Introduction and Application of Differential Equations

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Abstract-*This review paper deals with introduction and applications of differential equations in abbreviation it is written as DE. In this paper definition, types along with examples on solving differential equation will be disussed as this topic is of huge interest in Engineering as well as real life.*

Keywords-Differential Equations (DE), Order, Degree

I. INTRODUCTION

With the invention of calculus Newton and Leibniz first invented Differential equations .

Issac Newton gave three types of differential equations

1) $\frac{dy}{dx} = f(x)$

2) $\frac{dy}{dx} = f(x, y)$

3) $x_1 \frac{\partial y}{\partial x_1} + x_2 \frac{\partial y}{\partial x_2} = y$

Above examples were solved by Newton using infinite series and discussed the non-uniqueness of solutions.

Jacob Bernoulli proposed the Bernoulli differential equation in 1695. This is an ordinary differential equation of the form

$y' + P(x)\,y = Q(x)\,y^n$, this equation was solved by Leibniz.

Jean le Rond d'Alembert, Leonhard Euler, Daniel Bernoulli, and Joseph-Louis Lagrange studied problems on vibrating string . D'Alembert found the one-dimensional wave equation in 1746, and Euler found the three-dimensional wave equation after ten years.

Euler and Lagrange developed Euler–Lagrange equation in connection with their studies of the tautochrone problem in 1750. Concept behind this problem is determining a curve on which a weighted particle will fall to a fixed point in a fixed amount of time, independent of the starting point. This problem was solved by Lagrange in 1755 and forwarded to Euler. After that both discovered Lagrange's method and used in to mechanics which results into formulation of

Lagrangian mechanics

Fourier discussed on Newton's law of cooling, which says the rate of change of temperature is proportional to very small difference between body temperature and surrounding medium temperature.

The motion of a body is described by its position and velocity as the time varies in classical mechanics,. Newton's laws permitted (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically in a differential equation for the unknown position of the body as a function of time.

Real world problem can be framed by using differential equations which is determination of the velocity of a stone falling in air under gravity and air resistance. Acceleration on stone is one gravity with negative sign n second air resistance with negative sign..Gravitational force will be constant and air resistance may be proportional to its velocity. With this differential equation can be modeled and velocity can be obtained as a function of time.

Differential equation is of huge importance in pure and applied mathematics, physics, and engineering. In pure mathematics existence and uniqueness of differential equation is taken into interest whether in applied mathematics modeling and approximating solution of differential equations emphasize. All real life problems reduced to differential equation can not necessarily solved instead numerical methods can also used there.

Laws of chemistry and physics can be modeled in a differential equation. Behavior of complex systems in biology and economics, differential equations are used. Joseph Fourier developed Conduction of heat theory which is governed by second-order partial differential equation i.e. heat equation. It results that many diffusion processes, while seemingly different, are described by the same equation.

II. DEFINITION OF DIFFERENTIAL EQUATION

Differential Equation is the equation in which function and its derivative occurs.

Example: $$\frac{dy}{dx}+8y=e^x$$ (Derrivat) Here $\frac{dy}{dx}$ is derivative of function y with independent variable x.

III. CLASSIFICATION OF DIFFERENTIAL EQUATION

Ordinary Differential Equation (ODE): : A Ordinary differential equation (ODE) is a differential equation that contains single variable and its derrivatives. ordinary differential equations often model one-dimensional dynamical systems,

$$Example \ : \frac{dy}{dx}+\frac{d^2y}{dx^2}=\sin x$$

Partial Differential Equation (PDE) : A partial differential equation (PDE) is a differential equation that contains unknown multivariable functions and their partial derivatives. PDEs are used to formulate problems involving functions of several variables, and are either solved in closed form, or used to create a relevant computer model. partial differential equations often model multidimensional systems. PDEs can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid flow, elasticity, or quantum mechanics.

$$Example \ : \frac{\partial z}{\partial x}=a^2\frac{\partial z}{\partial t}$$

IV. ORDER OF DIFFERENTIAL EQUATION

Order of DE is described by the highest derivative present in that differential equation.

