Designing A Network Topology For Controllability Problem

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Abstract- *It is difficult to control all nodes in a network system with a large number of nodes.This probem can be solved by designing a network topology.The nodes in the network system are controlled by a few nodes.The basic link between structural controllability of network systems and topology design is based on theoretical framework.It gives how new nodes are added in to the network system without the introduction of any new control nodes in the network. Based on this results helps in dealing with a controllable network of topology design.Moreover, the results also show under what circumstances a network system with multiple identical nodes is uncontrollable.Groups of identical nodes are connected to each other in many applications termed as network of groups.Here, we address the structural controllability problem for multiple groups of network systems which provides information on proper topology design at both network level and node level .*

I. INTRODUCTION

The advanced technology in communication,computing and sensing have arised new challenges in the network system based on topology.To accomplish a common objective in the network systems consisting of interconnected nodes augmented with sensing,actuating computing and communicating capabilities.The networked systems play a great role in the field of sensor networks,multirobot systems,healthcare systems,smart urban systems etc.Network control is provided for the simplification and facilitation in supervision and control of networks with large number of nodes in the network system.The network controllability in most real systems are driven by nonlinear processes.Controllability problem in the network system arises as a fundamental problem.In the network science. Structural controllability has emerged as a major interests in the network system.The word controllability can be posed as identifying the minimum number of driver nodes in the equation.Structural controllability is not depending on degree distribution.Topology design is an interesting challenges in network systems.Inside the network the nodes are interconnected and arranged in a proper way.In this paper we design a topology based network in which the nodes in the network can be controlled by few nodes.A theoretical framework is developed which illustrates the connecting link between the structural controllability and topology design problem of network system.The existence of identical nodes in the network topology may lead to the condition of uncontrollability.Inorder to overcome this problems we introduce a topology based method for the examination of structural controllability with multiple nodes in a network system.The group of networks which provides information on appropriate topology design for network level and node level are discussed and address their structural controllability problem.

II. LITERATURE REVIEW

The controllability problem arises as a fundamental problem in the network system.

Liu,Slotine and Barabasi proposed tools to address controllability for arbitrary network topologies,but the results are based on the assumption that each node has an infinite time constant donot represent the dynamics of biological and physical systems.[1]

The technique proposed in [2] explains that structural controllability doesnot depend on degree distribution and can always be conferred with a single independent control input or driver that is connected directly to all nodes. It is difficult to connect the single driver directly to all nodes in the network when the signals are time varying.

M.Ji and M.Egerstedt proposed conditions that reder a multi agent networked system.The applicable conditions for networks in which a collection of the agents take on leader roles.Local consensus like control laws are executed by remaining agents.[3]

M.Newman, Barabasi,D.J Watts describes some of the driving applications of cooperative control,some relevant technology surveys that have been developed over past decades [4]

The technique proposed in [5] explains pinning controllability of networks of coupled dynamic systems, asymptotically driving a network.The technique proposed in[6] describes a simple controllability condition for discrete –time ,single leader networks.It lacks to derive continuous-time,single leader switching networks controllability condition.

III. EXISTING SYSTEM

The infrastructure of a network system consist of a wide range of physical nodes which are connected.In the existing system the graph topology explains the formation stabilization of a group of unicycles.

3.1. LIMITATIONS

- Existence of identical nodes leads to uncontrollability
- Leader node detection in multiagent systems are based on graph topology which makes the problem simpler since it rely on the degree Laplacian matrix.
- In large scale network systems due to the large number of nodes in the network it is difficult to apply input signals to all the nodes.
- The results are based on the assumption that each node has an infinite time constant which donot represent the dynamics of physical and biological systems.

IV. PROPOSED SYSTEM

The proposed system in this paper overcomes the limitations of existing system.The classical controllability concept is used to prove the structural controllability in a network system.Multiple driver nodes controllability is discussed in this paper.Laplacian matrix based concept is used on self loop,nodes dynamics and interconnection weights and gives a rigorous mathematical proof on structural controllability.

