Oscilaltion Properties of Higher Order Delay Differential Equations With Impulsive

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Abstract- *In this paper, we investigate the oscillation of higher order delay differential equations with impulse of the form*

$$x^{(m)}(t)+(t)x^{(m-1)}(t)+\sum_{i=1}^{n} p_i(t)x(g_i(t))=0,\ t\geq t_0,\ t\neq t_k,$$

$$x^{(j)}(t_k)-x^{(j)}(t_k^-)=a_k\, x^{(j)}(t_k^-),\ j=0,1,2,\dots,m-1$$

$$\text{Where } x'(t^+)=\lim_{x\to 0^+}\frac{x(t_k+\text{h})-x(t_k)}{\text{h}},$$

$$x'(t_k^-)=x'(t_k)=\lim_{x\to 0^-}\frac{x(t_k+\text{h})-x(t_k)}{\text{h}}$$

An example is given to illustrate the main results.

Keywords- Oscillation, Delay differential equations, Impulse effect, Higher order, Fixed times.

I. INTRODUCTION

In recent years, there has been an increasing interest on the oscillatory and non-oscillatory behavior of the solutions of impulsive delay differential equations attracted the attention of many growing researchers. Then there are only a few papers on higher order impulsive delay differential equations.

In this paper, we consider a kind of higher order impulsive delay differential equation. Some sufficient conditions for all bounded solutions of this kind of higher order impulsive delay differential equation to be non-oscillatory are obtained by using a comparison theorem with a corresponding non-impulsive differential equation. Our results generalize and improve several known-results.

$$x^{(m)}(t)+(t)x^{(m-1)}(t)+\sum_{i=1}^{n} p_i(t)x(g_i(t))=0,\ t\geq t_0,\ t\neq t_k,$$

$$x^{(j)}(t_k)-x^{(j)}(t_k^-)=a_k\, x^{(j)}(t_k^-),\ j=0,1,2,3\dots\dots,m-1 \quad (1.1)$$

$$\text{Where } x'(t^+)=\lim_{x\to 0^+}\frac{x(t_k+\text{h})-x(t_k)}{\text{h}},$$

$$x'(t_k^-)=x'(t_k)=\lim_{x\to 0^-}\frac{x(t_k+\text{h})-x(t_k)}{\text{h}}$$

And the delay differential problem

$$y^{(m)}(t)+a(t)y^{(m-1)}(t)+\sum_{i=1}^{n} p_i(t)\prod_{g_i(t)<t_k\leq t}(1+\alpha_k)^{-1}y(g_i(t))=0,\ \ t\geq t_0$$

We assume the following:

(H_1) $0\leq t_0<t_1<\dots..<t_k<\dots..$are fixed point with $\lim_{k\to\infty} t_k=\infty;$

(H_1) $f:[t_0-\tau,+\infty)\times R\times R\to R$ *is continuous*

(H_2) $a,\ p_i\in c([0,\infty),\text{R}),\ i=1,2,..,n,$ are Lebesgue measurable and Locally essentially bounded functions, R is the real axis;

(H_3) $g_i\in c([0,\infty),\text{R}),\quad i=1,2,\dots..,n,$ are Lebesgue measurable functions and $g_i(\text{t})\leq t$ satisfies $\lim_{t\to\infty} g_i(\text{t})=\infty;$

$(H_3{}')$ $\lim_{t\to\infty}\int_{t_0}^{t}\prod_{t_0<t_k<s}\frac{c_k}{d_k}ds=+\infty.$

(H_4) $\{\alpha_k\}$ is a sequence of constants and $\alpha_k>-1.$

Definition: 1.2

For nay $\tau_0\geq 0$ and $\phi\in\Psi$, a function $x:[\tau_0^-,\infty)\to\text{R}$ is said to be a solution of (1.1) on $[\tau_0^-,\infty)$ satisfying the initial value condition

$$x(\text{t})=\phi(\text{t}),\ \phi(\tau_0)>0,\ \text{y}(t)=\prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}x(t)$$

if the following conditions are satisfied:

* x satisfies (1.1) ;
* x is absolutely continuous in each interval $(\tau_0, +t_k)$, (t_k, t_{k+1}),

  $k \geq k_0$, $k_0 = \min\{k/ t_k > \tau_0\}$, $x(t_k^-)$ exist and $x(t_k^-) = x(t_k)$,

  the second condition in (1.1) holds;
* x satisfies the first equation in (1.1) almost everywhere in (τ_0^-, ∞).

