Mathematical Modelling And Vibration Analysis of Cylindrical Shell With Ring Support

Mr. Shivaji Deshmukh[1], Dr. P. Nandakumar[2]
[1, 2] Dept of Mechanical Engineering
[2]Associate professor, Dept of mechanical Engineering
[1, 2] SRM University Kattankulathur, Chennai

Abstract- *Free vibration of thin cylindrical shell with ring support is studied. cylindrical shell found in many industrial applications. They are often used in load bearing structure for aircraft, drilling gigs, Ships etc. The cylindrical shell has a ring support which are arbitrarily placed along the shell. A mathematical modelling is established using Love's first approximation shell theory and with beam functions used as axial modal functions. The governing equation of motion are derived using an Lagrangian function with Ritz method. Using an appropriate set of displacement functions, the energy equation leads to an eigenvalue problem whose roots are the natural frequencies of vibration. The influence of simply supported boundary condition and the effect of change in shell geometrical parameter and variation in ring support position on natural frequency of vibration are studied.*

Keywords- Vibration Analysis, Natural Frequency, Love's shell Theory, simply supported Boundary Condition.

I. INTRODUCTION

The dynamic behaviour cylindrical shell studied by many researchers. It was first study by Arnold and Warburton [1]. Thereafter Loves modified the Kirchhoff's plate hypothesis and established the classical theory of thin shell, which is now commonly known as Loves first approximation theory also known as Kirchhoff's Love's assumptions [2]. The work on the free vibration of cylindrical shell is very rare as per my knowledge, so it is very important to study the vibration of cylindrical shell. The paper deals with free vibration of cylindrical shell with ring support along the axial direction based on Love's first approximation shell theory. The displacement field equation chosen consist of axial function, which guarantee satisfies edge boundary condition, trigonometric functions that guarantee periodicity in the circumferential shell direction.In this Mathematical modelling of cylindrical shell with ring support is carried out by using energy approach by use of Ritz method. Studies are carried out for simply supported boundary condition with an arbitrary ring support along the axial direction of the cylindrical shell. The influence of simply supported boundary condition and the effect of change in shell geometrical parameter and variation in ring support position on natural frequency of vibration are studied.

II. MATHEMATICAL MODELLING

2.1 THEOROTICAL FORMULATION

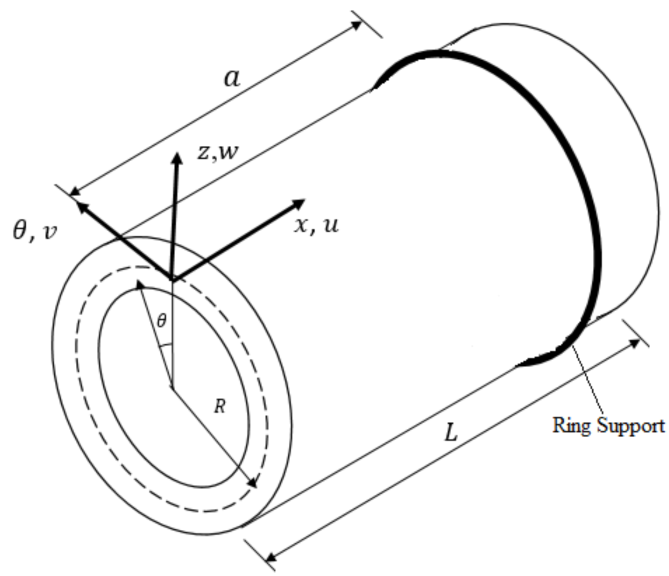

Fig. 1: Nomenclature of cylindrical shell

The nomenclature of cylindrical shell is given in Fig. 1. R is radius, L is length, $h$is thickness which bisect the two surface called middle surface of cylindrical shell ring support, where x, θ, z are axial, circumferential, radial orthogonal coordinate system.u,v, w are displacement at x, θ, $z$axial, circumferential and radial direction respectively. $a$is position of the ring support along the axial direction.

From the Love's first approximation shell theory[2], the strains, axial e_x, circumferentiale_θ and share $e_{x\theta}$at a distance z from the middle surface of the shell can be defined as

$$e_x = e_1 + zk_1 ,\ e_\theta = e_2 + zk_2,\ e_{x\theta} = \gamma + 2z\tau \qquad (1)$$

Where,

$e_1 = \frac{\partial u}{\partial x}$, $e_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right)$, $\gamma = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}$, $k_1 = \frac{\partial^2 w}{\partial x^2}$,
$k_2 = \frac{1}{R}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v^2}{\partial \theta^2}\right)$, $\tau = \frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right)$
(2)

Where, e_1, e_2 is unit elongation of middle surface in x and θ direction, γ is sharingstrain of the middle surface of the shell. k_1, k_2, 2τ denotes the change of curvature and twisting of the middle surface.