Example:

$$i)\ \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=x; \quad order=2$$

$$ii)\ \frac{dy}{dx}=\sqrt{\left(1+\frac{d^2y}{dx^2}\right)} \ ; \quad order=2$$

$$iii)\ \frac{dy}{dx}+1=\frac{1}{\frac{dy}{dx}} \ ; \quad order=1$$

$$iv)\ \frac{\partial z}{\partial x}=a^2\frac{\partial z}{\partial t} \ ; \quad order=1$$

V. DEGREE OF DIFFERENTIAL EQUATION

Degree of DE is described by degree of highest derivative free from radical and fractions.

$$i)\ \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=x; \quad Degree=1$$

$$ii)\ \frac{dy}{dx}=\sqrt{\left(1+\frac{d^2y}{dx^2}\right)} \ ; \quad Degree=1$$

$$iii)\ \frac{dy}{dx}+1=\frac{1}{\frac{dy}{dx}} \ ; \quad Degree=2$$

$$iv)\ \frac{\partial z}{\partial x}=a^2\frac{\partial z}{\partial t} \ ; \quad Degree=1$$

VI. SOLUTION OF DIFFERENTIAL EQUATION

A Solution or Particular Solution of DE of order n consists of function defined and n times differentiable on a domain D having property that functional equation obtained by substituting the function and its n derivatives in to DE holds for every point in D.

Example:

$$i)\ y=x^2 \ \ is \ \ solution \ of \ \ \frac{dy}{dx}=2x$$

$$ii)\ y=\sqrt{x} \ \ is \ \ solution \ \ of \ \ 2y\frac{dy}{dx}=1$$

VII. METHODS AND VARIOUS TYPES OF DIFFERENTIAL EQUATION

a) Variable Seperable Method:

This method is defined as

If DE is of the form

$$a_1(x)b_1(y)+a_2(x)b_2(y)\frac{dy}{dx}=0 \ or \ a_1(x)b_1(y)\,dx+a_2(x)b_2(y)\,dy=0$$

$$then \ \ solution \ is \ \int_x \frac{a_1(x)}{a_2(x)}\,dx + \int_y \frac{b_2(y)}{b_1(y)}dy = c$$

Example:

Solution of $x\frac{dy}{dx}-y=0 \ is \ \ x=c\,y$

b) Homogenous Differential Equation

Form of DE is

$$\frac{dy}{dx}=f(x,y) \ where \ \ f(x,y) \ is \ \text{hom}ogenous \ \ funcion \ \ of \ \ x \ \ and \ \ y \ \ either \ of \ \ the \ \ form \ \ x^n \ f\left(\frac{y}{x}\right) \ or \ \ y^n \ f\left(\frac{x}{y}\right)$$

It can solved by substituting y= vx or x=vy.

Example: $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$

Above is homogenous DE with order zero and its solution can be found by substituting y=vx and it is $c\,x\cos\left(\frac{y}{x}\right)=1.$

c) Exact Differential Equation

An expression in the following form F(x,y)dx+G(x,y)dy is known as first order differential form.

A differential form F(x,y)dx+G(x,y)dy is said to be exact it there exists a function r (x,y) such that dr=Fdx+Gdy.

If α= Fdx+Gdy is an exact differential form then α=0 is called an exact differential equation and its solution is r=c where α=dr.

Example:

$$Solution \ of \ \left(y-x^3\right)dx+\left(x+y^3\right)dy=0 \ is \ 4\,x\,y-x^4+y^4=c$$

VIII. INTEGRATING FACTOR (IF)

Let the equation α=0.If α is not exact there may be exists a function D(x,y) such that Dα is exact, hence r=0 can be solved by multiplying both sides by D.

Here the function D is called an Integrating factor for equation α=0.

IX. CONCLUSION

This review paper deals with the study of differential equation. As this topic is of huge importance in Engineering as well as real life, Economics, Biology, Electrodynamics, General relativity, Quantum mechanics, Classical mechanics etc. In this paper basic definitions related to differential equation and various types of differential along with their solution have been discussed.

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