On Some Connections in Sasakian Manifold

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Abstract *- In this chapter, we have defined two quarter symmetric metric-F-T-connections in a Trans sasakian manifold, we have shown, by following the patterns of K. Yano (1976) and Mishra and Pandey (1980), that if the curvature tensors with respect to these connections vanishes in a a-sasakian manifold, then the Contact Bochner Curvature Tensor also vanishes in the same manifold, also we have shown that if the curvature tensors with respect to these quarter symmetric metric F-T- connection vanishes and if a=0, then the manifold is either a cosymplectic one, or b-Kenmotsu manifold with a specific value of b. Similar results we have also obtained analogously for the contact conformal connections in Trans sasakian manifold.*

I. INTRODUCTION

Let V_n (n= 2m+1) be an almost contact metric manifold equipped with a structure tensor F, of type (1.1) a contravariant vector T, a-1-form A and a metric tensor g, satisfying

(1.1)(a) $F^2X = -X + A(X)T$

(1.1)(b) $FT = 0$ and

(1.2)(c) $g(T,X) = A(X)$

Also, a fundamental 2-form 'F in V_n is defined as

(1.3) ${}'F(X,Y) = g(\bar{X},Y) = -g(X,\bar{Y}) = -{}'F(Y,X)$

Then we call the structure bundle {F,T,A,g} an almost contact metric structure. An almost contact metric manifold V_n is said to be normal if

(1.4)(a) $(dA)(X,Y) + 2\,{}'F(X,Y) = 0$

where,

(1.4)(b) $(dA)(X,Y) = (D_XA)(Y) - (D_YA)(X)$

Here D is is the Riemannian Connection and 'd' is the exterior derivative in V_n with regard to the metric tensor g.

Now, an almost contact metric structure {F,T,A,g} on V_n is called trans-sasakian structure [2], [3], if

(1.5) $(D_XF)(Y) = \alpha\left(g(X,Y)T - A(Y)X\right) + \beta\{{}'F(X,Y)T - A(Y)\bar{X}\}$

It can easily be seen that a trans-sasakian manifold is normal and in view of (1.5) one can easily obtain in V_n the relations

(1.6)(a) $D_XT = -\alpha\bar{X} + \beta\left(X - A(X)T\right)$

and

(1.6)(b) $(D_XA)(Y) = -\alpha F(X,Y) + \beta\, g(\bar{X},\bar{Y})$

REMARK

(1) In the above and in what follows, X,Y,Z,......, etc. are tangent vector fields in V_n.

II. CONTACT CONFORMAL CONNECTION IN A TRANS-SASAKIAN MANIFOLD

Let us consider a conformal change of the metric tensor g which induces a new metric tensoer , given by

(4.2.1) $\tilde{g}(X,Y) = e^{2p}\, g(X,Y)$

with regard to this metric we take an affine connection B, which satisfies:-

(4.2.2) $(B_z\tilde{g})(Y,Z) = B_z\{e^{2p}\, g(Y,Z)\} = e^{2p}\, p(X)A(Y)A(Z),$

where p is a scalar point function in V_n and

(4.2.3) being covariant derivative of the scalar p with respect to the metric tensor g, is a 1-form in V_n, where contravariant vector is P. Further, we assume that the torsion tensor of the connection B satisfies:-

(4.2.4) $S(X,Y) = -2\,{}'F(X,Y)U$

where U is certain contravariant vector field. In view of (4.2.2) and (4.2.4), we can easily obtain a relation between the connection B and the Riemannian Connection D[1], given by

(4.2.5) $B_yZ = D_yZ + \{Y - A(Y)T\}\, p(Z) + \{Z - A(Z)T\}\, p(Y)$

$-g(\bar{Y},\bar{Z})\, p + u(Y)\bar{Z} + u(Z)\bar{Y} - {}'F(Y,Z)U,$

where, $u(X) \underset{=}{def}\ g(u,X)$

Now, we suppose B an F-connection so that

$(B_yF)(Z) = 0 = (D_yF)(Z) + \{Y - A(Y)T\}\, p(\bar{Z}) - p(Z)\bar{Y} + p(Y)\bar{Z} - p(Y)\bar{Z}$

