Different Types of Numbers

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Abstract- *We human were not meant to fiddle with numbers, we were just like other mammals -Hunt-Eat-Sleep, but still somehow we did created numbers to satisfy the creative needs of human brain. The number system is usually taken for granted in even the most basic of all courses. This might lead some mathematics majors to graduate with the notion that the number system is the foundation of mathematics. Basically, this assumes the existence of a 10 digit numbers which can be intuitively accepted. Along the way, the integers, rationals and the real numbers will be constructed systematically. Another aim of this paper is to present fundamental concepts of number sat the undergraduate level in an intuitive manner.*

Keywords- Algebraic Numbers, Aliquot Numbers, Almost Perfect Numbers, Alphametic Numbers, Amicable Numbers, Apocalypse Number, Arrangement Numbers, Automorphic Numbers, Binary Numbers

I. INTRODUCTION

Those ten simple numbers, pictures , digits that we all learn early in our life that influence our lives in far more ways than we could ever imagine. Have you ever wondered what our lives would be like without these 10 graceful digits . These numbers are useful for representing a. Human Birthdays, b. Human , animal , and plant ages, c. height, weight, dimensions of any objects, d. Addresses, telephone numbers, book page number, vehicle plate numbers, credit/ debit card numbers, PIN numbers, bank account numbers, radio/TV station numbers, e. Time, dates, years, directions, wake up times, sports scores, prices, accounting, sequences/series of numbers, magic squares, polygonal numbers, factors, squares, cubes, Fibonacci numbers, perfect, deficient, and abundant numbers, and the list goes on ad infinitum. Mathematicians, Statistician , Scientists , All branches ofEngineers, Accountants, clerks, store keepers , Manufacturers, Cashiers, Bankers, Stock brokers, Carpenters, , and so on, could not survive without them. In a sense, it could easily be concluded that we would not be able to live without 10 elegant numbers

In the interest of stimulating a broader interest in number theory and recreational mathematics, this collection will endeavour to present basic definitions and brief descriptions for several types of number. It is sincerely hoped that the material provided herein will definitely stimulate you to read and explore further about this topic . I also hope that after reading, understanding, and digesting the material provided herein, that you will have enjoyed the experience and that you

will never speek about Mathematics that , "I hate math."Definitly You will say “I Love Mathematics”.

II. SOME BASIC DEFINITIONS

2.1 Digits - The 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, used to create numbers in the base 10 decimal number system.

2.2 Numerals - The symbols used to denote the natural numbers. The Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are those used in the Hindu-Arabic number system to define numbers.

2.3 NaturalNumbers- The set of numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,....., that we see and use every day. The natural numbers are often referred to as the counting numbers and the positive integers.

2.4 WholeNumbers - The natural numbers in addition to that the zero are called as Whole numbers (0,1,2,3,4,5,6,7,8,9,…………….)

2.5 Rational Numbers - Any number that is in the ratio of two integers, that is in the form of $\frac{x}{y}$.

The numerator, "x", may be any whole number, and the denominator, "y ", may be any positive whole number greater than zero i.e. $y \neq 0$.

2.6 Fractional Numbers - Any number expressible in the form of quotient of two numbers i.e.$\frac{x}{y}$,. "$y > 1$".

If "x" is smaller than "y" it is a called “Proper fraction”. If "a" is greater than "b" it is called an “Improper fraction” which can be broken up into an integer and a proper fraction.

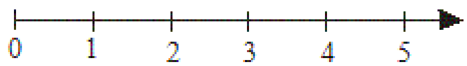
2.7 Irrational Numbers - An **irrational number** is a number that cannot be written as a ratio (or fraction). In decimal form, it never ends or repeats. The ancient Greeks discovered that not all numbers are rational; there are equations that cannot be solved using ratios of integers. Irrational numbers are expressible only as decimal fractions where the digits continue forever with no repeating pattern. Some examples of irrational numbers are .$\sqrt{2}$ *and* $\sqrt{3}$.

2.8 Transcendental Numbers - Any number that cannot be the root of a polynomial equation with rational coefficients.

They are a subset of irrational numbers
Example of transcendental no. are Pi = 3.14159... and e = 2.7182818..., the base of the natural logarithms.

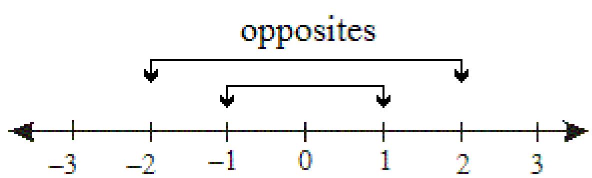
2.9 Real Numbers - The rational numbers and the irrational numbers together make up the **real numbers**.
Draw a line.

Put on it all the whole numbers 1,2,3,4,5,6,7.....etcbthen put 0



then put all the negatives of the whole numbers to the left of 0

........-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0



Then put in all of the fractions .