Definition: 1.3

The solution x of system (1.1) is said to be non-oscillatory if it is eventually negative or eventually positive. Otherwise, it is said to be oscillatory.

By a solution y of (1.2) on $[\tau_0^-, \infty)$ we mean a function which has an absolutely continuous derivative y' on $[\tau_0^-, \infty)$, satisfies (1.2) a.e. on $[\tau_0^-, \infty)$ and satisfies (1.5) on $[\tau_0^-, t_0]$. In this paper, we always suppose $\tau_0^- = \tau_0^-, \tau_0 = t_0$.

II. MAIN RESULTS

In this section we shall establish theorems which enable us to redu-oscillation and non-oscillation of (1.1) to the corresponding problem (1.2).

Theorem: 2.1

Assume that $(A_1)-(A_4)$ hold.

i).If y is a solution of (1.2) on $[t_0^-, \infty)$, then $x(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)y(t)$ is a solution of (1.1) on $[t_0^-, \infty)$.

ii). If x is a solution of (1.1)on $[t_0^-, \infty)$, then $y(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}x(t)$ is a solution of (1.2)on $[t_0^-, \infty)$.

Proof:

First we shall prove (i). let y be a solution of (1.2) on $[t_0^-, \infty)$. Then $x(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)y(t)$ has an absolutely continuous derivative x' on $(t_0^-, t_0), [t_k, t_{k+1})$, $k \geq 0$. For any, $t \neq t_k, t > t_0^-$, it is easy to prove that

$$x^{(m)}(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)y^{(m)}(t)$$

$$= \prod_{t_0<t_k\leq t}(1+\alpha_k)\left\{-a(t)y^{(m-1)}(t) - \sum_{i=1}^{n}P_i(t)\prod_{g_i(t)<t_k\leq t}(1+\alpha_k)^{-1}y(g_i(t))\right\}$$

$$= -a(t)\prod_{t_0<t_k\leq t}(1+\alpha_k)y^{(m-1)}(t) - \sum_{i=1}^{n}P_i(t)\prod_{t_0<t_k\leq g_i(t)t}(1+\alpha_k)y(g_i(t))$$

$$= -a(t)x^{(m-1)}(t) - \sum_{i=1}^{n}P_i(t)x(g_i(t)).$$

So we have

$$x^{(m)}(t) + a(t)x^{(m-1)}(t) + \sum_{i=1}^{n}P_i(t)x(g_i(t)) = 0, \quad t \geq t_0, \ t \neq t_k,$$

and

$$x^{(j)}(t_m^-) = \prod_{t_0<t_k\leq t_{m-1}}(1+\alpha_k)y^{(j)}(t_m)$$

then

$$x^{(j)}(t_m) = (1+\alpha_k)x^{(j)}(t_m^-),$$

Which implies that x solves the second condition in (1.1). Hence

$x(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)y(t)$ is a solution of (1.1) on $[t_0^-, \infty)$.

Next we prove (ii). Let x be a solution of (1.1). we prove that

$$y^{m(t)}(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}x^{(m)}(t)$$

$y(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}x(t)$ is a solution of (1.2) on $[t_0^-, \infty)$.

for any $t \geq t_0$, $t \neq t_k$ $t \geq t_0^-$,

$$y^{m(t)}(t) = \prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}x^{(m)}(t)$$

$$= \prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}\left\{-a(t)y^{(m-1)}(t) - \sum_{i=1}^{n}P_i(t)x(g_i(t))\right\}$$

$$= -a(t)\prod_{t_0<t_k\leq t}(1+\alpha_k)^{-1}x^{(m-1)}(t) - \sum_{i=1}^{n}P_i(t)\prod_{g_i(t)<t_k\leq t}(1+\alpha_k)^{-1}x(g_i(t))$$

$$= -a(t)y^{(m-1)}(t) - \sum_{i=1}^{n}P_i(t)\prod_{g_i(t)<t_k\leq t}(1+\alpha_k)^{-1}y(g_i(t))$$

