Magnetic Effect on Mixed Convection Melting From A Vertical Plate With Variable Wall Temperature Embedded In Darcy Porous Media

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Abstract- *In this study, the effects of melting on heat transfer in Newtonian fluid over an infinite vertical plate in Darcy porous medium in the presence of the magnetic effect are examined. The important similarity transformations are used to reduce the governing partial equations into a system of nonlinear ordinary differential equations and those are then solved numerically using the Runge–Kutta fourth order method together with shooting technique. The effects of different controlling parameters, namely, the melting and magnetic parameters, on flow and heat transfer are investigated. The numerical results for the dimensionless velocity and temperature are presented graphically and discussed. It is found that the rate of heat transfer decreases with increases of melting parameter as well as magnetic numbers.*

Keywords- Melting effect, Newtonian fluids, Darcy porous medium, Mixed convection

I. NOMENCLATURE

B_0	Magnetic field strength
Cs	specific heat of solid phase
f	Dimensionless stream function
g	Acceleration due to gravity
h_x	Local heat transfer coefficient
L	Latent heat of melting of solid
K	Permeability of the porous medium
k	Thermal conductivity
Ha	the Hartmann number
M	Melting parameter
Nu	Local Nusselt
q_x	Wall heat flux
T	Temperature in thermal boundary layer
T_s	Temperature at the solid region
u	Velocity in x-direction
v	Velocity in y- direction
x	Coordinate along the melting plate
y	Coordinate normal to melting plate
η	dimensionless similarity variable

Greek symbols

α	Thermal diffusivity
β_T	Coefficient of thermal expansion
μ	Viscosity of fluid
υ	Kinematic viscosity
ρ	Density
f	Stream function
σ	Electrical conductivity
θ	Dimensionless temperature

Subscripts

m	Melting point
∞	Condition at infinity

II. INTRODUCTION

Convective heat transfer study in porous media in the presence of melting effect has gained some significant attention in recent years due to important application in permafrost melting, frozen ground thawing, casting and welding processes as well as phase change material. The mixed convection problem on melting from a vertical plate embedded in porous medium studied by Gorla et al. (1999) [1] to achieve the conclusion about melting process which is equivalent to mass injection or blowing near boundary deducing the information relating to the distribution of temperature, stream function, velocity and heat transfer rate by solving the problem using fourth order Runge-Kutta method. They also conclude that heat transfer rate decreases at the solid liquid interface which is maintained at uniform temperature. Epstein and Cho (1976) [2] studied the effect of melting considering the laminar film condensation over a vertical to confer the melting rate using Nusselt's method. They suggested that transient effects can be neglected until the thickness of the thermal boundary layer is smaller than melting solid. Kazmierczak *et al.* (1986,1987) [3, 4] analyzed the velocity, temperature and Nusselt number from a flat plate embedded in porous medium in case of steady natural convection considering in the melting region. Concerning with natural convection Sparrow *et al*. (1977) [5] studied the heat transfer in the melting region of a non- porous medium providing the information regarding the velocity, temperature fields and the heat transfer rate by applying finite difference method. Pozovonkov *et al.* (1968) [6] also calculated the heat transfer rate near a melting surface in the same medium using Karman-Pohlhausen scheme. Bakier (1997) [7] obtained the solution using analytical homotopy analysis for the case, aiding and opposing flow in a porous medium. He pointed out that in the liquid solid interface the melting phenomenon decreases the local Nusselt number. Obtaining similarity solution Cheng (1977) [8] studied the melting phenomenon for mixed convection over a flat plate. Epstein (1975) [9] analyzed the effect of melting of submerged bodies. Ishak et al. (2010) [10] studied melting heat transfer from a moving flat plate. Reid (1975) [11] analyzed the melting and solidification heat transfer in presence of convection in porous media for vertical flat plate. Several studies relating to effect of magnetic field on melting are analyzed [12-16] in porous and non porous media. The effect of Magnetic and buoyancy on melting from a vertical plate embedded in saturated porous media is studied by Tashtoush [17]. He observed that the melting parameters increase with the decrease of temperature as well as Nusselt number at the solid liquid interface. The problems of melting effect on mixed convective heat transfer from a porous vertical plate with uniform wall temperature in the liquid-saturated porous medium with aiding and opposing external flows is numerically examined at steady state Cheng and Lin(2008) [18].

