Queuing system with Feed back

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Abstract- *Queuing theory is playing a vital role in the management of various systems involving congestions. Customers' dissatisfaction and impatience are the key areas to be looked into. Feedback in queueing literature represents customer dissatisfaction because of inappropriate quality of service. In case of feedback, after getting partial or incomplete service, customer retries for service. Such customers are termed as retained customers. Keeping in mind these concepts, a single-server Markovian feedback queuing model with discouraged arrivals, reneging and retention of reneged customers is studied. Finally, some important queuing models are derived as special cases of this model.*

Keywords- Service Channels, Queue Discipline, Queuing Model.

I. INTRODUCTION

The queueing theory had its origin in 1909, when A.K. Erlang published his fundamental paper relating to the study of congestion in telephone traffic. A queue or waiting line is formed when units needing some kind of service arrive at a service channel that offers such facility. A queueing system can be described by the flow of units for service, forming or joining the queue, if service is not immediately available and leaving the system after being served.

The queue may be considered as customers at a bank counter or at a reservation counter, calls arriving at a telephone exchange, vehicular traffic at a traffic interaction machines for repair before a repairman, airplanes waiting for take off at a busy airport.

Below we briefly describe some situations in which queueing is important and Queueing theory tries to answer some questions related to the situations. For example in Super market; how long do customers have to wait at the checkouts? What happens with the waiting time during peak-hours? Are there enough checkouts?, in Production system; that is a machine produces different types of products. What is to production lead time of an order? What is the reduction in the lead time when we have an extra machine? Should we assign priorities to the orders?, in Data communication; that is in computer communication network standard packages called cells are transmitted over links from one switch to the next. In each switch incoming cells can be buffered when the incoming demand exceeds the link capacity. Once the buffer is full incoming cells will be lost. What is the cell delay at the switches? What is the traction of cells that will be lost? What is a good size of the buffer?. and in Mainframe Computer; that is many customers are connected to a big main frame computer rendered all financial transactions. Is the capacity of the main frame computers sufficient? What happens when the use of customers increases?

II. CHARACTERISTICS OF QUEUEING SYSTEM

In queueing theory, a queueing model is used to approximate a real queueing situation. So queueing behavior can be analysed mathematically. The basic factures which characterise a system are ; The input process, the service mechanism, the queueing discipline and the number of service channels. These are briefly described below:

1. The Input Process

The input describes the manner in which units arrive and join the system. The system is called a delay or loss system, depending on whether a unit who, on arrival, finds the service facility occupied, join or leaves the system. The system may have either a limited or an unlimited capacity for holding units. The source from which the units come may be finite or infinite. A unit may arrive either single or in a group. The interval between two consecutive arrivals is called the interarrival time or interval.

2. The service mechanism

The service mechanism describes the manner in which service is rendered. A unit may be served either singly or in a batch. The time required for servicing a unit is called the service time.

3. The Queue Discipline

The queue discipline indicates the way in which the units form a queue and are served. The usual discipline is first come first served [FCFS] or first in first out [FIFO], other rules, such as last come first served or random ordering before service are also adopted.

4. The number of Service Channels

The system may have a single channel or a number of parallel cannels for service.

III. NOTATION

A very convenient notation designed by Kendall in 1951. It consists of a three-part descriptor A/B/C. The first and second symbols denote the interarrival and service time distributions respectively, and the third denotes the number of service channels or servers.

A and B usually take one of the following symbols:

M	:	for exponential (Markovian) distribution
Ek	:	for Erlang-k distribution
G	:	for arbitrary (General) distribution
D	:	for fixed (deterministic) interval

IV. THE QUEUEING MODEL M/M/1

The single server model envisages Poisson input with parameter λ and exponential service time with parameter μ and with FCFS queue discipline. The interarrival time and service time distribution are

$$A(x) = 1 - e^{-\lambda x} = x \geq 0 \text{ and}$$

$$B(x) = 1 - e^{-\mu x} = x \geq 0 \text{ respectively.}$$

Let N(t) be the number in the system at the time instant t ≥ 0, then { N(t), t ≥ 0 } is a Markov process in continuous time with denumerable number of states {0, 1, 2, ...}. Here transitions take place only to two neighboring states. This is a type of birth and death process with the birth parameter $\lambda_n = \lambda$, n ≥ 0 and the death parameter $\mu_n = \mu$, n ≥ 1, $\mu_0 = 0$.

Let p_r {N(t)=n/N(0)=0}= p_n (t),n≥0.

Using probabilistic arguments, we get, the following differential – difference equation.

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t) \quad ...(1.1)$$

$$P_n'(t) = (\lambda+\mu) P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t), n \geq 1 \qquad (1.2)$$

Let us assume the existence of steady-state for the system. Then as t tends to infinity, $p_n(t)$ tends to a limit p_n, independent of t. The equations of steady – state probabilities

p_r(N = n) = p_n can be obtained by putting P_n'(t) = 0 and replacing $P_n(t)$ by P_n, applying limit t → ∞ on both sides of (1.1) and (1.2) we get the difference equation

$$0 = -\lambda P_0 + \mu P_1 \qquad (1.3)$$

$$0 = -(\lambda+\mu) P_n + \lambda P_{n-1} + \mu P_{n+1}, \; n = 1,2... \qquad (1.4)$$