And $y^{(j)}(t_m) = \prod_{t_0<t_k\le t_m} (1+\alpha_k)^{-1} x^{(j)}(t_m),$

$$y^{(j)}(t_m^-) = \prod_{t_0<t_k\le t_{m-1}} (1+\alpha_k)^{-1} x^{(j)}(t_m^-)$$

$$= \prod_{t_0<t_k\le t_{m-1}} (1+\alpha_k)^{-1}(1+\alpha_m)^{-1} x^{(j)}(t_m)$$

$$= \prod_{t_0<t_k\le t_m} (1+\alpha_k)^{-1} x^{(j)}(t_m) = y^{(j)}(t_m).$$

So $y(t) = \prod_{t_0<t_k\le t} (1+\alpha_k)^{-1} x(t)$ is a solution of (1.2) on $[t_0^-,\infty)$. The proof is therefore complete.

Using Theorem 2.1, we obtain the following results.

Theorem: 2.2

Assume that $(H_1)-(H_4)$ hold. Then all solutions of (1.1) are oscillatory (non-oscillatory) if and only if all solutions of (1.2) are oscillatory (non-oscillatory).

Theorem: 2.3

Assume that $(H_1)-(H_4)$ hold. Then all solutions of (1.1) asymptotically approach to zero if and only if all solutions of (1.2) asymptotically approach to zero.

Let $r(t) = \exp(\int_0^t a(s)ds)$. then (2.1) reduces to the problem

$$(ry^{(m-1)})^1(t) + r(t)\sum_{i=1}^n P_i(t) \prod_{g_i(t)<t_k\le t} (1+\alpha_k)^{-1} y(g_i(t)) = 0,\ t>0 \quad (2.1)$$

Theorem: 2.4

Assume that $(H_1)-(H_4)$ hold. Moreover, suppose that

(H_5) $\prod_{t_0<t_k\le t} (1+\alpha_k)$ is bounded and $\liminf_{x\to\infty} \prod_{t_0<t_k\le t} (1+\alpha_k) > 0;$

(H_6) $p_i(t) \ge 0, i = 1,2,...,$

(H_7) $\int_{t_0}^{t}\int_{t_0}^{\sigma_{m-2}} ....\int_{t_0}^{\sigma_1} \frac{1}{r(s)}\int_s^\infty r(u)\sum_{i=1}^n p_i(u) \prod_{g_i(u)<t_k\le u} (1+\alpha_k)^{-1} du...$

$...d\sigma_{m_{-3}} d\sigma_{m_{-2}} < \frac{1}{4}$ (2.2)

Let y denote the locally convex space of all continuous functions $y \in C([To,\infty), \mathrm{R})$ with the topology of uniform convergence on compact subintervals of $[T_o,\infty)$. Let $\Gamma = \left\{ y \in Y : \frac{\gamma}{2} \le y(t) \le \frac{2\gamma}{3}, t \ge T_o \right\}$, where $\gamma > 0$ is an orbitrary given constant.

We note that Γ is a closed and convex subset of Y and it is non-empty.

Now, we define a map $v : \Gamma \to Y$ by

$$\le 4\varepsilon.\frac{1}{4} = \varepsilon.$$

$$(vy)(t) = \begin{cases} \frac{\gamma}{2} + (\Omega y)(t), t > T, \\ \frac{\gamma}{2}, T_o \le t < T, \end{cases}$$

Where $(\Omega y)(t) = \int_T^t \int_0^{\sigma_{m-2}} ......\int_0^{\sigma_1} \frac{1}{r(s)}\int_s^\infty r(u)\sum_{i=1}^n p_i(u)$ x

$$\text{x} \prod_{g_i(u)<t_k\le u} (1+\alpha_k)^{-1} y(g_i(t))du.....d\sigma_{m-2}.$$