The main aim of the present investigation is to illustrate the effects of melting with magnetic field from a vertical plate in a Darcy porous medium. The velocity and temperature profiles of various parameters under the influence of magnetic field with melting effect and the results so obtained are compared with relevant results in the existing literature and are found to be in good agreement in the absence of melting and magnetic effect.

III. MATHEMATICAL FORMULATION

Let us consider the free convection heat transfer from a vertical flat plate embedded in a non Darcy porous media saturated with Newtonian fluid. It is assumed that this plate constitutes the interface between the liquid and solid phases during melting inside the porous matrix. The plate temperature, T_m, is the melting temperature of the material occupying the porous matrix, which is regarded as constant. The temperature of the solid phase far from the interface is T_s and the liquid phase temperature is T_∞ ($T_\infty < T_m$). The origin of the coordinate system is placed at the leading edge of the interface surface between the solid and liquid phases. x is the coordinate along the surface of the plate measured from the origin, and y is the coordinate normal to the surface. u and v are the components of the non-Darcy velocity in the x and y directions, respectively, is shown in the Figure 1.

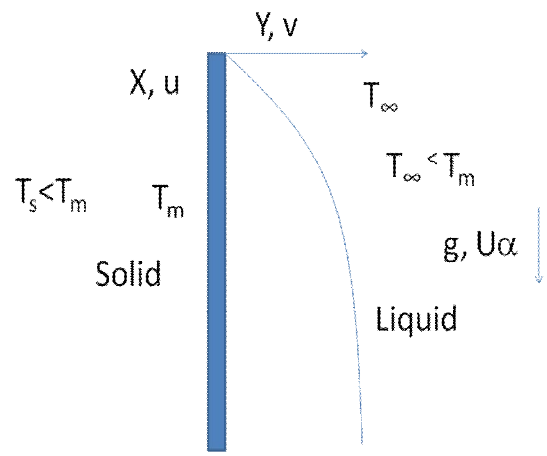

Figure 1. Schematic model and system of coordinates.

A magnetic field of strength B_0 is applied in the y direction which is normal to the flow direction. The flow is steady, laminar and two dimensional. The fluid and porous medium are in local thermal equilibrium and constant except density. The Boussiensq approximation is valid and the boundary layer approximation is applicable. The governing equation, namely the equation of continuity, momentum equation and energy equation for isotropic and homogeneous porous medium can be written as (see Bear [19]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial y} = -\frac{Kg\beta_T}{\mu}\left(\frac{\partial T}{\partial y}\right) \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y} + \frac{\sigma B_0^2 u^2}{\rho C_p} \quad (3)$$

And

$$\rho = \rho_\infty[1 - \beta_T(T - T_\infty)]$$

In the above equations, u and v are the component of darcian velocity along x and y direction, ρ is the density, g and σ are the acceleration and electrical conductivity of the fluid respectively. $\alpha = \frac{k}{\rho c}$ is the equivalent thermal diffusivity.

The appropriate boundary conditions necessary to complete the problem are as follows

$$v = 0,\ T \to T_m = T_\infty + Ax^\lambda,\ k\frac{\partial T}{\partial y} = \rho[L + C_s(T_m - T_s)]v \quad \text{at } y = 0 \quad (4)$$

$$u = U_\infty,\ T \to T_\infty \quad \text{at } y \to \infty \quad (5)$$

Where L and C_s are the latent heat of solid and specific heat capacity of solid phase, respectively.

The boundary condition (4) refers that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the melting heat and the heat required for raising the temperature of solid to its melting temperature [Epstein and Cho (1976) [2], Carslaw et al. (1959) [20], Roberts (1958) [21]].