solving (1.3) and (1.4) we have

$$P_n = P_0 \rho^n, \; n \geq 1, \qquad (1.5)$$

where $\rho = \frac{\lambda}{\mu}$ and

$$\sum_{n=0}^{\infty} P_n = 1 \qquad (1.6)$$

solving (1.5)and (1.6) we have

$$P_0 = 1 - \rho$$

$$P_n = (1-\rho)\rho^n, \; n \geq 0 \qquad (1.7)$$

Some of the performance measures related to the model are

$$E(N) = \sum_{n=0}^{\infty} n P_n = \frac{\rho}{1-\rho} \qquad (1.8)$$

$$E(N^2) = \sum_{n=0}^{\infty} n^2 P_n$$

$$E(N^2) = \frac{\rho^2 + \rho}{(1-\rho)^2} \quad \text{and}$$

$$Var(N) = E(N^2) - [E(N)]^2 = \frac{\rho}{(1-\rho)^2} \qquad (1.9)$$

V. THE QUEUING MODEL M/G/1

The input process is Poisson with intensity λ and that the service times are independent by and identically distributed random variables having an arbitrary distribution with mean 1/μ. Denote the service by V, its distribution function by B(t), its probability density function, when it exists, by b(t) = $B'(t)$ dt and is Laplace Transform (L.T) by

$$B^*(S) = \int_0^{\infty} e^{-st} dB(t)$$

Let t_n be the instant at which the n^{th} unit completes his service and leaves the system. These points t_n are the regeneration points of the process {N(t)=t≥0}]. Where N(t), the number of customers in the system at time t.

Write $X_n \equiv N(t_{n+0})$, n = 0,1,2,... and A_n the random variable corresponds to the number of arrivals during the service time of the n^{th} unit. Then

$$X_{n+1} = \begin{cases} X_{n-1} + A_{n+1}, & \text{if } X_n \geq 1 \\ A_{n+1}, & \text{if } X_n = 0 \end{cases} \qquad (1.10)$$

Now the service times of all the units have some distribution so that $A_n = A$ for n = 1,2 ... we have

p_r {A=r/service time of a units is t}

$$= \frac{e^{-\lambda t}(\lambda t)^r}{r!} \text{ and}$$

denote $k_r = p_r$ {A=r}

$$= \int_0^\infty \frac{e^{-\lambda t}(\lambda t)^r}{r!} dB(t), \; r = 0, 1, 2, \ldots$$

gives the distribution of A, the number of arrivals during the service time of a unit. The transition probabilities

$$p_{ij} = p_r \{X_{n+1} = j / X_n = i\}$$

given by

$$\left.\begin{aligned} p_{ij} &= K_{j-i+1}, \; i \geq 1, \; j \geq i-1 \\ &= 0, i \geq 1, \; j < i-1 \\ p_{oj} &= p_{ij} = k_j, \; j \geq 0 \end{aligned}\right\} \qquad (1.11)$$

The stochastic processes $\{X_n. \; n \geq 0\}$ is a Markov chain having transition probability matrix.

$$p = \begin{bmatrix} K_o & K_1 & K_2 & \ldots\ldots\ldots. \\ K_o & K_1 & K_2 & \ldots\ldots\ldots \\ 0 & K_o & K_1 & \ldots\ldots\ldots \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots & \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots & \end{bmatrix}$$

As every state can be reached from every other state, the Markov chain $\{X_n\}$ is irreducible. Again as $P_{ii} \geq 0$, the chain is aperiodic. It can also be shown that, when the traffic intensity $\rho = \frac{\lambda}{\mu} < 1$, the chain is persistent, non-null and hence ergodic. Apply the ergodic theorem of Markov chain, the limiting probabilities.

$$V_j = \lim_{n\to\infty} P_{ij}^n, \; j = 0,1,2. \ldots.$$

exist and are independent of the initial state i. The probabilities $V = (V_o, V_1 \ldots.)$, $\sum V_j = 1$, are given as the unique solutions of

$$V = VP$$

Let $K(s) = \sum K_j S^j$ and $V(s) = \sum V_j S^j$ denote the probability generating function of the distributions of $\{K_j\}$ and $\{V_j\}$ respectively.

We have $K(S) = B^*(\lambda - \lambda S)$

$$E(A) = k'(1) = -\lambda B^*(0)$$

$$E(A) = \rho \qquad \ldots(1.12)$$

Now V = VP gives an infinite system of equations. Multiplying the $(K + 1)^{st}$ equation by S^K, K = 0,1, ... and adding over K and using (1.12),

We get

$$V(S) = \frac{(1-\rho)(1-s)B^*(\lambda - \lambda s)}{B^*(\lambda - \lambda s) - s}$$

The mean number of customers in the system is

$$V'(1) = E\{N\}$$

$$= \rho + \frac{\lambda^2}{2(1-\rho)} E(V^2)$$

VI. THE FEEDBACK QUEUE

The feedback queue is defined as follows:

Upon completion of a service, the customer engages a switch. At this point a decision is made to feed the customer for more service or to have the customer depart the system. A queueing model with above designed service policy is called a feedback queueing model.

VII. CONCLUSION

This paper studies a single server Markovian feedback queuing model with Characteristics of queueing system, The service mechanism, The Queue Discipline, Service Channels, Notation, Queueing Models. We obtain the steady-state solution and different measures of effectiveness. Some queuing models are derived as special cases of this model. This model finds its applications in businesses and industries facing the problems of customers impatience and dissatisfaction in service.

REFERENCES

[1] Takacs, L., A single server queue with feed back, the Bell sys. Tech. Jr., 42, 509 – 519, 1963.

[2] D' Avignen, G.R and Disney, R.L, Queues with instantaneous feed back, mgt. Sci., 24, 2, 168 – 179, 1977.

[3] Disney, R.L., A. note on Sojourn times in M/G/1 queue with instantaneous Bernoulli feedback, Nov. Res. Logi. Qrt., 28, 679 – 683, 1981.

[4] Disney, R.L. McNickle, D.C, and Simon, B., The M/G/1 queue with instantaneous Bernoulli feedback, Nov. Res. Logi. Qrt., 27, 635 – 644, 1980.