Modeling and Analysis of Different Tuning Methodologies of PID Controller for a Linearly Parameterized Non Linear System

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Abstract- *The primary objective of process control is to maintain a process at the desire operating conditions safely and efficiently, while satisfying environment and product quality requirements [1]. This paper, propose the modeling and comparison between different tuning methodologies of PID controller for conical tank.Many process industries use conical tanks because of its shape contributes to better drainage of solid mixtures, slurries and viscous liquids. So control of conical tank presents a challenging problem due to its non-linearity and constantly changing cross-section. PID controller is the most commonly used controller in industries due to its simple and robustness. Selection of the tuning method also very important during design and sometimes it will vary plant to plant. In this paper, PID performance are compared in terms of settling time, rise time, overshoot, Integral of Squared Error (ISE), Integral of Absolute Error (IAE) and Integral Time Absolute Error (ITAE).The performance of the controller for different tuning rules has been investigated in a MATLAB simulation environment.*

Keywords- Conical Tank, Modeling, PID controller, Tuning Methodologies, performance indices.

I. INTRODUCTION

In process control applications, control of process parameters is a challenging task, because of processparameters are uncertain, time-varying, and load or set point variations. Maintaining the level and temperature of the liquid in the tank is a most common problem in Process control industries. If height is too low or too high problems may arise because of spillage of material or improper chemical reaction or penalty for sequential operations. In this paper conical tank is used as a plant. Conical tank is highly nonlinear system due to the variation of area of cross section with height. Nonlinear processes are difficult to control and it provides a most challenge to control engineers.

A proportional integral derivative (PID) controller is the most commonly used controller in industries due to its simple structure, robust nature and easy implementation. In spite of all the advances in control over the past 50 years the PID controller is still the most common controller [6]. The ability of PI and PID controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications [2]. It has been stated that 98% of control loops in the pulp and paper industries are controlled by SISO PI controllers [3] and that, in process control applications, more than 95% of the controllers are PID type [4]. In order for the controllers to work satisfactory, controller must be tuned appropriately. Fine-tuning of controllers can be done in a number of ways, depending on the dynamics of the system and several methods have been developed in latest years [5].

II. PROCESS DESCRIPTION

1. Mathematical modeling of the system

The process considered here is a conical tank system shown in Figure 1 in which the level of the liquid is desired to maintain a constant value. This can be achieved by controlling the input flow rate into the tank. Here Fi is the inlet flow and Fo is the outlet flow

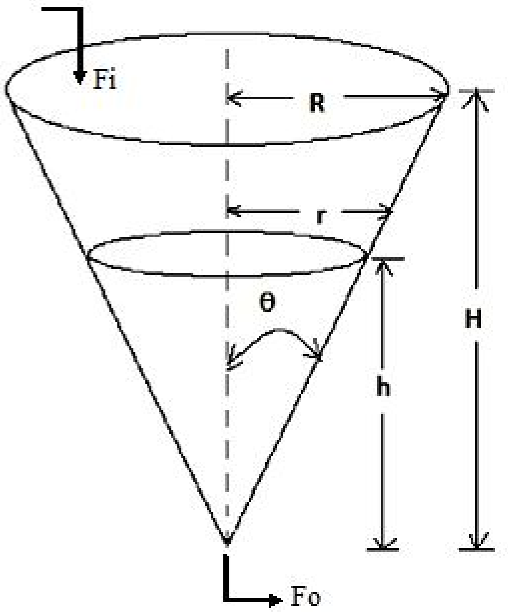

Figure 1. schematic of conical tank system

The model of the conical tank is determined withthe following assumptions level as the controlledvariable and Inflow to the tank as the manipulatedvariable [8]

The parameters of the model are as follows,

H – Total height of the tank = 50 cm

R – Outer radius of tank = 18 cm

r – Inner radius of the tank (cm)

h – Current height of fluid in the tank (cm)

A – Area of the conical tank (cm^2)

θ – Angle difference which relates the current

height of fluid to the total height of the tank

(Degrees)

F_i– In-flow rate = 13.3cm^3/s

F_o– out-flow rate = 8.8 cm^3/s

C – Valve coefficient = 4.3

The objective is to control the level of tank, which can be achieved by controlling the input flow of the conical tank by using valve at inlet. At steady state both the in-flow and out-flow rates remain the same.

At each height of the conical tank, the radius willvary due to the non-linear nature which is due tothe shape of the tank. The difference between theinflow and the out flow rate will be based on thecross section area of the tank and level of the tankwith respect to time. The flow and the level of thetank can be regulated by properly modelling thetank.