The displacement field equation for vibrating cylindrical shell is assumed
As

$$u = A\frac{\partial \varphi(x)}{\partial x}\cos(n\theta)\sin(\omega t)$$
$$v = B\varphi(x)\sin(n\theta)\cos(\omega t) \quad (3)$$
$$w = C\varphi(x)\prod_{i=1}^{N}(x - a_i)^{\zeta_i}\cos(n\theta)\cos(\omega t)$$

which satisfy the boundary condition at $x = 0$, $x = L$. Where A, B and C are the amplitudes of cylindrical shell in axial, circumferential and radial direction, n is circumferential wave number, ω is angular frequency of cylindrical shell, a_i is position of i^{th} ring support, N denotes number of ring support, ζ_i is parameter having value 1 when ring support exist and 0 when no ring support. $\varphi(x)$ is axial function from Eq. (3), which satisfy boundary condition.
The kinetic energy of cylindrical shell is given by

$$KE = \frac{1}{2}\int_0^L\int_0^{2\pi}\int_{-h/2}^{h/2}\rho\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right]Rdzd\theta dx \quad (4)$$

Where, ρ is mass density of cylindrical shell.
Similarly, potential energy of cylindrical shell is given by

$$PE = \frac{1}{2}\int_0^L\int_0^{2\pi}\int_{-h/2}^{+h/2}\{\varepsilon\}^T\{\sigma\}Rdzdx\, d\theta \quad (5)$$

From the hook's law and stress strain equation the final equation of potential energy for cylindrical shell is as follow

$$PE = \frac{1}{2}\int_0^L\int_0^{2\pi}\{\varepsilon\}^T[s]\{\varepsilon\}R\, dx\, d\theta \quad (6)$$

Where , $\{\varepsilon\}$ is strain vector and $[s]$ is stiffness matrix

2.2 BOUNDARY CONDITION

The axial function is given by

$$\varphi(x) = \alpha_1\cosh\left(\frac{\lambda_m x}{L}\right) + \alpha_2\cos\left(\frac{\lambda_m x}{L}\right) - \zeta_m\left(\alpha_3\sinh\left(\frac{\lambda_m x}{L}\right) + \alpha_4\sin\left(\frac{\lambda_m x}{L}\right)\right) \quad (7)$$

Where, $\alpha_i = (i = 1..4)$, $\alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 0$, $\alpha_4 = -1$ are some constants with value 0 or 1 chosen according simply supported boundary condition. Also the values of $\zeta_m = 1$, $\lambda_m = m\pi$ for simply supported boundary condition (3).

The geometric boundary condition for simply supported can be expressed in mathematical term of $\varphi(x)$ for simply supported boundary condition.

$$\varphi(x) = \frac{\partial^2\varphi(x)}{\partial x^2} = 0 \quad (8)$$

Using Eq.(1), Eq.(2) and Eq.(3) Kinetic and potential energy is derived, the derived equations used in Lagrangian function for finding equation motion. The Lagrangian energy function for cylindrical shell is given by

$$L = KE - PE \quad (9)$$

Minimizing the Lagrangian energy function L with respect to unknown A, B and C.We get three equation of motions.

$$\frac{\partial L}{\partial A} = \frac{\partial L}{\partial B} = \frac{\partial L}{\partial C} = 0 \quad (10)$$

The derived equationsof motion arranged in matrix form as follows

$$\begin{bmatrix} \rho h\omega^2 - a_{11} & a_{12} & a_{13} \\ a_{21} & \rho h\omega^2 - a_{22} & a_{23} \\ a_{31} & a_{32} & \rho h\omega^2 - a_{33} \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0 \quad (11)$$

Where, $[\chi]^T = [A \;\; B \;\; C]$ (12)

For solution, the determinant of Eq.(11) has to be zero. Expanding determinant gives even power polynomial.

$$\omega^6 + b_1\omega^4 + b_2\omega^2 + b_3 = 0 \quad (13)$$

Where b_1, b_2 and b_3 are some constants. Eq. [13] is solved and get three positive and three negative roots.

III. NUMERICAL EXAMPLE

[1] The cylindrical shell having radius $R = 5.08cm$, length $L = 20.3\ cm$, thickness $h = 0.25\ cm$, Poisson's ratio $\upsilon = 0.3$, the mass density $m = 8166\ kg/m^2$, modulus of

elasticity $E = 2.065 \times 10^{11}\ N/m^2$, circumferential wave number $n = 1$, mode shape number $m = 1$ to 4 and fundamental frequency $\omega\ Hz$ [7]. Shows in table 1.

[2] For an isotropic cylindrical shell having radius $R = 1\ cm$, length $L = 20\ cm$, mass density $m = 1314\ kg/m^2$, modulus of elasticity $E = 4.8265 \times 10^{11}\ N/m^2$ and Poisson's ratio $\vartheta = 0.3$ [8], shown in table 2.

IV. RESULTS AND DISCUSSION

For validating result with available literature, we considering cylindrical shell with ring support. For no ring support we considering the parameter $\zeta_i = 0$ in Eq. (3).

Table 1: Comparison of natural frequency of cylindrical shell without ring support ω in Hz.

Circumferential wave number (n)	Mode shape number (m)	Proposed ω in Hz	M.R. Isvandzibaei [7] ω in Hz
1	1	2044.84	2043.6
	2	5526.39	5635.2
	3	9072.48	8932.1
	4	11771.22	11407.2
	5	13848.50	13252.8

Table 1. Shows the comparison of fundamental frequency ω in Hz from literature M.R. Isvandzibaei [7]. The result compared with the third order deformation theory.

