Bifurcation Analysis Of Permanent Magnet Dc Motor

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Abstract- *To ease human errors and to reduce the time consumed to perform a specific task, we have designed various systems. The working of various such systems is based on the certain parameters that act as an input to the system. The output of such systems is satisfactory until a certain range of input parameters. If a sudden unexpected change occurs in any one of the parameters, the system may stop functioning properly and the results obtained may be not acceptable. Hence changes parameters may make the system unstable. Analysis of systems is therefore very important in ensuring its stable state. There are various techniques that can analyse the behaviour of the systems. Bifurcation theory is one such mathematical analysis technique that can help us in obtaining the equilibrium points of the system. Whenever a system becomes unstable these equilibrium points change or vanish, bifurcation analysis provides us with the change in input parameters that correspond to the vanishing of the equilibrium points. Hence the exact cause behind the irregular behaviour of the system is known. A DC motor is an electrical equipment that is used in various applications in industries and transportation. It can be considered as a system, whose rotational output depends on system parameters such as current, flux, angular velocity etc. in this underlying paper the bifurcation theory is been applied to the DC motor for the purpose of analysing the equilibrium points and obtaining various parameter changes that can lead to breakdown of the dc motor..*

Keywords- Bifurcation, Equilibrium Points, Eigen Values, Stability, Differential Equations etc.

I. INTRODUCTION

DC motors drives are one of the most widely used electric machines in industries. The DC motor drives are used in applications such as conveyors, rolling mills, textile mills and in crushers. DC motors provide high staring torque, operate on low Starting Current and have higher efficiency.

The stability of voltage, current and load are extremely important for the secure operation of a DC Motor. Random failures may occur from time to time due to extreme operating conditions, any damage in electrical components or damage in mechanical coupling. Even during these extreme conditions, it is not acceptable that the DC motor breakdown.

A mathematical model describing the operations of a DC motor contains both differential and algebraic equations. A DC motor, like any other dynamic system, is normally subjected to continuous perturbations. For convenience, we can assume that at a given operating level the system is at rest, i.e. equilibrium exists. This implies that we can find a steady-state solution to the equations describing the operation of the DC motor.

To study the bifurcation analysis of the DC motor we first need to understand the concept of stability and equilibrium points. Since the dc motor is a non-linear system, special analysis techniques are used

II. SYSTEM MODELLING

The qualitative structure of the flow can change as parameters are varied. In particular, fixed points can be created or destroyed, or their stability can change. These qualitative changes in the dynamics are called bifurcations, and the parameter values at which they occur are called bifurcation points.

1. PM DC Motor.

In a DC motor, an armature rotates inside a magnetic field. Basic working principle of DC motor is based on the fact that whenever a current carrying conductor is placed inside a magnetic field, there will be mechanical force experienced by that conductor. All kinds of DC motors work in this principle only. Hence for constructing a DC motor it is essential to establish a magnetic field. The magnetic field is obviously established by means of magnet. The magnet can by any types i.e. it may be electromagnet or it can be permanent magnet. When permanent magnet is used to create magnetic field in a DC motor, the motor is referred as permanent magnet DC motor or PMDC motor. These types of motor are essentially simple in construction. These motors are commonly used as starter motor in automobiles, windshield wipers, washer, for blowers used in heaters and air conditioners, to raise and lower windows, it also extensively used in toys.

As the magnetic field strength of a permanent magnet is fixed it cannot be controlled externally, field control of this type of dc motor cannot be possible. Thus permanent magnet

DC motor is used where there is no need of speed control of motor by means of controlling its field. Small fractional and sub fractional kW motors now constructed with permanent magnet.

The input of the system is the voltage source (V) applied to the motor's armature, while the output is the rotational speed of the shaft dθ/dt.

2. Mathematical Modelling.

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current I by a constant factor Kt as shown in the equation below. This is referred to as an armature-controlled motor.

$$T=K_t i$$

The back emf, e, is proportional to the angular velocity of the shaft by a constant factor Ke.

$$e= K_e \frac{d\theta}{dt}$$

In SI units, the motor torque and back emf constants are equal, that is, $K_t = K_e$; therefore, we will use K to represent both the motor torque constant and the back emf constant.

From the figure we can derive the following governing equations based on Newton's 2nd law and Kirchhoff's voltage law.

