Near Skolem Difference Mean Labeling of Some Cycle Related Graphs

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Abstract- *Let G be a (p,q) graph and $f:V(G)\to\{1,2,\dots,p+q-1,p+q+2\}$ be an injection. For each edge $e=uv$, the induced edge labeling f^* is defined as follows:*

$$f^*(e)=\begin{cases}\dfrac{|f(u)-f(v)|}{2} & \text{if } |f(u)-f(v)| \text{ is even}\\ \dfrac{|f(u)-f(v)|+1}{2} & \text{if } |f(u)-f(v)| \text{ is odd}\end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and from $\{1,2,3,\dots,q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, we investigate the Near Skolem Difference Mean labeling of the cycle related graph C_n@ $K_{1,m}$ and the graph $C_n \otimes K_{1,m}$.

Keywords- Cycle, graph labeling, Near Skolem difference mean labeling, Near Skolem difference meangraph.

I. INTRODUCTION

The graphs considered in this paper are finite, undirected and simple graphs. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary [1].

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. A detailed survey of several types of graph labeling is found in [2]. The notion of skolem difference mean labeling was due to Murugan and Subramanian [3]. It motivates us to define near skolem difference mean labelling.

In this paper, we extend the study on Near skolem difference mean labeling and show that the graph C_n@ $K_{1,m}$ *and the graph* $C_n \otimes K_{1,m}$ are Near Skolem difference mean graphs. We use the following definitions in the subsequent section.

Definition 1.1: Let G be any graph and $K_{1,m}$ be a star with m spokes. We denote by G @ $K_{1,m}$, the graph obtained from G by identifying one vertex of G with the central vertex of $K_{1,m}$.

Definition 1.2: Let G be any graph and $K_{1,m}$ be a star with m spokes. We denote by $G \otimes K_{1,m}$, the graph obtained from G by identifying one vertex of G with any vertex of $K_{1,m}$ other than the center.

II. MAIN RESULT

Definition 2.1: A graph $G=(V,E)$ with p vertices and q edges is said to have Nearskolem difference mean labeling if it is possible to label the vertices $x\in V$ with distinct elements $f(x)$ from $\{1,2,\dots,p+q-1,p+q+2\}$ in such a way that each edge $e=uv$, is labeled as $f^*(e)=\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $f^*(e)=\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and from $\{1,2,\dots,q\}$. A graph that admits a Near skolem difference mean labeling is called a Near skolem difference mean graph.

Theorem 2.2: *The graph C_n @ $K_{1,m}$ is Near skolem difference mean for every $n\ge 3, m\ge 1$.*

Proof: We consider two cases:

Case(i): *Let* $n=2k+1$.

Let G be the graph C_{2k+1} @ $K_{1,m}$.

Let $V(G)=\{u_i,v_j,w_s/1\le i\le k+1, 1\le j\le k, 1\le s\le m\}$.

Identify w with the vertex u_{k+1} of C_{2k+1}. Then

$E(G)=\{u_iu_{i+1},v_jv_{j+1},u_1v_1,u_{k+1}v_k,ww_s/1\le i\le k, 1\le j\le k-1, 1\le s\le m\}$

.

Then $|V(G)|=2k+m+1$ and $|E(G)|=2k+m+1$

Define $f:V(G) \to \{1,2,\dots,4k+2m+1,4k+2m+4\}$ as follows:

***Subcase (i)** When k is odd:*

$f(u_{2i+1}) = 4i+2, \quad 0 \le i \le \frac{k-1}{2}.$

$f(u_{2i}) = 4k+2m+8-4i, 1 \le i \le \frac{k-1}{2}.$

$f(u_{k+1}) = 2k+2m+5$

$f(v_{2i+1}) = 4k+2m+1-4i, 0 \le i \le \frac{k-1}{2}.$

$f(v_{2i}) = 4i+1, \quad 1 \le i \le \frac{k-1}{2}.$

$f(w_s) = 2k+2m+3-2s, \quad 1 \le s \le m.$

***Subcase (ii)** When k is even:*

$f(u_{2i+1}) = 4i+2, \quad 0 \le i \le \frac{k}{2}.$

$f(u_{2i}) = 4k+2m+8-4i, \quad 1 \le i \le \frac{k}{2}.$

$f(v_{2i+1}) = 4k+2m+1-4i, \quad 0 \le i \le \frac{k-2}{2}.$

$f(v_{2i}) = 4i+1, \quad 1 \le i \le \frac{k}{2}.$

$f(w_s) = 2k+4+2s, \quad 1 \le s \le m.$

In both the subcases, let f^* be the induced edge labeling of f.