The stream function $\Psi(x,y)$ is chosen in such a way that the continuity equation automatically satisfied i.e.,

$$u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \Psi}{\partial x} \quad (6)$$

Introducing the following similarity transformation equation, the above system of partial differential equations (1-3) are transferred into the non-linear ordinary differential equations Bakier (1997) [7]

$$\eta = \left(\frac{U_\infty}{\alpha}\right)^{1/2} y x^{\frac{\lambda-1}{2}} \quad (7)$$

$$f(\eta) = \frac{\Psi}{(\alpha U_\infty)^{1/2} x^{\frac{\lambda-1}{2}}} \quad (8)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (9)$$

Introducing Eqs. (7), (8), (9) into Eqs. (2) and (3), we obtain the following transformed governing equations:

$$f'' + \frac{Gr}{Re}\theta' = 0 \quad (10)$$

$$\theta'' + \frac{1}{2}(1+\lambda)f\theta' - \lambda f'\theta + \frac{Ha^2 Ec}{Da}f'^2 = 0 \quad (11)$$

Where, $Gr = \frac{kg\beta_T(T_\infty - T_m)x}{v^2}$, Grashof number $Ha = \frac{\sigma B_0^2}{\rho c_p}$, Hartmann number, $\frac{Gr}{Re}$, buoyancy parameter, $M = \frac{C_p(T_\infty - T_m)}{L + C_s(T_m - T_\infty)}$, melting parameter, $Re = \frac{U_\infty x}{v}$, Reynolds number, $Ec = \frac{u_0^2}{C_p(T_m - T_\infty)}$, Eckart number, $Da = \frac{k}{x^2}$, Darcy number.

In the present problem the parameter $\frac{Gr}{Re}$, known as buoyancy parameter express the free and forced convection. The forms of non linear differential are as follows:

$$f' + 2M\theta' = 0, \quad \theta = 0 \quad at \quad \eta = 0 \tag{12}$$

And

$$f' \to 1, \ \theta \to 1 \quad at \quad \eta \to \infty \tag{13}$$

Physical quantities of interest is the local Nusselt number Nu_x which are defined as

$$Nu_x = \left(\frac{q_w{}''x}{k}\right) \tag{14}$$

Further, $q_w{}''$ is the heat transfer from the surface of the plate, and is given by

$$q_w{}'' = -k\left(\frac{\partial T}{\partial y}\right) \tag{15}$$

Using equation (13), equation (12) becomes

$$Nu_x = Ra^{1/2}\theta' \tag{16}$$

Where, $Ra = \frac{kg\beta_T\rho(T_\infty - T_m)x}{\alpha\upsilon}$

IV. SOLUTION PROCEDURE

The Eq. (9) together with Eq. (8) is split into system of first order ordinary differential equations. Using boundary conditions (10) and (11) they are solved numerically by means of the fourth order Runge- Kutta method coupled with a shooting technique and by giving appropriate initial guess values for $\theta'(0)$. The solution, thus, obtained is matched with the given values at $f'(\infty)$ and $\theta(0)$. An accuracy upto 4th decimal place is considered for convergence.

V. RESULTS AND DISCUSSION

Numerical calculations are carried out for flow for a range of values of the buoyancy parameter Gr/Re, the melting parameter *M* and magnetic parameter. The effects of buoyancy parameter and melting parameter on the dimensionless stream function *f*, velocity *f'*(0) and temperature *θ*(0) at the plate are presented in Tables 1. With increases of melting parameter *(M)* the local heat transfer rate decreases. This result is in good agreement with the results reported by Gorla et al. (1999) [1]. Additionally, theheat transfer rates decreases with increase of the buoyancy parameter Gr/Re is reported.

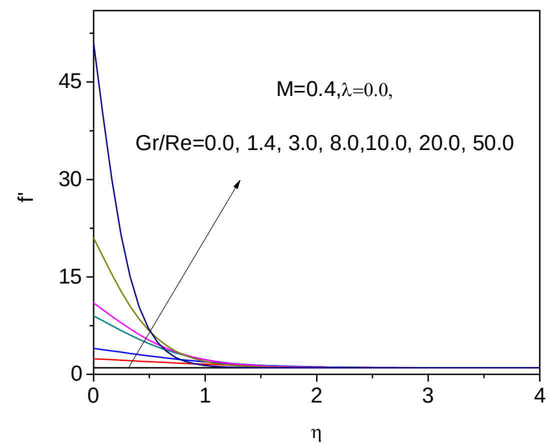

Figure 2. Dimensionless velocity profiles for various values mixed convection parameter with M=0.4, λ=0.0, Ha = 0.0, Ec =0.0, Da=0.1.