First we verify $\mathrm{V}\Gamma \subset \Gamma$. For all $y \in \Gamma$, it is obvious that $v\ y \subset \Gamma$ for $T_o \le t < T$. When $t > T$, combining (2.2) we get

$$vy \le \frac{\gamma}{2} + \frac{2\gamma}{3}\int_T^t \int_0^{\sigma_{m-2}} ......\int_0^{\sigma_1} \frac{1}{r(s)}\int_s^\infty r(u)\sum_{i=1}^n p_i(u) \text{ x}$$

$$\text{x} \prod_{g_i(u)<t_k\le u} (1+\alpha_k)^{-1} du......d\sigma_{m-2}.$$

$$\le \frac{\gamma}{2} + \frac{2\gamma}{3}.\frac{1}{4} = \frac{2\gamma}{3}.$$

So v maps Γ into Γ. On the other hand, $\{vy\}$ is uniformly bounded. The continuity of $v\ \Gamma \to \Gamma$ is verified as follows: let $y_n \in \Gamma$, $y \in \Gamma$ with $N \in \{1,3,...,\mathrm{m}-1\}$ $\lim_{x\to\infty} y_n = y.$ for any $\varepsilon > 0$, there exists a positive integer N_ε such that $|y_n = y| < 4\varepsilon$ for nay $n > N_\varepsilon$. In particular,

$$|y_n g_i(t) - \mathrm{y}\, g_i(t))| < 4\varepsilon,\quad n > N_\varepsilon,\ t > T_o.$$

Hence

$$|\, v\, y_n(t)-(v\, y)(t)\,| \le \int_T^t \int_0^{\sigma_{m-2}} \ldots\ldots \int_0^{\sigma_1} \frac{1}{r(s)} \int_s^\infty r(u) \sum_{i=1}^n p_i(u) \times$$

$$\times \prod_{g_i(u)<t_k\le u} \left(1+\alpha_k\right)^{-1} |\, y_n g_i(t) - y\, g_i(t))\,|\, du \ldots d\sigma_{m-3} d\sigma_{m-2}$$

$$\le 4\varepsilon \int_T^t \int_0^{\sigma_{m-2}} \ldots\ldots \int_0^{\sigma_1} \frac{1}{r(s)} \int_s^\infty r(u) \sum_{i=1}^n p_i(u)\, x$$

$$\times \prod_{g_i(u)<t_k\le u} \left(1+\alpha_k\right)^{-1} du \ldots d\sigma_{m-3} d\sigma_{m-2}$$

$$\le 4\varepsilon . \frac{1}{4} = \varepsilon .$$

So we know that v maps Γ continuously into a compact subset of Γ. Therefore, by Schauder-Tychonov's fixed point theorem, v has a fixed point y in Γ. It is easy to check that the fixed point y in Γ. it is easy to check that the fixed point y is a solution of (2.1). so (1.2) has a boundary non-oscillatory solution y. By Theorem 2.1, $x(t) = \prod_{t_0<t_k\le t} (1+\alpha_k) y(t)$ is a bounded non-oscillatory solution of (1.1). using condition (H_5),we eventually get $\liminf_{t\to\infty} |\, x(t)\,| > 0.$ the proof is therefore complete.

Next we shall give an oscillation criterion for (1.1) . suppose m is a given even number. First we some lemmas whose proofs are omitted, because their proofs are similar to [14] but without impulses.

Lemma: 2.5.

Let y be a given solution of (2.1). suppose that $\exists\, T > 0$ s.t $x(t) > 0, x^{(i)}(t) \le 0$ $fort \ge T.$ Moreover suppose that $(H_1)-(H_4)$ and (H_6) hold and $(H_8) \int_{t_0}^{\infty} \frac{1}{r(s)} ds = \infty$. Then $\exists\, T_1 > 0$ such that $x^{(i-1)}(t) \ge 0 (<0), t > T_1.$

Lemma: 2.6

Let y be a given solution of (2.1). Suppose that $\exists\, T > 0$ such that $x(t) > 0, x^{(i)}(t) \le 0$ $fort \ge T.$ moreover suppose that $(H_1)-(H_4)$ and (H_8) hold. Then $\exists\, T_2 > 0$ such that $x^{(i-1)}(t) > 0, t > T_2.$

Lemma: 2.7.