$+{}'F(Y,Z)P + g(\bar{Y},\bar{Z})\, p + u(Y)\bar{\bar{Z}} - u(Y)\bar{\bar{Z}} + U(\bar{Z})\bar{Y} + u(Z)$

$\{Y - A(Y)T\} - g(\bar{Y},\bar{Z})\, U + {}'F(Y,Z)\bar{U}$

Using (1.5), the above relation becomes

$\alpha\{g(Y,Z)T - A(Z)Y\} + \beta\{{}'F(Y,Z)T - A(Z)\bar{Y}\} + p(\bar{Z})\{Y - A(Y)T\}$

$-p(Z)\bar{Y} + {}'F(Y,Z)P + g(\bar{Y},\bar{Z})\bar{P} + u(\bar{Z})\bar{Y} + u(Z)\{Y - A(Y)T\}$

$-g(\bar{Y},\bar{Z})U + {}'F(Y,Z)\bar{U} = 0$

Contracting the above equation with respect to Y, we have

$-2m\alpha A(Z) + 2mp(\bar{Z}) - p(\bar{Z}) + 2mu(Z) - u(Z) + A(Z) - u(Z)$

$+A(u)A(Z) = 0$

or $2(m-1)\,p(\bar{Z}) + 2(m-1)U(Z) - 2A(Z)\{m\alpha - A(u)\} = 0$

If we put

$A(U) = u(T) = \alpha,$ then

(4.2.6)(a) $u(Z) = \alpha A(Z) - p(\bar{Z})$ or

(4.2.6)(b) $U = \alpha T + \bar{P}$

here, we put $Q = \bar{P}$ so that $q(Z) = g(Q,Z) = -p(\bar{Z})$ and $p(Q) = q(P) = 0,$ then (4.2.6) becomes

(4.2.7)(a) $u(Z) = \alpha A(Z) + q(Z)$ and

(4.2.7)(b) $U = \alpha T + Q$

Using (4.2.7) in (4.2.5), we have

(4.2.8) $B_YZ = D_YZ + (Y - A(Y)T)\, p(Z) + \{Z - A(Z)T\}\, p(Y)$

$-g(\bar{Y},\bar{Z})\, P + \{\alpha A(Y) + q(Y)\}\bar{Z} + \{\alpha A(Z) + q(Z)\}\bar{Y}$

$-{}'F(Y,Z)\{\alpha T + Q\}$

Further, we suppose that B is a T-connection, Then

$B_YT = 0 = D_YT + p(T)\{Y - A(Y)T\} + \alpha\bar{Y}$

Using (1.6)(a) in this equation, we have

$-\alpha\bar{Y} + \beta\{(Y - A)T\} + p\{Y - A(Y)T\} + \alpha\bar{Y} = 0$

which implies that

(4.2.9) $p(T) = A(P) = -\beta$

Proposition (4.2.1): In a Trans-sasakian manifold the affine connection B, which is an F-T- connection and whose torsion tensor satisfies (4.2.4), is given by (4.2,8) with the conditions $u(T) = a = A(U)$, $p(T) =$ -b$= A(P)$ and $\bar{P} = Q,\ g(Y) = -p(\bar{Y})$

4.3 CURVATURE TENSOR OF CONTACT CONFORMAL CONNECTION

The curvature tensor of the contact conformal connecion B given by (4.2.8) is given by

(4.3.1) $R(X,Y,Z) = B_XB_YZ - B_YB_XZ - B_{[X,Y]}Z$

Using (4.2.8), (4.2.9), (1.1), (1.2), (1.3), (1.5) and (1.6) in the above equation and after a straight forward computation, we obtain

(4.3.2) $R(X,Y,Z)=K(X,Y,Z)-\{X-A(X)T\}{}'P(Y,Z)+\{Y-A(Y)T\}$

${}'P(X,Z)-g(\bar{Y},Z)P(X)+g(\bar{X},\bar{Z})P(Y)-{}'Q(Y,Z)\bar{X}+{}'Q(X,Z)\bar{Y}$

$-{}'F(Y,Z)Q(X)+{}'F(X,Z)Q(Y)-V(X,Y)\bar{Z}-{}'F(X,Y)W(Z)+[(\alpha^2+\beta^2)$

${}'F(Y,Z)\bar{X}-(\alpha^2+\beta^2){}'F(X,Z)\bar{Y}-2\alpha^2{}'F(X,Y)\bar{Z}]$; where