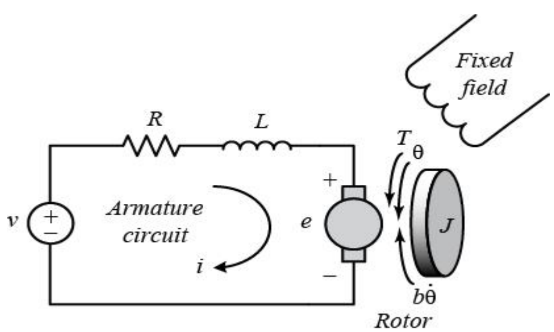

Figure 1. Basic Circuit Diagram for PM DC motor

$$J\frac{d\theta}{dt} + B\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

3. Transfer Function.

Applying the Laplace transform, the above modelling equations can be expressed in terms of the Laplace variable *s*.

$$s(Js + B)\theta(s) = K_t\, I(s)$$

$$(Ls + R)I(s) = V(s) - K_e\, s\theta(s)$$

We arrive at the following open-loop transfer function by eliminating *I*(*s*) between the two above equations, where the rotational speed is considered the output and the armature voltage is considered the input.

$$P(s) = \frac{\theta(s)}{V(s)} = \frac{K}{(Js+B)(Ls+R)+Kt\,Ke}$$

4. State Space.

In state-space form, the governing equations above can be expressed by choosing the rotational speed and electric current as the state variables. Again the armature voltage is treated as the input and the rotational speed is chosen as the output.

$$\frac{d}{dt}\begin{bmatrix}\theta \\ i\end{bmatrix} = \begin{bmatrix}\frac{-B}{J} & \frac{K}{J} \\ \frac{-K}{L} & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}\theta \\ i\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{L}\end{bmatrix} V$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta \\ i\end{bmatrix}$$

5. PM DC Motor Analysis.

For the purpose of bifurcation analysis, a Permanent Magnet DC Motor has been used. The motor used is MCO C23 Series PM DC Motor with following specifications.

Table 1. PM DC motor Specifications

MOTOR PARAMETERS	C23 SERIES PM DC MOTOR
Terminal Voltage (V)	48 V
Current (I)	1.4 A
Speed (N)	2100 RPM
Torque (T)	0.16 Nm
Resistance (R)	8.13 Ω
Inductance (L)	10.24 H
Back EMF (E_b)	17.25 V
Friction Torque (T_f)	0.04 Nm
Rotor Inertia (J_m)	0.00003672 $Kg.m^2$
Torque Constant (K_t)	0.1659 Nm/A
Voltage Constant (K_e)	0.0228 V/RPM
Viscous Friction (B_m)	0.0000192 N
Electrical Time Constant (T_e)	1.2595 msec
Mechanical Time Constant (T_m)	1.91 msec

III. CALCULATION

Consider the Differential Equations,

$$J\frac{d\theta}{dt} + B\dot{\theta} = Ki \ldots\ldots\ldots\ldots\ldots(1)$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta} \ldots\ldots\ldots\ldots\ldots(2)$$

Therefore,

$$\frac{d\theta}{dt} = \frac{-b}{J}\dot{\theta} + \frac{Kt}{J}i \ldots\ldots\ldots\ldots\ldots(3)$$

$$\frac{di}{dt} = \frac{-R}{L}i - \frac{Ke}{L}\dot{\theta} + \frac{V}{L} \ldots\ldots\ldots\ldots(4)$$

For finding the Equilibrium points, using the parameter values mentioned in Table.1 and equating RHS of (3) and (4) to zero, therefore

$$\frac{-B}{J}\dot{\theta} + \frac{Kt}{J}i = 0 \ldots\ldots\ldots\ldots\ldots (5)$$

$$\frac{-R}{L}i - \frac{Ke}{L}\dot{\theta} + \frac{V}{L} = 0 \ldots\ldots\ldots\ldots\ldots (6)$$

Using the values of parameters mentioned in above table and solving equations (5) and (6),

We get solutions of differential equations,

$$x_1 = 0.23 \quad \& \quad x_2 = 2046.06$$

Now, the stability of the system is calculated by finding out the Eigen values of the system equations.

For Eigen values λ_1 and λ_2,

$$\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}\frac{-B}{J} & \frac{K}{J}\\ \frac{-K}{L} & \frac{-R}{L}\end{bmatrix} = 0$$

Substituting and solving the above equation we get the Eigen values as,

$$\lambda_1 = -\,0.6583 + 3.1490i$$

$$\lambda_2 = -\,0.6583 - 3.1490i$$

Since λ_1 and λ_2 both are complex conjugate and have negative real part the system is stable.

IV. BIFURCATION IN PM DC MOTOR

The PM DC motor operates in stable equilibria for the rated values of parameters. To check whether bifurcation occurs or not in PM DC motor Armature resistance R, and Viscous friction Bm are chosen as the bifurcating parameter, as changes to these parameters can cause large variations in armature current and speed of the motor consequently affecting the equilibrium of the system.

The rated value of armature resistance is 8.13 Ω, this value is varied to analyse the change in current. The below figure represent the equilibrium conditions for various values of R.

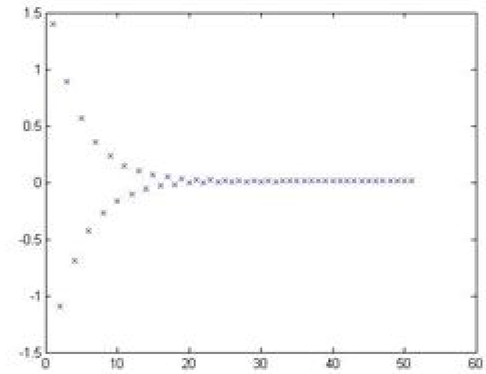

Figure 2. Plot for R= 8.13 Ω.

