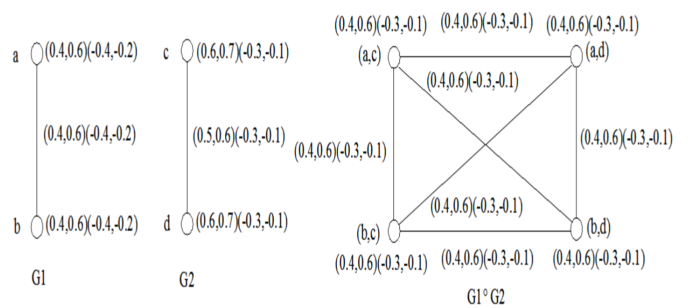

Figure 4

Proposition 3.3 *Let G_1 be a strong interval valued bipolar fuzzy graph. Then for any interval valued bipolar fuzzy graph G_2,*
$G_1 \times G_2$ is strong interval valued bipolar fuzzy graph iff
$P_{A_1L}(x_1) \le P_{B_1L}(x_2y_2)$, $N_{A_1L}(x_1) \ge N_{B_1L}(x_2y_2)$,
$P_{A_1U}(x_1) \le P_{B_1L}(x_2y_2)$, $N_{A_1U}(x_1) \ge N_{B_1L}(x_2y_2)$
$\forall x_1 \in V_1, x_2y_2 \in E_2$

Definition 3.4 Let A_1 and A_2 be interval-valued bipolar fuzzy subsets of V_1 and V_2 respectively. Let B_1 and B_2 interval-valued bipolar fuzzy subsets of E_1 and E_2 respectively. The ***composition*** of two strong interval valued bipolar fuzzy graphs G_1 and G_2 is denoted by $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ and is defined as follows:

1)

$$(P_{A_1L} \circ P_{A_2L})(x_1,x_2) = min\left(P_{A_1L}(x_1), P_{A_2L}(x_2)\right)$$
$$(P_{A_1U} \circ P_{A_2U})(x_1,x_2) = min\left(P_{A_1U}(x_1), P_{A_2U}(x_2)\right)$$
$$(N_{A_1L} \circ N_{A_2L})(x_1,x_2) = max\left(N_{A_1L}(x_1), N_{A_2L}(x_2)\right)$$
$$(N_{A_1U} \circ N_{A_2U})(x_1,x_2) = max\left(N_{A_1U}(x_1), N_{A_2U}(x_2)\right)$$
$\forall x_1 \in V_1, x_2 \in V_2$

2)

$$(P_{B_1L} \circ P_{B_2L})((x,x_2)(x,y_2)) = min\left(P_{A_1L}(x), P_{B_2L}(x_2y_2)\right)$$

$$(P_{B_1U} \circ P_{B_2U})((x,x_2)(x,y_2)) = min\left(P_{A_1U}(x), P_{B_2U}(x_2y_2)\right)$$

$$(N_{B_1L} \circ N_{B_2L})((x,x_2)(x,y_2)) = max\left(N_{A_1L}(x), N_{B_2L}(x_2y_2)\right)$$

$$(N_{B_1U} \circ N_{B_2U})((x,x_2)(x,y_2)) = max\left(N_{A_1U}(x), N_{B_2U}(x_2y_2)\right)$$
$\forall x \in V_1, x_2y_2 \in E_2$

3)

$$(P_{B_1L} \circ P_{B_2L})((x_1,z)(y_1,z)) = min\left(P_{B_1L}(x_1y_1), P_{A_2L}(z)\right)$$

$$(P_{B_1U} \circ P_{B_2U})((x_1,z)(y_1,z)) = min\left(P_{B_1U}(x_1y_1), P_{A_2U}(z)\right)$$

$$(N_{B_1L} \circ N_{B_2L})((x_1,z)(y_1,z)) = max\left(N_{B_1L}(x_1y_1), N_{A_2L}(z)\right)$$
$$(N_{B_1U} \circ N_{B_2U})((x_1,z)(y_1,z)) = max\left(N_{B_1U}(x_1y_1), N_{A_2U}(z)\right)$$
$\forall z \in V_1, x_1y_1 \in E_1$

4)

$$(P_{B_1L} \circ P_{B_2L})((x_1,x_2)(y_1,y_2)) = min\left(P_{A_2L}(x_2), P_{A_2L}(y_2), P_{B_1L}(x_1y_1)\right)$$

$$(P_{B_1U} \circ P_{B_2U})((x_1,x_2)(y_1,y_2)) = min\left(P_{A_2U}(x_2), P_{A_2U}(y_2), P_{B_1U}(x_1y_1)\right)$$

$$(N_{B_1L} \circ N_{B_2L})((x_1,x_2)(y_1,y_2)) = max\left(N_{A_2L}(x_2), N_{A_2L}(y_2), N_{B_1L}(x_1y_1)\right)$$

$$(N_{B_1U} \circ N_{B_2U})((x_1,x_2)(y_1,y_2)) = max\left(N_{A_2U}(x_2), N_{A_2U}(y_2), N_{B_1U}(x_1y_1)\right)$$
$\forall (x_1,x_2)(y_1,y_2) \in E^0 - E$ where

$E^0 = E \cup \{(x_1,x_2)(y_1,y_2) | x_1y_1 \in E_1, x_2 \neq y_2\}$

The following propositions are stated without their proof.