$f^*(u_iu_{i+1}) = 2k+m+3-2i, \quad 1 \le i \le k$

$f^*(u_1v_1) = 2k+m$

$f^*(v_iv_{i+1}) = 2k+m-2i, \quad 1 \le i \le k-1$

$f^*(u_{k+1}v_k) = 1$

$f^*(u_{k+1}w_s) = 1+s, \quad 1 \le s \le m$

The induced edge labels are all distinct and are $\{1,2,\dots,2k+m+1\}$.

Case(ii): $Let\ n = 2k$

Let G be the graph C_{2k} @ $K_{1,m}$.

Let $V(G) = \{u_i, v_j, w_s / 1 \le i \le k-1, 1 \le j \le k-1, 2 \le s \le m\}$.

Then $|V(G)| = 2k+m$ and $|E(G)| = 2k+m$.

Define $f:V(G) \to \{1,2,\dots,4k+2m-1,4k+2m+2\}$ as follows:

***Subcase (i)** When k is odd:*

$f(u_{2i+1}) = 4i+2, \quad 0 \le i \le \frac{k-3}{2}.$

$f(u_k) = 2k-1$

$f(u_{2i}) = 4k+2m+6-4i, \quad 1 \le i \le \frac{k-3}{2}.$

$f(u_{k-1}) = 2k+2m+12$

$f(v_{2i+1}) = 4k+2m-1-4i, 0 \le i \le \frac{k-3}{2}.$

$f(v_k) = 2k+2m+2$

$f(v_{2i}) = 4i+1, \quad 1 \le i \le \frac{k-3}{2}.$

$f(v_{k-1}) = 2k$

$f(w_s) = 2k-1+2s, \quad 1 \le s \le m.$

***Subcase (ii)** When k is even:*

$f(u_{2i+1}) = 4i+2, \quad 0 \le i \le \frac{k-2}{2}.$

$f(u_{2i}) = 4k+2m+6-4i, \quad 1 \le i \le \frac{k-2}{2}.$

$f(u_k) = 2k+2m+5$

$f(v_{2i+1}) = 4k+2m-1-4i, \quad 0 \le i \le \frac{k-2}{2}.$

$f(v_{2i}) = 4i+1, \quad 1 \le i \le \frac{k}{2}.$

$f(w_s) = 2k+2m+6-2s, \quad 1 \le s \le m.$

Let f^* be the induced edge labeling of f. Then,

$f^*(u_iu_{i+1}) = 2k+m+2-2i, \quad 1 \le i \le k-1.$

$f^*(v_iv_{i+1}) = 2k+m-1-2i, \quad 1 \le i \le k-1.$

$f^*(u_1w_1) = 2k+m-1$

$f^*(u_kv_k) = m+2$

$f^*(u_kw_s) = s, \quad 1 \le s \le m.$

The induced edge labels are all distinct and are $\{1,2,..,2k+m\}$.

Hence, the given graph is Near skolem difference mean.

***Example 2.3:** The Near skolem difference mean labeling of C_{11} @ $K_{1,6}$, C_{11} @ $K_{1,7}$, C_{13} @ $K_{1,6}$, $C_{1,3}$ @ $K_{1,7}$, C_{14} @$K_{1,7}$, C_{14} @ $K_{1,8}$, C_{12} @ $K_{1,7}$, C_{12} @$K_{1,8}$ are given fig 1, fig 2 , fig 3, fig 4 , fig 5 , fig 6, fig 7 and fig 8 respectively,*