Let y be a given solutions of (2.1). suppose that $\exists\, T > 0$ such that $x(t) > 0$ $fort \ge T$. moreover suppose that $(H_1)-(H_4)$, (H_6) and (H_8) hold. Then $\exists\, T_3 > T$ and $N \in \{1,3,\ldots, m-1\}$ such that for $t > T_3$

$$x^{(i)}(t) > 0, \qquad i = 0,1,2,.., N:$$

$$(-1) x^{(i-1)} x^{(i)}(t) > 0, \qquad i = N+1, N+2, \ldots, m-2;$$

$$x^{(m-1)}(t) > 0.$$

Let $g(t) = \min_{1\le i\le n0} g_i(t).$

Theorem: 2.8 Assume that $(H_1)-(H_4)$, (H_6) and (H_8) hold, and

(H_9) g_i has an absolutely continuous derivative g_i' on $(t\bar{o}\infty)$,and $g_i' \ge 0$;

(H_{10}) $\int_{t_0}^{\infty} s^{m-1} r(s) \sum_{i=1}^n p_i(t) \prod_{g_i(s)<t_k\le s} \left(1+\alpha_k\right)^{-1} ds = \infty$

(H_{11}) $\exists\, G > 0$ such that $r(t) < G$.

Then all bounded solutions of (1.1) are oscillatory.

Proof:

We only need to prove that all bounded solutions of (2.1)are oscillatory. Suppose that the assertion is not true. Without of generality, we may suppose that there exists $T > 0$ such that $y(t) > 0$ for $t \ge T.$

First we consider the case when N=1. From Lemma 2.7, we get

$$y'(-t) > 0,\ y''(t) < 0,\ y'''(t) > 0, \ldots, y^{(m-1)}(t) > 0, t > T'.$$

so $(y(g_i(t)))^1 = y'(g_i(t)) g_i'(t) > 0,$

Which implies $y(g_i(t))$ is increasing in t for $t > T'.$ therefore, for $t > T'$

$$(ry^{(m-1)})^{1}(t) = -r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(s)<t_k\le t} \left(1+\alpha_k\right)^{-1} y(g_i(t))$$

$$\le -r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(s)<t_k\le t} \left(1+\alpha_k\right)^{-1} y(g_i(T'))$$

$$\le -R\,r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(s)<t_k\le t} \left(1+\alpha_k\right)^{-1}$$

Where $R = y(g(T')) > 0$. We by t^{m-1} and integrate on $[T', t)$ to find

$$\int_{T'}^{t} s^{m-1}(ry^{(m-1)})'(s)ds \le -R\int_{T'}^{t} t^{m-1} r(s)\sum_{i=1}^{n} p_i(s) \prod_{g_i(u)<t_k\le s} \left(1+\alpha_k\right)^{-1} \qquad 2.5$$

On the other hand, combining (H_{11}), we have

$$= s^{m-1}\, y^{(m-1)}(s)\Big|_{T'}^{t} - G(m-1)\times$$

$$\times\left\{ s^{m-2}\, y^{(m-2)}(s)\Big|_{T'}^{t} - (m-2)\int_{T'}^{t} s^{m-3} dy^{(m-3)}(s) \right\}$$

$$= s^{m-1}\, y^{(m-1)}(s)\Big|_{T'}^{t} - G(m-1)s^{m-2} y^{(m-2)}(s)\Big|_{T'}^{t}$$

$$-(m-1)(m-2)s^{m-3} y^{(m-3)}(s)\Big|_{T'}^{t}$$

$$+(m-1)(m-2)(m-3)\int_{T'}^{t} s^{m-4} y^{(m-4)}(s) \}$$

....