(4.3.3)(a) ${}'P(X,Z)=(D_Y p)(Z)+\alpha^2 A(Y)A(Z)+\alpha A(Z)q(Y)+\alpha A(Y)q(Z)$

$-p(Y)p(Z)+q(Y)q(Z)+\tfrac{1}{2}p(P)g(Y,Z)$

(4.3.3)(b) $P(Y)=D_Y P+\alpha^2 A(Y)T+q(Y)T+\alpha A(Y)Q-p(Y)P+q(Y)Q+\tfrac{1}{2}p(P)\{(Y-A)(Y)T\}$

(4.3.4)(a) ${}'Q(Y,Z)=(D_Y q)(Z)-q(Z)p(Y)-q(Y)p(Z)-\alpha A(Z)p(Y)$

$-\alpha A(Y)p(Z)+\tfrac{1}{2}p(P){}'F(Y,Z)$

(4.3.5) $V(X,Y)=-(dq)(X,Y)=-\{(D_X q)(Y)-(D_Y q)(X)\}$ and

(4.3.6) $W(Z)=2[p(Z)Q-q(Z)P+\beta A(Z)Q-\beta(q(Z)T)]$

Since, $p(X)$ is a gradient vector, then

(4.3.7) $(D_X p)(Y)-(D_Y p)(X)=0$

and consequently is symmetric, i.e.

(4.3.8) ${}'P(X,Y)={}'P(Y,X)$

Also, we have $p(T)=-\beta$, then $(D_Y p)(T)+p(D_Y T)=0$ or

$(D_Y p)(T)=-p(-\alpha\bar{Y}+\beta\{Y-A(Y)T\}$

$=\alpha p(\bar{Y})-\beta p(Y)-\beta^2 A(Y)$ or

(4.3.9) $(D_Y p)(T)=-\alpha q(u)-\beta p(Y)-\beta^2 A(Y)$;

which is obtained by using (1.6) (a) and

Now, from (4.3.3) (a), we have

${}'P(Y,T)=(D_Y p)(T)+\alpha^2 A(Y)+\alpha q(Y)+\beta p(Y)$

$=-\alpha q(Y)-\beta p(Y)-\beta^2 A(Y)+\alpha^2 A(Y)+\alpha q(Y)+\beta p(Y)$ or,

(4.3.10) ${}'P(Y,T)=(\alpha^2-\beta^2)A(Y)={}'P(T,Y)$

From which, we get

(4.3.11) ${}'P(\bar{Y},T)=0={}'P(T,\bar{Y})$

Again, by taking covariant derivative of and using (1.6)(a), we obtain

$(D_Y q)(T)=-q(D_Y T)=-q(-\alpha\bar{Y}+\beta\{Y-A(Y)T\}$

$=\alpha q(\bar{Y})-\beta q(Y)=\alpha p(Y)+\alpha\beta A(Y)-\beta q(Y)$

Using above equation in (4.3.4) (a), we obtain

${}'Q(Y,T)=(D_Y q)(T)+\beta q(Y)-\alpha p(Y)+\alpha\beta A(Y)$

$=\alpha p(Y)+\alpha\beta A(Y)-\beta q(Y)+\beta q(Y)-\alpha p(Y)+\alpha\beta A(Y)$ or

(4.3.12) ${}'Q(Y,T)=2\alpha\beta A(Y)$

Further, differentiating covariantly with respect to X, the expression $p(\bar{Z})=-q(Z)$, we have

$(D_Y p)(\bar{Z})+p(D_Y F)(Z)+p(\overline{D_Y Z})=-(D_Y q)(Z)=q(D_Y Z)$

Using (1.5) here in the above equation, we get

$(D_Y p)(\bar{Z})+p(\alpha\{g(Y,Z)T-A(Z)Y\}+\beta\{{}'F(Y,Z)T-A(Z)\bar{Y}\}=-(D_Y q)(Z)$

or

$(D_Y p)(\bar{Z})-\alpha\beta g(Y,Z)-\alpha A(Z)p(Y)-\beta^2\,{}'F(Y,Z)+\beta A(Z)q(Y)=-(D_Y q)(Z)$