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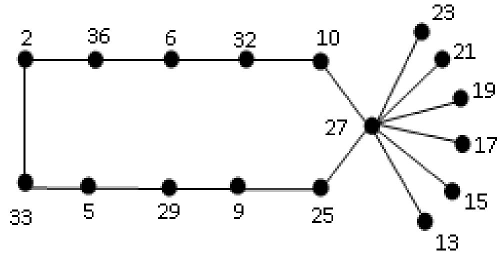


Fig 1

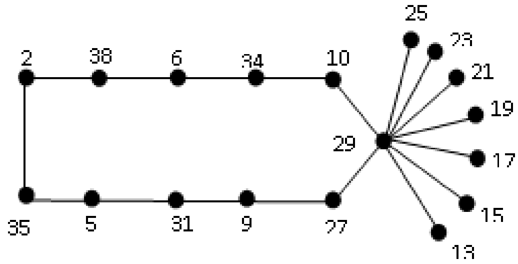


Fig 2

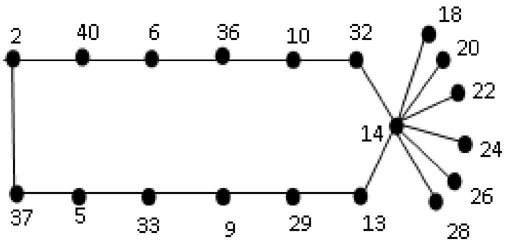


Fig 3

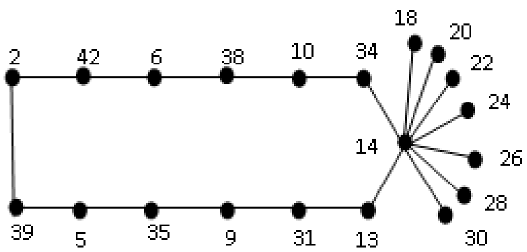


Fig 4

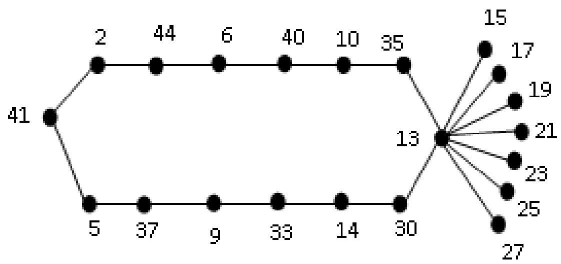


Fig 5

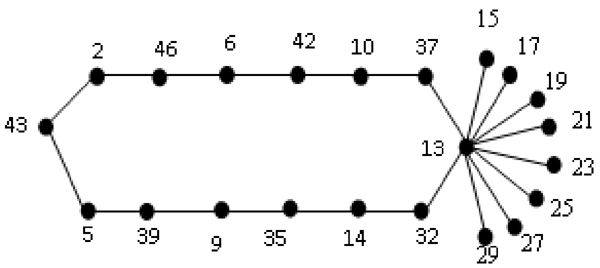


Fig 6

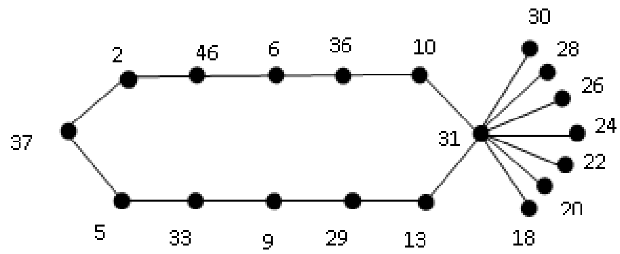


Fig 7

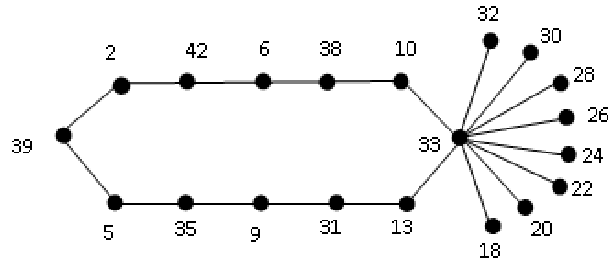


Fig 8

Theorem 2.4: *The graph* $C_n \otimes K_{1,m}$, *is Near skolem difference mean for every* $n \geq 3, m \geq 1$.