Proposition 3.4 *If* G_1 and G_2 *are the strong interval valued bipolar fuzzy graph then the composition* $G_1 \circ G_2$ *is a strong interval valued bipolar fuzzy graph.*

Proposition 3.5 *If* $G_1 \circ G_2$ *is strong interval valued bipolar fuzzy graph then atleast* G_1 *or* G_2 *must be strong.*

Example 3.5

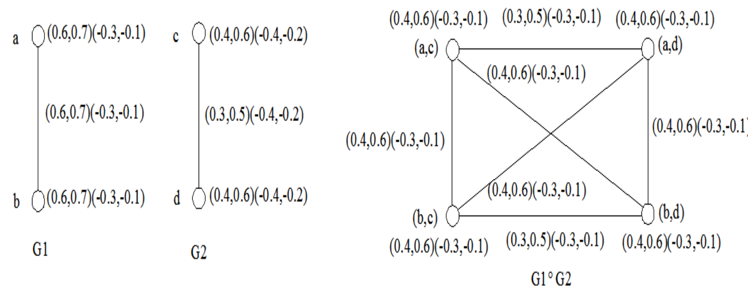

Figure 5

In this example, G_1 is an SIBFG and G_2 is not a SIBFG, then $G_1 \circ G_2$ is not an SIBFG.

Example 3.6

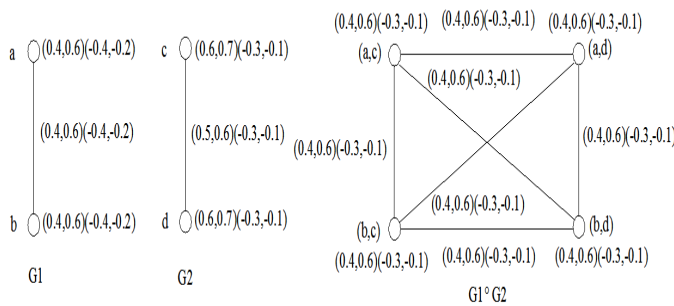

Figure 6

In this example, G_1 is an SIBFG and G_2 is not a SIBFG, then $G_1 \circ G_2$ is a SIBFG.

Proposition 3.6 *Let* G_1 *be a strong interval valued bipolar fuzzy graph. Then for any interval valued bipolar fuzzy graph* G_2, $G_1[G_2]$ *is strong interval valued bipolar fuzzy graph iff*
$P_{A_1L}(x_1) \leq P_{B_1L}(x_2y_2)$, $N_{A_1L}(x_1) \geq N_{B_1L}(x_2y_2)$,
$P_{A_1U}(x_1) \leq P_{B_1L}(x_2y_2)$, $N_{A_1U}(x_1) \geq N_{B_1L}(x_2y_2)$
$\forall x_1 \in V_1, x_2y_2 \in E_2$.

Definition 3.5 Let A_1 and A_2 be interval – valued bipolar fuzzy subsets of V_1 and V_2 respectively. Let B_1 and B_2interval valued bipolar fuzzy subsets of E_1 and E_2 respectively. The ***join*** of two strong interval valued bipolar fuzzy graphs G1 and G2 is denoted by $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ and is defined as follows:

1) $(P_{A_1L} + P_{A_2L})(x) = (P_{A_1L} + P_{A_2L})(x)$
$(P_{A_1U} + P_{A_2U})(x) = (P_{A_1U} + P_{A_2U})(x)$
$(N_{A_1L} + N_{A_2L})(x) = (N_{A_1L} + N_{A_2L})(x)$
$(N_{A_1U} + N_{A_2U})(x) = (N_{A_1U} + N_{A_2U})(x)$
if $x \in V_1 \cup V_2$

2) $(P_{B_1L} + P_{B_2L})(xy) = (P_{B_1L} \cup P_{B_2L})(xy)$
$(P_{B_1U} + P_{B_2U})(xy) = (P_{B_1U} \cup P_{B_2U})(xy)$
$(N_{B_1L} + N_{B_2L})(xy) = (N_{B_1L} \cap N_{B_2L})(xy)$
$(N_{B_1U} + N_{B_2U})(xy) = (N_{B_1U} \cap N_{B_2U})(xy)$
if $xy \in E_1 \cup E_2$

3) $(P_{B_1L} + P_{B_2L})(xy) = min(P_{B_1L}(x), P_{B_2L}(x))$
$(P_{B_1U} + P_{B_2U})(xy) = min(P_{B_1U}(x), P_{B_2U}(x))$
$(N_{B_1L} + N_{B_2L})(xy) = max(N_{B_1L}(x), N_{B_2L}(x))$
$(N_{B_1U} + N_{B_2U})(xy) = max(N_{B_1U}(x), N_{B_2U}(x))$
if $xy \in E'$

The join of graphs G_1^* and G_2^* is the simple graph $G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ where E' is the set of all edges joining the nodes of V_1 and V_2. In this construction it is assumed that $V_1 \cap V_2 = \emptyset$.

Proposition 3.7 *If* G_1 *and* G_2 *are the strong interval valued bipolar fuzzy graphs, then* $G_1 + G_2$ *is a strong interval valued bipolar fuzzy graph.*

V. CONCLUSION

In this paper, Cartesian product, Composition and Join of two SIVBFGs are discussed. Our future plan to extend our research to some other operations on interval valued bipolar fuzzy graph.

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