Then put in all of the decimals [some decimals aren't fractions]

Now you have what is called the "real number line"

III. TYPES OF NUMBERS

3.1 ALGEBRAIC NUMBERS

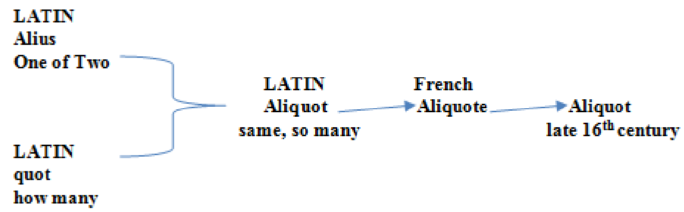
Algebraic numbers are the real or complex number which are roots to polynomial equations of the form:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots a_1 x + a_0 = 0$$

where the $a_i's$ are <u>integers</u> (or equivalently, rational numbers)

3.2 ALIQUOT NUMBERS

Origin of Aliquot Numbers



An **aliquot number** is set of divisors of a number, not equal to the number itself.

The divisors are often referred to as proper divisors.
Example: 1. The set of aliquot numbers of 15 are 1,3, and 5
2. The set of aliquot number of 24 are 1, 2, 3,4, 6, 8 and 12.

ALMOST PERFECT NUMBER

A natural number n such that the sum of all divisors of n (the sum of divisors n is equal to $2n - 1$. The only known almost perfect numbers are the powers of 2 i.e.in the form of 2^k with non-negative exponents.

Example 16 is Almost perfect number because all divisors of 16 are 1,2,4,8,16 and sum is 31 which is equal to $2 \times 16 - 1 = 31$

3.3 ALPHABETIC NUMBERS

Alphabetic numbers form cryptarithms where a set of numbers are assigned to letters that usually spell out some meaningful thought. The numbers can form an addition, subtraction, multiplication or division problem. One of the first cryptarithms came into being in 1924 in the form of an addition problem the words being intended to represent a student's letter from college to the parents. The puzzle read SEND + MORE = MONEY. The answer was 9567 += 1085 = 10,652. Of course, you have to use logic to derive the numbers represented by each letter..

3.4 AMICABLE NUMBERS

Amicable numbers are pairs of numbers, each of which is the sum of the others aliquot divisors. For example, 220 and 284 are amicable numbers whereas all the aliquot divisors of 220, i.e., 110, 55, 44, 22, 10, 5, 4, 2, 1 add up to 284 and all the aliquot divisors of 284, i.e., 142, 71, 4, 2, 1 add up to 220. Also true for any two amicable numbers, N1 and N2, is the fact that the sum of all the factors/divisors of both, Sf(N1 + N2) = N1 + N2. Stated another way, Sf(220 + 284) = 220 + 284 = 504. Other amicable numbers are:

$$\frac{1184}{1210},\frac{2620}{2924},\frac{5020}{5564},\frac{6232}{6368},\frac{10{,}744}{10{,}856},\frac{17{,}296}{18{,}416}\ and\ \frac{9{,}363{,}584}{9{,}437{,}056}$$

There are more than 1000 known amicable pairs. Amicable numbers are sometimes referred to as friendly numbers.

There are various methods for discovering pairs of amicable numbers. For example, if *n* is a positive integer such that

$3 \cdot 2^n - 1$, $3 \cdot 2^{n+1} - 1$, and $3^2 \cdot 2^{2n+1} - 1$ are all prime, then $2^{n+1}(3 \cdot 2^n - 1)(3 \cdot 2^{n+1} - 1)$ and $2^{n+1}(3^2 \cdot 2^{2n+1} - 1)$ form an amicable pair.

3.5 APOCALYPSE NUMBER

The **Apocalypse number**, 666, often referred to as the beast number, is referred to in the bible, Revelations 13:18.While the actual meaning or relevance of the number remain unclear, the number itself has some surprisingly interesting characteristics.

The sum of the first 36 positive numbers is 666 which makes it the 36th triangular number.

The sum of the squares of the first seven prime numbers is 666.

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 4^3 + 3^3 + 2^3 + 1^3 = 666$$

$$6 + 6 + 6 + 6^3 + 6^3 + 6^3 = 666$$

Multiplying the sides of the primitive right triangle 12-35-37 by 18 yields non-primitive sides of 216-630-666.

Even more surprising is the fact that these sides can be written in the Pythagorean Theorem form:

$$(6 \times 6 \times 6)^2 + (666 - 6 \times 6)^2 = 666^2$$

3.6 ARRANGEMENT NUMBERS

Arrangement numbers, more commonly called permutation numbers, or simply permutations, are the number of ways that a number of things can be ordered or arranged. They typically evolve from the question how many arrangements of "n" objects are possible using all "n" objects or "r" objects at a time. We designate the permutations of "n" things taken "n" at a time as ${}_nP_n$ and the permutations of "n" things taken "r" at a time as ${}_nP_r$ where P stands for permutations, "n" stands for the number of things involved, and "r" is less than "n". To find the number of permutations of "n" dissimilar things taken "n" at a time, the formula is ${}_nP_n$ = n! which is "n" factorial which means:

$$n(n-1)(n-2)(n-3)\ldots3\times2\times1$$

Example: How many ways can you arrange the letters A & B. Clearly 2 which is 2 x 1 = 2, namely AB and BA.