Proof: We consider two cases:

Case(i): *When* $n = 2k + 1$.

Let G be the graph $C_{2k+1} \otimes K_{1,m}$, where $k \geq 1$ *and* $m \geq 1$

Let $V(G) = \{u_i, v_j, w, w_s \,/ 1 \leq i \leq k+1, 1 \leq j \leq k, 1 \leq s \leq m\}$.

Identify w_1 with the vertex u_1 of C_{2k+1}. Then,

$E(G) = \{u_1 v_1, u_{k-1} v_k, u_i u_{i+1}, v_j v_{j+1}, u_i w, w w_s \,/ 1 \leq i \leq k, 1 \leq j \leq k-1, 2 \leq s \leq m\}$

.

Then $|V(G)| = 2k + m + 1$ and $|E(G)| = 2k + m + 1$.

Let $f: V(G) \rightarrow \{1, 2, \ldots, 4k + 2m + 1, 4k + 2m + 4\}$ be defined as follows:

Subcase (i) *When* k *is odd*

$f(u_{2i+1}) = 4i + 2,\quad 0 \leq i \leq \frac{k-1}{2}$.

$f(u_{2i}) = 4k + 2m + 8 - 4i,\quad 1 \leq i \leq \frac{k+1}{2}$.

$f(v_{2i+1}) = 4k + 2m + 1 - 4i,\quad 0 \leq i \leq \frac{k-1}{2}$.

$f(v_{2i}) = 4i + 1,\quad 1 \leq i \leq \frac{k-1}{2}$.

$f(w) = 2m + 4$

$f(w_{2i}) = 4i,$ $1 \le i \le \frac{m-1}{2}, m \text{ is odd}$; $1 \le i \le \frac{m-2}{2}, m \text{ is even}$.

$f(w_m) = 2m+2$

$f(w_{2i+1}) = 4i+3$ $1 \le i \le \frac{m-3}{2}, m \text{ is odd}$; $1 \le i \le \frac{m-2}{2}, m \text{ is even}$.

***Subcase (ii)** When k is even*

$f(u_{2i+1}) = 4i+2,$ $0 \le i \le \frac{k}{2}$.

$f(u_{2i}) = 4k+2m+8-4i,$ $1 \le i \le \frac{k}{2}$.

$f(v_{2i+1}) = 4k+2m+1-4i,$ $0 \le i \le \frac{k-2}{2}$.

$f(v_{2i}) = 4i+1,$ $1 \le i \le \frac{k}{2}$.

$f(w) = 2m+4$

$f(w_{2i}) = 4i,$ $1 \le i \le \frac{m-1}{2}, m \text{ odd}$; $1 \le i \le \frac{m}{2}, m \text{ even}$.

$f(w_{2i+1}) = 4i+3,$ $1 \le i \le \frac{m-1}{2}, m \text{ odd}$; $1 \le i \le \frac{m-2}{2}, m \text{ even}$.

In both the subcases, let f^* be the induced edge labeling of f. Then,

***Case (a)** When k is odd*

$f^*(u_iu_{i+1}) = 2k+m+3-2i,$ $1 \le i \le k$
$f^*(v_iv_{i+1}) = 2k+m-2i,$ $1 \le i \le k-1$
$f^*(u_1v_1) = 2k+m$
$f^*(u_{k+1}v_k) = 2$
$f^*(u_1w) = m+1$
$f^*(ww_i) = m+2-i,$ $2 \le i \le m-1$
$f^*(ww_m) = 1$

***Case (b)** When k is even:*

$f^*(u_iu_{i+1}) = 2k+m+3-2i,$ $1 \le i \le k$
$f^*(v_iv_{i+1}) = 2k+m-2i,$ $1 \le i \le k-1$
$f^*(u_1v_1) = 2k+m$
$f^*(u_{k+1}v_k) = 1$
$f^*(u_1w) = m+1$
$f^*(ww_i) = m+2-i,$ $2 \le i \le m$

From both the cases, the induced edge labels are all distinct and are $\{1,2,\dots,2k+m+1\}$