Table 2. Shows the comparison of frequency parameter $\omega^* = R\omega\sqrt{(1-\mu^2)\rho/E}$ at different ration of $h/R = 0.002$ and $h/R = 0.05$ also the value $L/R = 20$taken from literature C. T. Loy [8]. The comparison shows that the result of present analysis based on Love's approximation theory have better argument than the third deformation theory [7], for low circumferential wave number.

Table 2. Comparison of frequency parameter $\omega^* = R\omega\sqrt{(1-\mu^2)\rho/E}$ for an cylindrical shell without ring support

Thickness to radius ratio (h/R)	Circumferential wave number (n)	Proposed (ω^*)	C.T. Loy [8] ω^*
0.05	0	0.9580	0.96231
	1	0.9642	0.956227
	2	0.9833	0.938425
	3	1.0173	0.910267
0.002	0	0.0929	0.09619
	1	0.0892	0.01634
	2	0.0273	0.03929
	3	0.1306	0.10978

But for higher circumferential wave number result tends to third order deformation theory. The results show that the frequency first decreases and increases as circumferential wave number increases.

In this paper we are studying ring which are arbitrary placed along the axial direction of the cylindrical, for that we making the setting the parameter $\zeta_i = 0$ in Eq. (3).

Fig. 2. Shows that the variation of frequency parameter$\omega^* = R\omega\sqrt{(1-\mu^2)\rho/E}$ with the circumferential wave number n and the position of ring support a/L at different L/R ratios. In Fig. 2(A). We considering position of ring support at $a/L = 0.3$ and Poisson's ratio $\upsilon = 0.3$

In Fig. 2(A). The frequency parameter subsequently increases between wave number $n = 0$, $n = 1$ and decreases at $n = 2$ and gradually increases as circumferential wave number increases. The minimum value of frequency parameter obtained at $n = 0$.

The Fig. 2(A), (B), (C), shows that variation of frequency parameter $\omega^* = R\omega\sqrt{(1-\mu^2)\rho/E}$ with the position of ring support at a/L along the axial direction of cylindrical shell and at different L/R ratios for simply supported boundary condition. The L/R ratio significantly influence on frequency parameter. For $L/R = 20$ show in Fig.2(B), the maximum frequency value is occurs when the ring support at middle of the cylindrical shell and decreases as the ring move towards the either side of cylindrical shell.

Also for $L/R = 6$ ratio show in Fig.2(C), the maximum fundamental frequency at position $a/L = 0.3$ and decreases as the ring move in the axial direction. Also $L/R = 10$ ratio show Fig 2(D), the fundamental frequency decreases along the axial direction. The study show that L/R ratio increases the frequency also increases, and fundamental frequency decreases as ring move either side of cylindrical shell. As the ratio increases the maximum value of fundamental frequency move toward the right end of cylindrical shell.

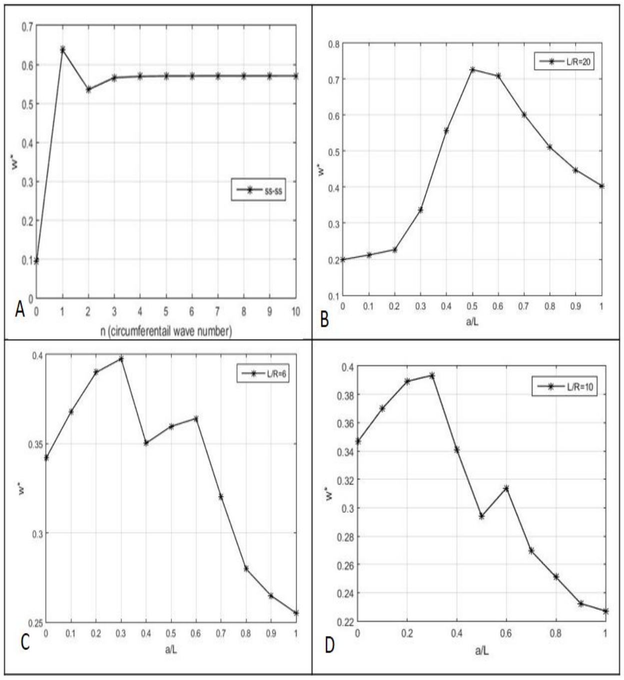

Fig. 2: Variation of frequency parameter

$\omega^* = R\omega\sqrt{(1-\mu^2)\rho/E}$ with circumferential wave number and the position of ring support $a\,/\,L$ at different $L\,/\,R$ ratios.

V. CONCLUSION

In this paper we presented the mathematical modelling and vibration analysis of cylindrical shell with ring support. The equation of motions for cylindrical shell is derived by using Lagrangian function and Ritz procedure. The boundary condition considered for deriving equations of motion is simply supported. Study was made on the frequency parameter, position of ring and geometrical parameters. The study showed that ring support has significant influence on the frequencies and this influence depend on position of ring support. The present analysis validates with the those available in literature.

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