From the velocity distribution shown in Figure 2, it is clear that in the absence of the melting parameter, the results are in good agreement with the reported value by Gorla et al. (1999) [1] and by Cheng and Lin(2008) [18] for λ=0. It is also observed that velocity is more important at the surface and increases rapidly with rise of mixed convection parameter. In forced convective flow the uniform velocity reached when mixed convection parameter treated as zero. Figure 3 divulges that the temperature increases from 0 (zero) to ∞ (infinity) for fixed value of melting parameter. It is quite obvious that with increase of buoyancy parameter temperature increases for fixed melting parameter. Figure 4 state that with increases of melting parameter velocity increases with fixed value of buoyancy parameter. The thermal boundary layer increases as shown in the Figure 5. It is clear that with increase of melting parameter thermal

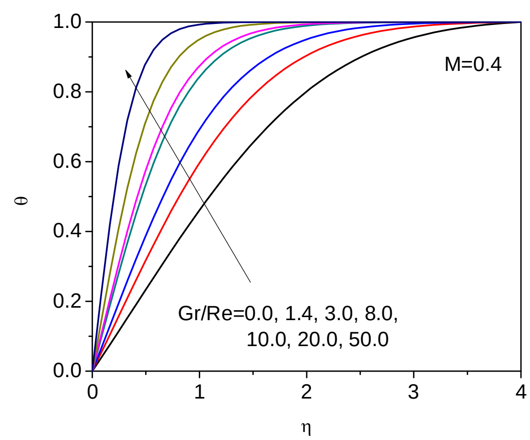

Figure 3. Dimensionless temperature profiles for various values mixed convection parameter with M=0.4, λ=0.0, Ha = 0.0, Ec =0.0, Da=0.1.

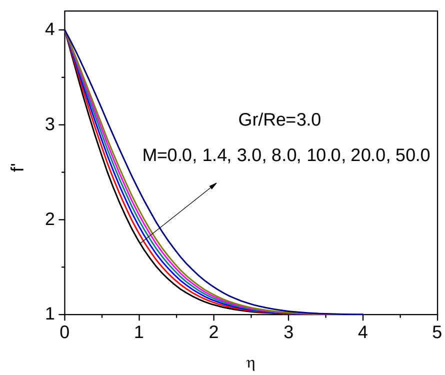

Figure 4. Dimensionless velocity profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.0, Ha = 0.0, Ec =0.0, Da=0.1.

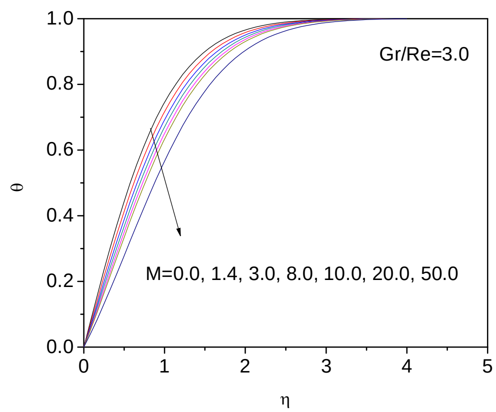

Figure 5. Dimensionless temperature profile for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.0, Ha = 0.0, Ec =0.0, Da=0.1.

boundary layer thickness increases for fixed mixed convection parameter. In Figure 6, the velocity increases with increases of melting parameter when magnetic field introduced with constant mixed convection parameter. Figure 7 indicates that temperature boundary layer increases with increase of melting parameter. The velocity increases with increases of melting parameter as shown in the Figure 8. Due to magnetic field temperature decreases with increment of melting parameter as shown in the Figure 9. It is obvious that velocity increases with increases of melting parameter as shown in the Figure 10. Due to magnetic field, temperature decreases with the increment of melting parameter shown in the Figure 11. It is observed that with the effect of high magnetic field the increment of velocity is much slow than that of low magnetic field.

Table 1 Comparison of present results with the values obtained by Gorla *et al.*[2] for different values of mixed convection and melting parameters.