***Case(ii):** When $n = 2k$*

Let G be the graph $C_{2k+1} \otimes K_{1,m}$.
Let $V(G) = \{u_i, v_j, w, w_s / 1 \le i \le k, 1 \le j \le k, 1 \le s \le m\}$.
Identify w_1 with the vertex u_1 of C_{2k}. Then,
$E(G) = \{u_1v_1, u_kv_k, u_iu_{i+1}, v_jv_{j+1}, u_1w, ww_s / 1 \le i \le k-1, 1 \le j \le k-1, 2 \le s \le m\}$
Then $|V(G)| = 2k+m$ and $|E(G)| = 2k+m$.
Define $f: V(G) \to \{1,2,\dots,4k+2m-1, 4k+2m+2\}$ as follows:

***Subcase (i)** When k is odd:*

$f(u_{2i+1}) = 4i+2,$ $0 \le i \le \frac{k-3}{2}$
$f(u_k) = 2k-1$
$f(u_{2i}) = 4k+2m+6-4i,$ $1 \le i \le \frac{k-3}{2}$.
$f(u_{k-1}) = 2k+2m+7$
$f(v_{2i+1}) = 4k+2m-1-4i,$ $0 \le i \le \frac{k-3}{2}$.
$f(v_k) = 2k+2m+2$
$f(v_{2i}) = 4i+1,$ $1 \le i \le \frac{k-3}{2}$.
$f(v_{k-1}) = 2k$
$f(w) = 2m+2$

$f(w_{2i}) = 4i,$ $1 \le i \le \frac{m-1}{2}, m \text{ odd}$; $1 \le i \le \frac{m}{2}, m \text{ even}$.

$f(w_{2i+1}) = 4i+3,$ $1 \le i \le \frac{m-1}{2}, m \text{ odd}$; $1 \le i \le \frac{m-2}{2}, m \text{ even}$

***Subcase (ii)** When k is even:*

$f(u_{2i+1}) = 4i+2,$ $0 \le i \le \frac{k-2}{2}$.

$f(u_{2i}) = 4k+2m+6-4i,$ $1 \le i \le \frac{k}{2}$.

$f(v_{2i+1}) = 4k+2m-1-4i,$ $0 \le i \le \frac{k-2}{2}$.

$f(v_{2i}) = 4i+1,$ $1 \le i \le \frac{k-2}{2}$.

$f(v_k) = 2k+2$
$f(w) = 2m+2$

$f(w_{2i}) = 4i,$ $1 \le i \le \frac{m}{2}, m \text{ even}$; $1 \le i \le \frac{m-1}{2}, m \text{ odd}$.

$f(w_{2i+1}) = 4i+3,$ $1 \le i \le \frac{m-2}{2}, m$ even; $1 \le i \le \frac{m-1}{2}, m \text{ odd}$.

In both the subcases, let f^* be the induced edge labeling of f. Then,

$f^*(u_iu_{i+1}) = 2k+m+2-2i,$ $1 \le i \le k-1$

$f^*(v_i v_{i+1}) = 2k + m - 1 - 2i, \qquad 1 \le i \le k-1$
$f^*(u_1 v_1) = 2k + m - 1$
$f^*(u_k v_k) = m + 2$
$f^*(u_1 w) = m$
$f^*(ww_i) = m - i, \qquad 1 \le i \le m.$

The induced edge labels are all distinct and are $\{1,2,.,2k+m\}$

Hence the graph $C_n \otimes K_{1,m}$ is Near skolem difference mean for every $n \ge 3, m \ge 1$.

Example 2.5: *The Near skolem difference mean of* $C_{11} \otimes K_{1,7}, C_{11} \otimes K_{1,8}, C_{13} \otimes K_{1,9}, C_{13} \otimes K_{1,8}, C_{14} \otimes K_{1,7}, C_{14} \otimes K_{1,8}, C_{12} \otimes K_{1,7}, C_{12} \otimes K_{1,8}$ *fig 9, fig 10, fig 11, fig 12, fig 13, fig 14, fig 15 and fig 16 respectively.*