$$+G\sum_{i=0}^{m-2} T' y^{(i)}(T')(-1)^{m+i\frac{(m-1)!}{i!}}$$

Considering this and the fact that m is an even number, we get

$$\int_{T'}^{t} s^{m-1}(ry^{(m-1)})'(s)ds$$

$$\ge t^{m-1} y^{(m-1)}(t) - (T')^{m-1} y^{(m-1)}(T') - G\, x(t)(-1)^{m+1}(m-1)!$$

$$+G\sum_{i=0}^{m-2} t^{i} y^{(i)}(t)(-1)^{m+i+1\frac{(m-1)!}{i!}} + G\sum_{i=0}^{m-2} (T')^{i} y^{(i)}(T')(-1)^{m+i+1\frac{(m-1)!}{i!}}$$

$$\ge (T')^{m-1} y^{(m-1)}(T') - Gy(t)(m-1)! + G\sum_{i=0}^{m-2} (T')^{i} y^{(i)}(T')(-1)^{m+i+1\frac{(m-1)!}{i!}}$$

multiply(2.4)

In view of (2.4)(2.5),we obtain

$$-(T')^{m-1} y^{(m-1)}(T') - Gy(t)(m-1)! + G\sum_{i=0}^{m-2} (T')^{i} y^{(i)}(T')(-1)^{m+i+1\frac{(m-1)!}{i!}}$$

$$\ge -R\int_{T'}^{t} t^{m-1} r(s)\sum_{i=1}^{n} p_i(s) \prod_{g_i(u)<t_k\le s} \left(1+\alpha_k\right)^{-1}$$

So using (H_{10}), we obtain $y(t) \to \infty$ as $t \to \infty$, which is a contradiction.

Next we consider the case when $N > 1$. since $y'(t) > 0, y''(t) > 0, t > T$, so y' is increasing in $t \in [T, \infty)$. We note

$$y(t) = y(T) + \int_{T'}^{t} y'(\tau)d\tau \ge y(T) + y'(T)(t-T).$$

So $y(t) \to \infty$, as $t \to \infty$, which is a contradiction. The proof is complete.

The oscillation of higher order non-linear impulsive differential equations are investigated

$$x^{(m)}(t) + (t)\, x^{(m-1)}(t) + \sum_{i=1}^{n} p_i(t)\, x(g_i(t)) = 0, \quad t \ge t_0, \quad t \ne t_k,$$

$$x^{(j)}(t_k) - x^{(j)}(t_k^-) = a_k\, x^{(j)}(t_k^-), \quad j = 0, 1, 2, 3\ldots\ldots, m-1$$

Where $x'(t^+) = \lim_{x\to 0^+} \frac{x(t_k + h) - x(t_k)}{h}$,

$$x'(t_k^-) = x'(t_k) = \lim_{x\to 0^-} \frac{x(t_k + h) - x(t_k)}{h}$$

When $m = 2$, (1.1) reduces to the impulsive differential problem

$$x^{11}(\mathrm{t}) + \mathrm{a}(\mathrm{t})x^{1}(\mathrm{t}) + \sum_{i=1}^{n} p_i(\mathrm{t})x(\mathrm{g}_i(\mathrm{t})) = 0, \quad t \geq t_0, \quad t \neq t_k$$

$$x^{(j)}(t_k) - x^{(j)}(t_k^-) = \alpha_k x^{(j)}(t_k^-), \qquad j = 0, 1,$$

Oscillation and non-oscillation has been extensively investigated in

When $m = 2$, $g_i(\mathrm{t}) = \mathrm{t}$, $n = 1$, reduces to the impulsive differential problem

$$x^{11}(\mathrm{t}) + \mathrm{a}(\mathrm{t})x^{1}(\mathrm{t}) + p(\mathrm{t})x(\mathrm{t}) = 0, \quad t \geq t_0, \quad t \neq t_k$$

$$x^{(j)}(t_k) - x^{(j)}(t_k^-) = \alpha_k x^{(j)}(t_k^-), \quad j = 0, 1,$$

Oscillation and non-oscillation has been investigated

For any $\tau_0 \geq 0$, let $\tau_0^- = \min_{1\leq i \leq n \to \infty} \inf_{t \geq \tau_0} g_i(\mathrm{t})$. Let Ψ denote the set of functions $\phi : [\tau_0^-, \tau_0] \to \mathrm{R}$, $k_0 = \min\{\mathrm{k}/ t_k > \tau_0\}$, which are bounded and Lebesgue measurable on $[\tau_0^-, \tau_0]$.