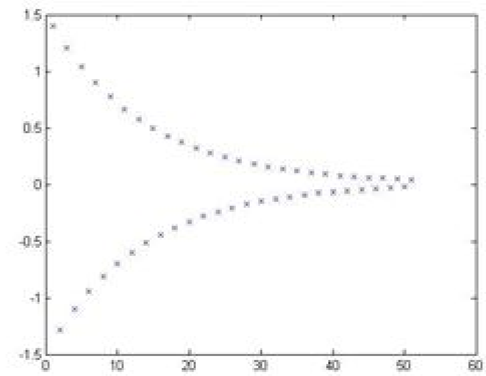

Figure 3. Plot for R= 9.5 Ω.

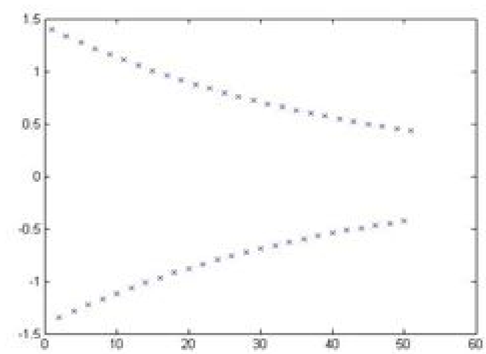

Figure 4. Plot for R= 10 Ω.

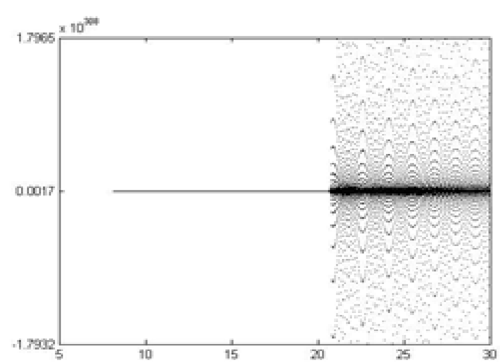

Figure 5. Plot for continuous variation in resistance

Now, the rated value of viscous friction Bm is varied to analyse the changes in speed of the motor. The following plots represent the equilibrium Continuous for various values of Bm.

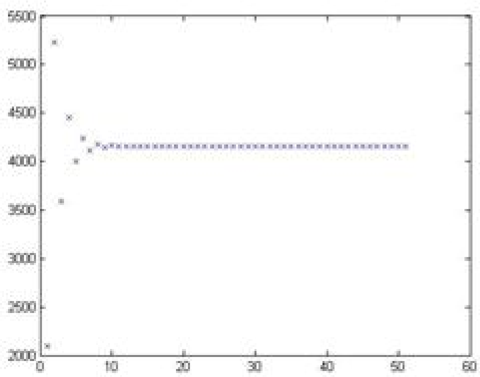

Figure 6. Plot for Bm = 0.0000192 N.

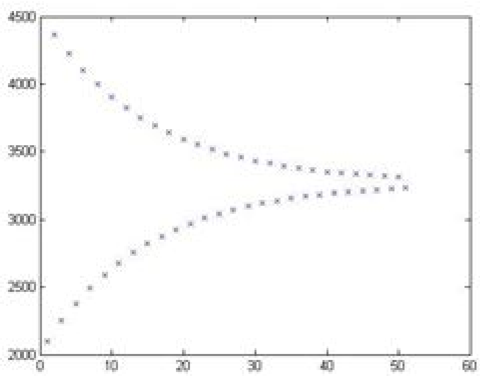

Figure 7. Plot for Bm = 0.0000343 N.

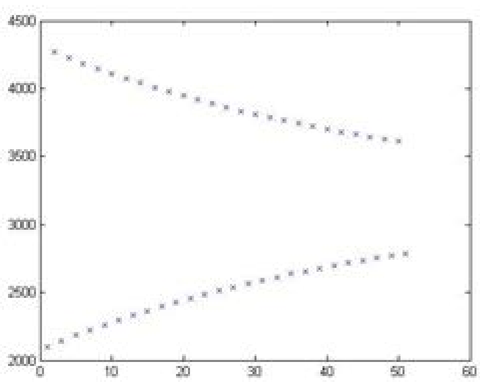

Figure 8. Plot for Bm = 0.0000360 N.

V. CONCLUSION

As the motor resistance changes from its standard value, the system response changes from stable equilibrium to recurrence jump between two values repeatedly, forming something called a 2-cycle. Somewhere between r = 8.13 and r = 25, there was a change in this behaviour of the solution. The r value for which this change occurs is the bifurcation Point. At R= 9.5, the stable response of the system changes from stable equilibrium to jump between two values. Similarly for the angular velocity equation, the change in viscous friction beyond the specified range will cause change in equilibrium points. At b = 0.0000343N, bifurcation occurs in the system response.

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