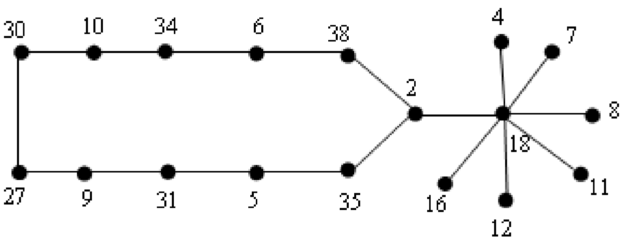


fig 9

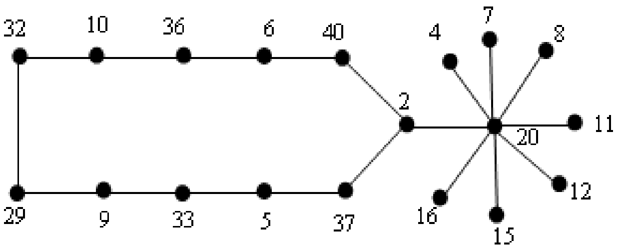


fig 10

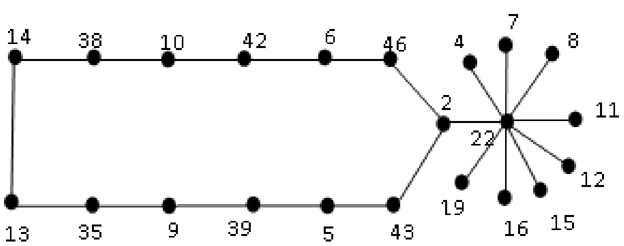
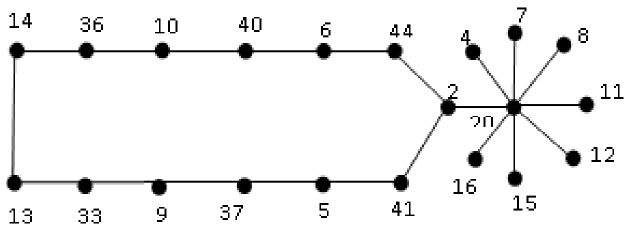


fig 11



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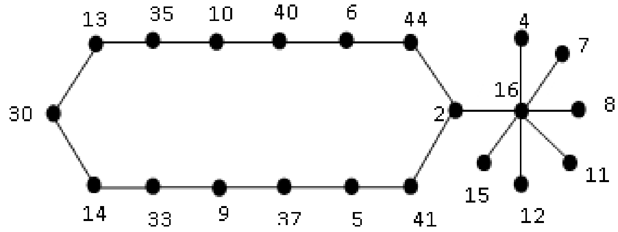

fig 13

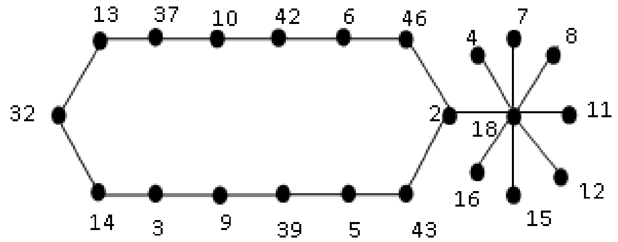

fig 14

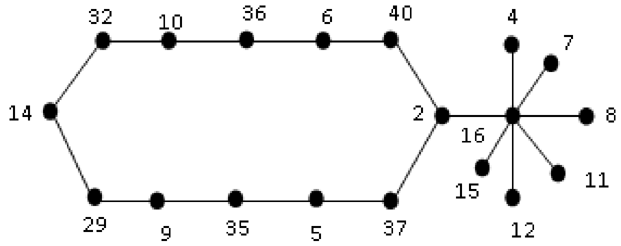

fig 15

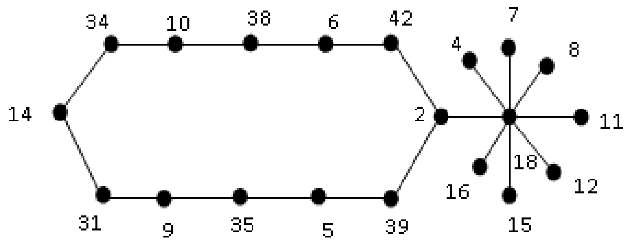

fig 16

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