AN EXAPLE:

Consider the impulsive delay differential equation

$$x''(\mathrm{t}) + x(\mathrm{t} - \tau) + \arctan\left|x'(\mathrm{t})\right| = 0, \quad t \geq 0, \quad t \geq 0,$$

$$x(\mathrm{t}_k) = \left(\frac{k+1}{k}\right)x(\mathrm{t}_k^-),\ x'(\mathrm{t}_k) = x'(\mathrm{t}_k^-),\ k = 1, 2, ...,$$

$$x(\mathrm{t}) = \phi(\mathrm{t}), \qquad -\tau \leq t \leq 0,$$

Where $t_{k+1} - t_k > \tau,\ k = 1, 2, ...,$ and $\phi, \phi' : [-\tau, 0] \to R$ are continuous.

Since $\phi(\mathrm{v}) = \mathrm{v}, \mathrm{p}(\mathrm{t}) = 1, \mathrm{a}_k = b_k = (\frac{k+1}{k})$ and $c_k = d_k = 1, k = 1, 2, ..,$ hypotheses (H_1) to (H_3) are satisfied. Note that

$$\lim_{x\to\infty} \int_{t_0}^{t} \prod_{t_{0<}t_{k<s}} \frac{c_k}{b_k} ds = \int_{t_0}^{\infty+} \prod_{t_{0<}t_{k<s}} \frac{k}{k+1} ds$$

$$= \int_{t_0}^{t_1} \prod_{t_{0<}t_{k<s}} \frac{k}{k+1} ds + \int_{t_1}^{t_2} \prod_{t_{0<}t_{k<s}} \frac{k}{k+1} ds + \int_{t_2}^{t_3} \prod_{t_{0<}t_{k<s}} \frac{k}{k+1} ds + ...$$

$$= (\mathrm{t}_1 - \mathrm{t}_0) + \frac{1}{2}(\mathrm{t}_2 - \mathrm{t}_1) + \frac{1}{3}(\mathrm{t}_3 - \mathrm{t}_2) + ...$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... = +\infty.$$

Thus, (H_4) is also satisfied.

Let $x(\mathrm{t})$ and $k_k = 1$ for all $k \geq 1$. And since

$$\int_{t_0}^{+\infty} p(\mathrm{u})du = \int_{t_0}^{+\infty} du = +\infty,$$

It follows from that all solutions $x(\mathrm{t})$ is oscillate

III. CONCLUSION

The theory of ordinary differential equation with impulses has been developed extensively over the past few years. Although there exists a well-developed oscillation theory of differential equation with and without delay, the oscillation of impulsive differential equations with and without delay seems to have rarely been considered. Finally in view of the known results obtained for differential equations without impulses new oscillation and non-oscillation criteria for higher order impulsive differential equations are derived.

The oscillation of higher order non-linear impulsive differential equations are investigated

$$x^{(m)}(t) + (t)\, x^{(m-1)}\ (\mathrm{t}) + \sum_{i=1}^{n} p_i(\mathrm{t})\, x\ (g_i(\mathrm{t})) = 0,\ \mathrm{t} \geq t_0, t \neq t_k,$$

$$x^{(j)}(t_k) - x^{(j)}(t_k^-) = a_k\, x^{(j)}(t_k^-), \quad j = 0, 1, 2, \ldots, m-1$$

Where $x'(t^+) = \lim_{x\to 0^+} \frac{x(t_k + \mathrm{h}) - x(t_k)}{\mathrm{h}}$,

$x'(t_k^-) = x'(t_k) = \lim_{x\to 0^-} \frac{x(t_k + \mathrm{h}) - x(t_k)}{\mathrm{h}}$

On applying the suitable impulses, the non-oscillation equation change into oscillation which is the purpose of this dissertation. Bring out many interesting aspects in the study of linear system.

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