Parameters		Gorla *et al.*(1999)			Present work		
M	Gr/Re	f(0)	f'(0)	$\theta'(0)$	f(0)	f'(0)	$\theta'(0)$
0.4	0.0	−0.3656	1.0	0.4570	−0.3635	1.0	0.4570
	1.4	−0.5022	2.4000	0.6278	−0.5022	2.4000	0.6277
	3.0	−0.6226	4.0000	0.7783	−0.6226	4.0000	0.7782
	8.0	−0.8998	9.0000	1.1248	−0.8998	9.0000	1.1247
	10.0	−0.9890	11.0000	1.2363	−0.9890	11.0000	1.2363
	20.0	−1.3492	21.000	1.6865	−1.3492	21.000	1.6865
	50	-2.0846	51.000	2.6058	-2.0845	51.000	2.6057
2.0	0.0	−1.1197	1.0000	0.2799	-1.09728	1.000000	0.274322
	1.4	−1.5291	2.4000	0.3823	−1.5231	2.400000	0.3807
	3.0	−1.9016	4.0000	0.4754	−1.89879	4.000000	0.4746
	8.0	−2.7611	9.0000	0.6902	−2.7607	9.0000	0.6901
	10.0	−3.0370	11.000	0.7594	−3.0375	11.000	0.7593
	20.0	−4.1530	21.000	1.0383	−4.1530	21.000	1.0383

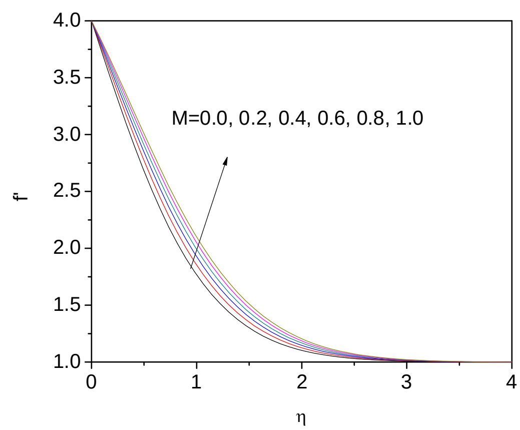

Figure 6. Dimensionless velocity profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.01, Ha = 0.10, Ec =0.10, Da=0.10.

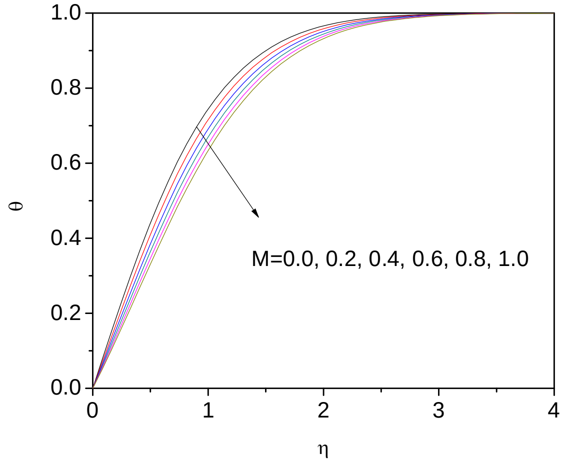

Figure 7. Dimensionless temperature profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.01, Ha = 0.10, Ec =0.10, Da=0.10.

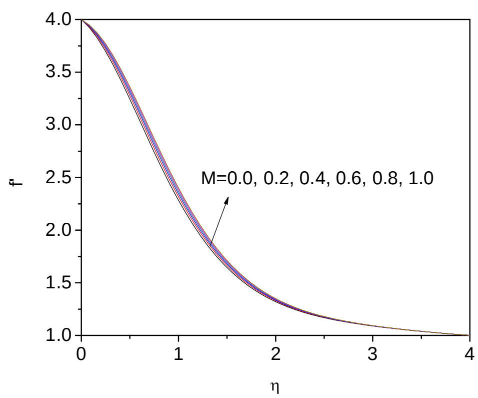

Figure 8. Dimensionless velocity profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.01, Ha = 0.40, Ec =0.10, Da=0.10.