Now, taking account of (4.3.3)(a) and (4.3.4)(a) in the above equation, we obtain

${}'P(Y,\bar{Z})-\alpha A(Y)p(Z)-\alpha\beta A(Y)A(Z)-p(Y)q(Z)-q(Y)p(Z)$

$-\beta q(Y)-A(Z)+\frac{p(P)}{2}{}'F(Y,Z)-\alpha\beta g(Y,Z)$

$=-\alpha A(Z)p(Y)-\beta^2,{}'F(Y,Z)+\beta A(Z)q(Y)$

$=-{}'Q(Y,Z)-q(Z)p(Y)-q(Y)p(Z)-\alpha A(Z)p(Y)$

$=-\alpha A(Y)p(Z)+\tfrac{1}{2}p(P){}'F(Y,Z)$ or

${}'P(Y,\bar{Z})=-{}'Q(Y,Z)+\alpha\beta A(Y)A(Z)-\alpha A(Z)p(Y)+\beta A(Z)q(Y)$

$+\alpha\beta g(Y,Z)+\alpha A(Z)p(Y)+\beta^2\,{}'F(Y,Z)q(Y)-\beta A(Z)q(Y)$

(4.3.13) ${}'P(Y,\bar{Z})=-{}'Q(Y,Z)+\alpha\beta g(Y,Z)+\alpha\beta A(Y)A(Z)+\beta^2\,{}'F(Y,Z)$

from which by barring Z, using (1.1) and (4.3.11) we get,

(4.3.14) ${}'Q(Y,\bar{Z})={}'P(Y,Z)-\alpha\beta\,{}'F(Y,Z)+\beta^2 g(Y,Z)$

Barring Y and using symmetricity of 'P and (4.3.13), we obtain-

(4.3.15) ${}'Q(\bar{Y},\bar{Z})={}'Q(Z,Y)+2\alpha\beta g(Y,Z)$ Also, in view of (4.3.14), we get

(4.3.16) ${}'Q(Y,\bar{Z})-{}'Q(Z,\bar{Y})=-2\alpha\beta\,{}'F(Y,Z)$

Further, putting Y=T in (4.3.13) and using (4.3.11), as (4.3.12), we have-

(4.3.17)(a) ${}'Q(T,Z)=2\alpha\beta A(Y,Z)={}'Q(Z,T)$

and

(4.3.17)(b) ${}'Q(T,\bar{Z})=-{}'Q(\bar{Z},T)=0$

Now, putting Y=T in (4.3.5) and using (4.3.17)(a), we can easily obtain

(4.3.18) $V(X,T)=0$

Also from (4.3.6), we have

(4.3.19)(a) $W(T)=0$ and

(4.3.19)(b) $W(Z,T)=g(w(Z),T)=0$

Now, from (4.3.5) and (4.3.6), we obtain

(4.3.20)(a) $V^*=-2D_\lambda P+4m\beta^2$ and

(4.3.20)(b) $w^*=4(p(P)-\beta^2)$

where we have taken $V^*=F^{ij}V_{ij}$ and $w^*=F^{ij}w_{ij}$. Then, we get

(4.3.21) $V^*-w^*=-2\{D_\lambda P+2p(P)-2(m+1)\beta^2\}$

Now, taking account of ${}'K(X,Y,Z,U)={}'K(Z,U,X,Y)$, where ${}'K(X,Y,Z,U)\,\underline{def}\,g(K(X,Y,Z),U)$, we obtain from (4.3.2)

(4.3.22) ${}'F(X,U)\{{}'Q(Y,Z)+{}'Q(Z,Y)\}-{}'F(Y,U)\{{}'Q(X,Z)+{}'Q(Z,X)\}$

$+{}'F(Y,Z)\{{}'Q(X,U)+{}'Q(U,X)\}-{}'F(X,Z)\{{}'Q(Y,U)+{}'Q(U,Y)\}$

${}'F(Z,U)\{V(X,Y)-{}'w(X,Y)\}-{}'F(X,Y)\{V(Z,U)-{}'w(Z,U)\}=0$

From (4.3.22), we obtain, after a straight forward computation,

(4.3.23) ${}'Q(Y,Z)+{}'Q(Z,Y)=\frac{4\alpha\beta}{(m+1)}A(Y)A(Z)$

Using this in (3.15), we have

(4.3.24) ${}'Q(\bar{Y},\bar{Z})+{}'Q(Y,Z)=2\alpha\beta g(Y,Z)-\frac{4\alpha\beta}{(m+1)}A(Y)A(Z)$

Again using (4.3.23) in (4.3.22), we obtain after a straight forward computation.