The area of the conical tank is given by

$$A = \pi r^2 \quad (1)$$

From the figure 1,

$$\tan\theta = \frac{r}{h} = \frac{R}{H} \quad (2)$$

$$r = R * \frac{h}{H} \quad (3)$$

Therefore,

$$A = \frac{\pi * R^2 * h^2}{H^2} \quad (4)$$

According to law of conservation of mass, Inflow rate – outflow rate =

Accumulation in the tank

$$F_i - F_o = A\frac{dh}{dt} \quad (5)$$

$$F_o = C\sqrt{h} \quad (6)$$

Where, C is the valve coefficient

On solving,

$$F_i - C\sqrt{h} = A\frac{dh}{dt} \quad (7)$$

$$\frac{dh}{dt} = \frac{F_i - C\sqrt{h}}{A} \quad (8)$$

On substituting (4) in (8)

$$\frac{dh}{dt} = \frac{F_i H^2}{\pi (Rh)^2} - \frac{C\sqrt{h}H^2}{\pi (Rh)^2} \quad (9)$$

$$\frac{dh}{dt} = \alpha F_i h^{-2} - \beta h^{(-3)/2} \quad (10)$$

Where,

$$\alpha = \frac{1}{\pi}\left(\frac{H}{R}\right)^2$$

$$\beta = \alpha C$$

The above equation shows the mathematical model of a conical tank system. Improving or understanding process operation is a major overall objective for developing a dynamic process model.

2. Linearization of system

Linearization is the process by which we approximate nonlinear systems with linear ones. It is widely used in the study of process dynamics and design of control systems for the following reasons:

1) We can have closed-form, analytic solutions for linear systems. Thus we can have a complete and general picture of a process behavior independently of the particular values of the parameters and input variables. This is not possible for nonlinear systems, and computer simulation provides only with the behavior of the system at specified values of inputs and parameters.
2) All the Significant developments towards the design of effective control systems have been limited process [9].

In conical tank system is a highly nonlinear process. The tank area various continuously with change in height.The Taylor series is used,for the linearization of the nonlinearity of the conical tank. In the aboveequation (10), a nonlinear terms appears, which can be linearized usingthe Taylor series expansion. The linearization is successfully applied inprocess control, as the whole purpose is to keep the controlled variable nearthe steady state value; so, a nonlinear term can be approximated by the slopeof the tangent at the operating point

In conical tank system model has two types of nonlinear$F_i h^{-2}$, a product of two functions, and $h^{(-3)/2}$. We shall have to linearize each of the functions separately around the steady state (h_s,F_{is}).

The linearization of $f(h,F_i) = F_i h^{-2}$ proceed as following,

$$f(h,F_i) = f(h_s,F_{is}) + \left(\frac{\partial f}{\partial h}\right)_{(h_s,F_{is})}(h - h_s) + \left(\frac{\partial f}{\partial F_i}\right)_{(h_s,F_{is})}(F - F_{is}) + higher\ order\ terms \quad (11)$$

Whereupon carrying out the indicated operations now signals,

$$f(h,F_i) = f(h_s,F_{is}) - 2F_{is}h_s^{-3}(h - h_s) + h_s^{-2}(F_i - F_{is}) \quad (12)$$

We ignore the higher order terms in equation (11)

Now, the second for linearization is$h^{(-3)/2}$,

$$h^{(-3)/2} = (h_s)^{(-3)/2} - \frac{3}{2}(h_s)^{(-5)/2}(h - h_s) \quad (13)$$

We now introduce these expressions (12) & (13) in place of the corresponding nonlinear terms of equation (10),

$$\frac{dh}{dt} = \alpha\left[f(h_s,F_{is}) - 2F_{is}h_s^{-3}(h-h_s) + h_s^{-2}(F_i - F_{is})\right] - \beta\left[(h_s)^{(-3)/2}\frac{3}{2}(h_s)^{(-5)/2}(h-h_s)\right] \quad (14)$$

At steady state,

$$F_i = F_o \quad (15)$$

$$\frac{dh_s}{dt} = \alpha F_{is}h_s^{-2} - \beta(h_s)^{(-3)/2} = 0 \quad (16)$$

$$\frac{d(h-h_s)}{dt} = -2\alpha F_{is}h_s^{-3}(h-h_s) + \alpha h_s^{-2}(F_i - F_{is}) + \frac{3}{2}\beta\,(h_s)^{(-5)/2}(h-h_s) \quad (17)$$