How many ways can you arrange the letters A, B & C in sets of three? Clearly ${}_3P_3$ = 3 x 2 x 1 = 6, namely ABC, CBA, BAC, CAB, ACB, and BCA.

How many ways can you arrange A, B, C & D in sets of four? Clearly ${}_4P_4$ = 4 x 3 x 2 x 1 = 24.

To find the number of permutations of "n" dissimilar things taken "r" at a time, the formula is:

$$nPr = n(n-1)(n-2)(n-3)\ldots\ldots(n-r+1)$$

Example: How many ways can you arrange the letters A, B, C, and D using 2 at a time? We have ${}_4P_4$ = 4 x (4-2+1) = 4 x 3 = 12 namely AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, and DC.

How many 3-place numbers can be formed from the digits 1, 2, 3, 4, 5, and 6, with no repeating digit? Then we have ${}_6P_3$ = 6 x 5 x (6-3+1) = 6 x 5 x 4 = 120.

How many 3-letter arrangements can be made from the entire 26 letter alphabet with no repeating letters? We now have ${}_{26}P_3$= 26 x 25 x (26-3+1) = 26 x 25 x 24 = 15,600.

Lastly, four persons enter a car in which there are six seats. In how many ways can they seat themselves? ${}_6P_4$ = 6 x 5 x 4 x (6-4+1) = 6 x 5 x 4 x 3 = 360.

Another permutation scenario is one where you wish to find the permutations of "n" things, taken all at a time, when "p" things are of one kind, "q" things of another kind, "r' things of a third kind, and the rest are all different. Without getting into the derivation,

$$nPn(p,q,r,s) = \frac{n!}{(p! \times q! \times r!)}$$

Example: how many different permutations are possible from the letters of the word committee taken all together? There are 9 letters of which 2 are m, 2 are t, 2 are e, and 1 c, 1 o, and 1 i.

Therefore, the number of possible permutations of these 9 letters is:

$$9P9(2,2,2,1,1,1) = \frac{9!}{(2! \times 2! \times 2! \times 1 \times 1 \times 1)} = \frac{362{,}880}{8} = 45{,}360$$

Automorphic Numbers

Automorphic numbers are numbers of "n" digits whose squares end in the number itself. Such numbers must end in 1, 5, or 6 as these are the only numbers whose products produce 1, 5, or 6 in the units place. For example, the square of 1 is 1; the square of 5 is 25; the square of 6 is 36.

What about 2 digit numbers ending in 1, 5, or 6? It is well known that all 2 digit numbers ending in 5 result in a number ending in 25 making 25 a 2 digit automorphic number with a square of 625. No other 2 digit numbers ending in 5 will produce an automorphic number.

Is there a 2 digit automorphic number ending in 1? We know that the product of 10A + 1 and 10A + 1 is $100A^2$ + 20A + 1. "A" must be a number such that 20A produces a number whose tens digit is equal to "A". For "A" = 2, 2 x 20 = 40 and 4 is not 2. For "A" = 3, 3 x 20 = 60 and 6 is not 3. Continuing in this fashion, we find no 2 digit automorphic number ending in 1.

Is there a 2 digit automorphic number ending in 6? Again, we know that the product of 10A + 6 and 10A + 6 is $100A^2$ + 120A + 36. "A" must be a number such that 120A produces a number whose tens digit added to 3 equals "A". For "A" = 2, 2 x 120 = 240 and 4 + 3 = 7 which is not 2. For "A" = 3, 3 x 120 = 360 and 6 + 3 = 9 which is not 3. Continuing in this manner through A = 9, for "A" = 7, we obtain 7 x 120 = 840 and 4 + 3 = 7 = "A" making 76 the only other 2 digit automorphic number whose square is 5776.

By the same process, it can be shown that the squares of every number ending in 625 or 376 will end in 625 or 376.

The sequence of squares ending in 25 are 25, 225, 625, 1225, 2025, 3025, etc. The nth square number ending in 25 can be derived directly from $N(n)^2 = 100n(n - 1) + 25$. (This expression derives from the Finite Difference Series of the squares.)

BINARY NUMBERS

Binary numbers are the natural numbers written in base 2 rather than base 10. While the base 10 system uses 10 digits, the binary system uses only 2 digits, namely 0 and 1, to express the natural numbers in binary notation. The binary digits 0 and 1 are the only numbers used in computers and calculators to represent any base 10 number. This derives from the fact that the numbers of the familiar binary sequence, 1, 2, 4, 8, 16, 32, 64, 128, etc., can be combined to represent every number. To illustrate, 1 = 1, 2 = 2, 3 = 1 + 2, 4 = 4, 5 = 1 + 4, 6 = 2 + 4, 7 = 1 + 2 + 4, 8 = 8, 9 = 1 + 8, 10 = 2 + 8, 11 = 1 + 2 + 8, 12 = 4 + 8, and so on.

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REFERENCES

[1] file:///E:/Papaer/Differnt%20kind%20of%20No/Numbers,%20Part%20II.html