(4.3.25) $V(Z,U)-{}'w(Z,U)=\tfrac{1}{2}m(v^*-w^*){}'F(Z,U)$

and in consequence of (4.3.21), we also have

(4.3.26) $V(Z,U)-{}'w(Z,U)=-1/m[D_\lambda P+2p(P)-2(m+1)\beta^2]{}'F(Z,U)$

Now, we suppose the curvature tensor with respect to the connection b vanishes. i.e.

$R(X,Y,Z)=0$

Then from (4.3.2), we have

(4.3.32) $K(X,Y,Z)=\{X-A(Z)T\}{}'P(Y,Z)-\{Y-A(Y)T\}{}'P(X,Z)+g(\bar{Y},\bar{Z})P(X)$

$-g(\bar{X},\bar{Z})p(Y)+{}'Q(Y,Z)\bar{X}-{}'Q(X,Z)\bar{Y}+{}'F(Y,Z)Q(X)$

$-{}'F(X,Z)Q(Y)+{}'B(B(X,Y)\bar{Z})+{}'F(X,Y)W(Z)$

$-[(\alpha^2+\beta^2){}'F(Y,Z)\bar{X}-(\alpha^2+\beta^2){}'F(X,Z)\bar{Y}-2\alpha^2\,{}'F(X,Y)\bar{Z}]$

Now, using (4.3.32) in

${}'K(X,Y,Z,U)+{}'K(Y,Z,X,U)+{}'K(Z,X,Y,U)=0$

we obtain, in consequences of the equations (4.3.14), (4.3.23), (4.3.28) and (4.3.29)

(4.3.33) $\frac{1}{m}[{}'F(Z,U){}'F(X,Y)+{}'F(X,U){}'F(Y,Z){}'F(Y,U){}'F(X,Z)]$

$[P^*+2p(P)-\alpha^2-(2m+1)\beta^2+mp(P)-2m\beta^2]$

$+\frac{4\alpha\beta}{(m+1)}[A(U)A(Z){}'F(X,Y)+A(X)A(U){}'F(Y,Z)+A(Y)A(U){}'F(X,Z)]=0$

Baring U in the above, we get

(4.3.34) $P^*+(m+2)p(P)-\alpha^2-\beta^2=0$

Using this in (4.3.31), we have

(4.3.35) ${}'w(Z,U)=-2\,{}'Q(Z,U)-(p(P)+2\beta^2){}'F(Z,U)+\frac{4\alpha\beta}{(m+1)}A(U)A(Z)$

Baring Y in (4.3.23), we have

${}'Q(\bar{Y},Z)+{}'Q(Z,\bar{Y})=0$

Using it and (4.3.14) in (4.3.28), we get

(4.3.36) $V(\bar{Y},Z)=2{}'P(Y,Z)-2\alpha^2 A(Y)A(Z)-2\alpha\beta\,{}'F(Z,Y)+2\beta^2 g(Z,Y)-p(P)g(\bar{Y},\bar{Z})$

Also barring Z in (4.3.35) and using (4.3.14), we get

(4.3.37) ${}'w(\bar{Z},U)=2{}'P(U,Z)-2\alpha^2 A(U)A(Z)-2\alpha\beta\,{}'F(U,Z)+2\beta^2 g(U,Z)+(p(P)+2\beta^2)g(\bar{U},\bar{Z})$

Contracting with respect to X equation (4.3.32) and using (4.3.14), (4.3.36) and (4.3.37), we obtain

(4.3.38)(a) $Ric(Y,Z)=2(m+2){}'P(Y,Z)-(P^*+\alpha^2+3\beta^2)A(Y)A(Z)+\{P^*-3\alpha^2+7\beta^2\}g(Y,Z)$

or

(4.3.38)(b) $K(Y)=2(m+2)P(Y)-(P^*+\alpha^2+3\beta^2)A(Y)T+(P^*-3\alpha^2+7\beta^2)Y$