Introduce a deviation variable $y = (h - h_s)$and $u = (F_i - F_{is})$

$$\frac{dy}{dt} = -2\alpha F_{is}h_s^{-3}y + \alpha h_s^{-2}u + \frac{3}{2}\beta\,(h_s)^{(-5)/2}y \quad (18)$$

$$\frac{dy}{dt} = -\left(\frac{1}{2}\right)\beta(h_s)^{(-5)/2}y + \alpha h_s^{-2}u \quad (19)$$

$$(2/\beta)(h_s)^{(-5)/2}\left(\frac{dy}{dt}\right) = -y + \alpha h_s^{-2}u \quad (20)$$

$$\tau\frac{dy}{dt} + y = (2\alpha/\beta)(h_s)^{1/2}u \quad (21)$$

$$\tau\frac{dy}{dt} + y = Ku \quad (22)$$

Taking the Laplace transform,

$$\frac{Y(S)}{U(S)} = \frac{K}{\tau s+1} = \frac{h(s)}{F_i(S)} \quad (23)$$

Where,

$K = \left(\frac{2\alpha}{\beta}\right)(h_s)^{1/2}$ (Steady state gain)

$\tau = \left(\frac{2}{\beta}\right)(h_s)^{5/2}$ (Time constant)

For very good control response, piecewise linearization is required because system is highly nonlinear in real time analysis.

For analyzing purpose take, steady statelevel of 5 cm and the radius of 1.8cm.

The model obtained as shown in equation (23) is of first order. But the real time dynamic system can be approximated as FOPDT (First Order Process with Dead Time) for a higher order system

$$G_m = \frac{K_m e^{-s\tau_m}}{1+sT_m} \quad (24)$$

We can calculate dead time (τ_m), time constant (T_m) for the process using open loop response of the process.

Ziegler and Nichols (1942) have obtained the time constant and time delay of a FOPDT model by constructing a tangent to the experimental open loop step response at its point of inflection. The intersection of the tangent with the time axis provides the estimate of time delay. The time constant is estimated by calculating the tangent intersection with the steady state output value divided by the model gain [10].

Cheng and Hung (1985) have also proposed tangent and point of inflection methods for estimating FOPDT model parameters. The major disadvantage of all these methods is the difficulty in locating the point of inflection in practice and may not be accurate [11]. Prabhu and Chidambaram(1991) have obtained the parameters of the first order plus time delay model from the reaction curve obtained by solving the nonlinear differential equations model of a distillation column [12].

Sundaresan and Krishnaswamy (1978) have obtained the parameters of FOPDT transfer function model by collecting the open loop input-output response of the process and that of the model to meet at two points which describe the two parameters τ_p and θ. The proposed times t_1 and t_2, are estimated from a step response curve. This time corresponds to the 35.3% and 85.3% response times [13].

The time constant and time delay are calculated as follows:

$$\tau_m = 0.67(t_1 - t_2) \quad (25)$$

$$T_m = 1.3t_1 - 0.29t_2 \quad (26)$$

From the model, dead time obtained from the open loop response was 1.322 seconds. Thus the transfer function representation is given as G_p

$$G_p = \frac{1.04003 * e^{-1.322s}}{10.58623s+1} \quad (27)$$

III. MODEL OF PID CONTROLLER

The PID controller produces an output signal consisting of three terms: The proportional (P) action gives a change in the input (manipulated variable) directly proportional to the error signal. The integral (I) action gives a change in the input proportional to the integral of error, and its main purpose is to eliminate offset. Whereas the derivative (D) action is used to speed up the response or to stabilize the system and it gives a change in the input proportional to the derivative of the error signal [5].

$$u(t) = K_p\, e(t) + \frac{K_p}{T_i}\int e(t)\,dt + K_p T_d \frac{d}{dt} e(t) \quad (28)$$

On taking the Laplace transform of equation (28) with zero initial conditions we get,

$$U(s) = K_p\, E(s) + \frac{K_p}{T_i}\frac{E(s)}{s} + K_p T_d\, s\, E(s) \quad (29)$$

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \quad (30)$$

The equation (29) gives the output of the PID controller for the input E(s) and equation (30) is the transfer function of the PID controller. The block diagram of PID shown in figure

Where, K_p is the proportional gain, K_d is the derivative gain K_i is the integral gain, T_d is the derivative time and T_i is the integral time. The derivative term improves the transient response by adding a zero to the open loop plant transfer function. The integrator eliminates error by increasing the system type with additional pole at the origin. Generally, K_p will have the effect of reducing the rise time and it also reduce error but the steady-state error can never be eliminated. For eliminating the steady state error Integral gain K_i can be used, but it will make the transient response worse [7]