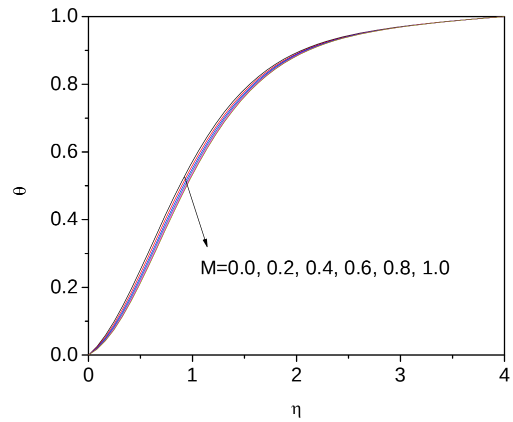

Figure 9. Dimensionless temperature profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.01, Ha = 0.40, Ec =0.10, Da=0.10.

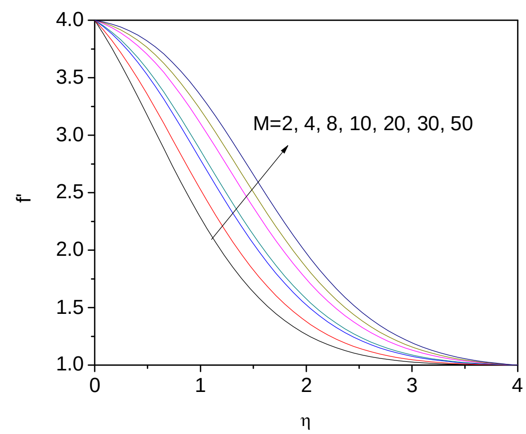

Figure 10. Dimensionless velocity profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.02, Ha = 0.40, Ec =0.10, Da=0.10.

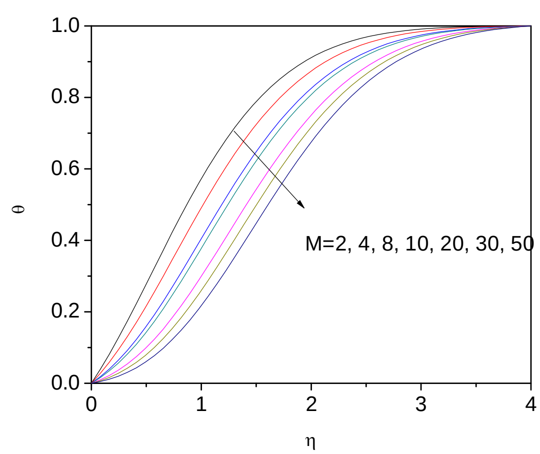

Figure 11. Dimensionless temperature profiles for various values of melting parameter with fixed values of mixed convection parameter, (Gr/Re=3.0), λ=0.02, Ha = 0.40, Ec =0.10, Da=0.10.

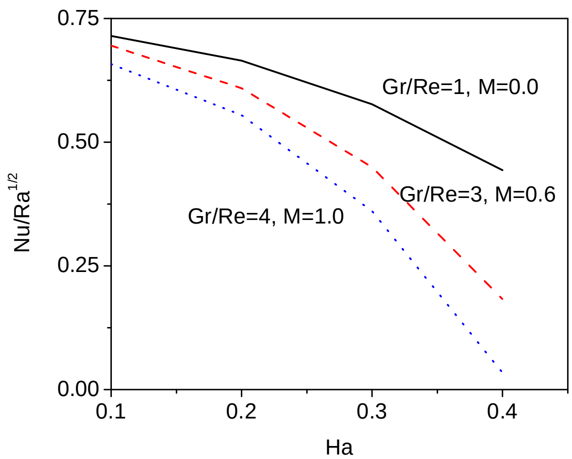

Figure 12. variation of Nusselt number with magnetic number for different values of melting parameter, mixed convection parameter along with λ=0.01, E=0.1, D=0.1.

VI. CONCLUSION

In this paper, the melting effect on mixed convective flow and heat transfer from a vertical embedded in the Darcy porous media with liquid-saturated porous medium is comprehensively studied in the magnetic field. The governing equation is derived by the boundary layer and Boussinesq approximation. A boundary condition to account for melting is used at the interface between the solid and liquid phases. These equations are then transferred using similarity transformation and solved by using Runge–Kutta fourth order method together with shooting technique. Graphical representation of results regarding the velocity and temperature distributions as well as the Nusselt number were presented and discussed for different melting parameters. The increment of velocity is slower at high magnetic field than that low magnetic field.

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