(4.3.40)(b) $${}'M(Y,Z)=-\frac{1}{2(m+2)}\left[Ric(\bar{Y},Z)+(L+3\alpha^2)\,{}'F(Y,Z)\right]$$

Now, from (4.3.38)(c) in (4.3.34), we get

(4.3.41) $$p(P)=\frac{1}{(m+2)}(L+\alpha^2)+\frac{\beta^2-(qm+4)}{2(m+1)(m+2)}$$

Now, using (4.3.41), (4.3.40)(a) in (4.3.28), we get

(4.3.42) $$V(X,Y)=2\,{}'M(X,Y)+\frac{(L+\alpha^2)}{(m+2)}\,{}'F(X,Y)$$
$$-\frac{2(m-2)}{(m+2)}\beta^2\,{}'F(X,Y)-2\alpha\beta\frac{(m-1)}{(m+1)}A(X)A(Y)\;-2\alpha\beta\, g(X,Y)$$

Similarly we can obtain

(4.3.43) $$w(Z,U)=2\,{}'M(Z,U)-\frac{(L+\alpha^2)}{(m+2)}\,{}'F(Z,U)$$
$$-\frac{\beta^2(4m^2+13m+4)}{(m+1)(m+2)}\,{}'F(Z,U)-\frac{2\alpha\beta(m-1)}{(m+1)}A(U)A(Z)-2\alpha\beta g(Z,U)$$

Now, putting result (4.3.39)(a), (4.3.40)(a), (4.3.42) and (4.3.43) in (4.3.32), we obtain

(4.3.44) $$B^{*}(X,Y,Z)-\frac{(3m+8)}{2(m+1)(m+2)}\beta^2\{A(Y)A(Z)X-A(X)A(Z)Y$$
$$+A(X)Tg(YZ)-A(Y)Tg(X,Z)\}$$
$$\frac{(7m+12)\beta^2}{2(m+1)(m+2)}\{g(Y,Z)X-g(X,Z)Y\}-\alpha\beta\{g(Y,Z)\bar{X}-g(X,Z)\bar{Y}\}\;+{}'F(Y,Z)X$$
$$-{}'F(X,Z)Y+A(Y)A(Z)\bar{X}-A(X)A(Z)\bar{Y}+A(X)T\,{}'F(Y,Z)-A(Y)T\,{}'F(X,Z)$$
$$-2g(X,Y)\bar{Z}-2\,{}'F(X,Y)\bar{Z}-2A(Z)T\,{}'F(X,Y)-2A(X)A(Y)\bar{Z}\}$$
$$-\frac{4\alpha\beta}{(m+1)}\{A(Z)T\,{}'F(X,Y)+A(X)A(Y)\bar{Z}\}$$
$$-\frac{(2m^2-m-8)\beta^2}{2(m+1)(m+2)}\{{}'F(Y,Z)-{}'F(X,Z)\bar{Y}\}\;+\frac{3m(2m-3)\beta^2}{(m+1)(m+2)}\,{}'F(X,Y)\bar{Z}=0$$

where,

(4.3.45) $$B^{*}(X,Y,Z)={}'K(X,Y,Z)+\{X-A(X)T\}\,{}'L(Y,Z)-\{Y-A(Y)T\}\,{}'L(X,Z)$$
$$+g(\bar{Y},\bar{Z})L(X)-g(\bar{X},\bar{Z})L(Y)+{}'M(Y,Z)\bar{X}-{}'M(X,Z)\bar{Y}+{}'F(Y,Z)M(X)$$
$$-{}'F(X,Z)M(Y)-2\{{}'F(X,Y)M(Z)+{}'M(X,Y)\bar{Z}\}$$
$$+\alpha^2\{{}'F(Y,Z)\bar{X}-{}'F(X,Z)\bar{Y}-2\,{}'F(X,Y)\bar{Z}\}$$

Here, $B^{*}(X,Y,Z)$ is contact Bochner Curvature in a-Sasakian Manifold

So, we have

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$$+\alpha^2\{{}'F(Y,Z)\bar{X}-{}'F(X,Z)\bar{Y}-2\,{}'F(X,Y)\bar{Z}\}$$