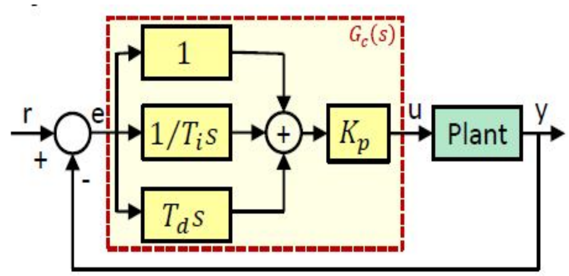

Figure 2. PID controller structure

IV. TUNING METHODOLOGIES

Tuning of a controller is a method of determining the parameters of a PID controller for a given system. Eight tuning methods discussed below have been used in this paper.

Table 1. Different tuning methodologies

Rule	K_p	T_i	T_d
Callender (1935) [14]	$\frac{1.066}{K_m \tau_m}$	$1.418\ \tau_m$	$0.353\ \tau_m$
Nichols (1942) [10]	$\frac{1.2\ T_m}{K_m \tau_m}$	$2\ \tau_m$	$0.5\ \tau_m$
Parr (1989) [15]	$\frac{1.25\ T_m}{K_m \tau_m}$	$2.5\ \tau_m$	$0.4\ \tau_m$
Borresen, Grindal (1990) [16]	$\frac{T_m}{K_m \tau_m}$	$3\ \tau_m$	$0.5\ \tau_m$
Connell (1996) [17]	$\frac{1.6\ T_m}{K_m \tau_m}$	$1.6667\ \tau_m$	$0.4\ \tau_m$
Chidambar-am (1995) [18]	$\frac{1.20\ T_m}{K_m \tau_m}$	$2.4\ \tau_m$	$0.38\ \tau_m$
Moros (1999) [19]	$\frac{1.2\ T_m}{K_m \tau_m}$	$2\ \tau_m$	$0.42\ \tau_m$
Liptak (2001) [20]	$\frac{0.85\ T_m}{K_m \tau_m}$	$1.6\ \tau_m$	$0.6\ \tau_m$

V. PERFORMANCE ANALYSIS

Performances of various controller tuning methods for PID controller are analyzed using MATLAB tested for a given stepinput and response are tabulated in table 2. For comparison purpose time domain specification (settling time, maximum overshoot, and delay time) and performance indices (ISE, IAE, ITAE) are tabulated from the response.

From the table (2) we can observed that, delay time of all tuning methods are almost same except callender method. In our analysis Connell and callender methods produce more sluggish and it take more time to settle, when compared to other methods.

Liptak method gives very good overshoot suppression (50%), than other technique. Good performance indices, can be observed in Chidambaram tuning method, but it produce overshoot around 80%.

In our, over all analysis Borresen, Grindal and Moros method gives low time settling time, reduced overshoot and very good performance indices.But suitable tuning methods are vary, plant to plant and it depends on dynamic of system, user experience on tuning.

Table 2. Performance analysis

Rule	K_p	T_i	T_d	Settling time (sec)	Maximum overshoot (%)	Delay time (sec)	ISE	IAE	ITAE
Callender	0.775	1.874	0.466	65	40	5.500	5.356	11.53	178.9
Nichols	9.239	2.644	0.661	13	90	1.300	1.956	2.902	8.498
Parr	9.624	3.305	0.528	14	85	1.320	1.977	2.892	7.811
Borresen, Grindal	7.699	3.966	0.661	10.6	55	1.318	1.590	2.528	7.124
Connell	12.31	2.203	0.528	32	150	1.322	4.222	6.601	43.58
Chidambaram	9.239	3.172	0.502	12	79	1.31	1.961	2.799	7.085
Moros	9.239	2.644	0.555	10	85	1.32	2.031	2.836	7.146
Liptak	6.544	2.115	0.793	16	50	1.318	1.841	3.447	14.75

VI. CONCLUSION

A different tuning methodologies are presented in this paper for controlling the level of a single conical tank system. In this paper nonlinear system as modeled as linear first order plus dead time system, and observed that Borresen, Grindal and Moros techniques gives better performance, when compared to other technique. Different tuning methodologies are very important and base for designing and analysis of control problem in real time for both, industry application and research purpose. Still classical tuning methods, gives confident result in process industries.

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