CASE1:Test for controllability

$(K \times , L \times)$ is controllable iff there is no eigenvector of K is orthogonal to L.

$$\forall w \text{ where } x^T K = \lambda x^T \Rightarrow x^T L = 0. \qquad \text{(i)}$$

Communications between nodes can be expressed by a weighted directed graph $\mathcal{G}(B,\mathcal{E},C)$, such that B= $\{b_1, b_2, \dots, b_n\}$ represents the set of nodes, $\mathcal{E} \subseteq B \times B$ is the edge set, C = $[c_{ij}]$ is the weighted adjacency matrix where $c_{ij} > 0$ if $(i, j) \in \mathcal{E}$ and $c_{ij} = 0$.

The following dynamic equation characterizes the physical nodes in a network system.

$$\dot{x}_i = -m_i x_i + k_{ij} x_j + l_i u_i \qquad \text{(ii)}$$

A dynamical system is controllable according to classical control theory iff any initial state there exists an input that can drive the system to any final state within a particular time period. It is well known that the system (K, L) is controllable iff the controllability matrix is:

$$M = [\, L, K^2 L, \dots, K^{n-1} L \,] \qquad \text{(iii)}$$

4.1CONTROLLABILITY:[On the basis of reachability of nodes in the network]

In this session explains a network system with a single node .(driver node i)which is expressed as given below.

$$t = Bx + \begin{pmatrix} 0_{(i-1)\times 1} \\ b_i \\ 0_{(n-i)\times 1} \end{pmatrix} u$$

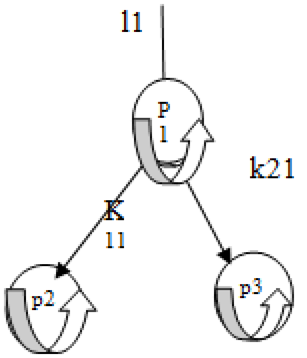
$$A \in R^{n \times 1}$$

Matrix K and matrix L can be expressed as follows for an arborescence diverging.

$$K = \begin{pmatrix} -\hat{k}_{11} & 0 & \cdots & 0 \\ k_{21} & -\hat{k}_{22} & & \vdots \\ \vdots & \vdots & \ddots & \\ \vdots & \vdots & & 0 \end{pmatrix}$$

$$^{k}N1 \quad ^{k}N2 \quad \cdots \quad -\hat{k}_{NN}$$

$$L = \begin{pmatrix} l_1 & 0 & \cdots & 0 \end{pmatrix}^T.$$



This figure is an example for proposed structural controllability and it is considering nodal dynamics, self loops.One input signal suffices for structural controllability of the network system.It explains a different controllability result.

$$K=\begin{pmatrix} k11 & 0 & 0 \\ k21 & k22 & 0 \\ k31 & 0 & k33 \end{pmatrix}$$

$$L=\begin{pmatrix} b1 \\ 0 \\ 0 \end{pmatrix}$$

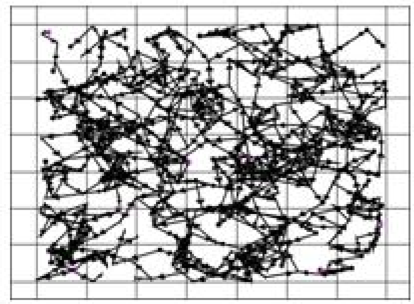
$$M=\begin{pmatrix} l1 & l1k11 & l1k11 \\ 0 & l1k21 & l1k21(k11+k22) \\ 0 & l1k31 & l1k31(k11+k33) \end{pmatrix}$$

4.2 CONTROLLABILITY:[All nodes are identical]

Let the network system consist of a single driver node and all the nodes are identical in the network represented as k11=….knn = a,when all the eigenvalues of matrix K are equal. $\lambda_1 = \cdot \cdot \cdot = \lambda_n = -\hat{k}$. Inorder to check structural controllability for this case, we invoke Popov–Belevitch–Hautus (PBH) controllability test given below:

$$\text{srank}\left[(-\hat{k})I - K \;\; L\right]$$

$$=\text{srank}\begin{pmatrix} 0 & 0 & \cdots & 0 & l_1 \\ ^{k}21 & 0 & \cdots & 0 & 0 \\ \vdots & & & \vdots & \vdots \\ \vdots & & \ddots & \vdots & \vdots \\ \vdots & & & \vdots & \vdots \\ ^{k}n1 & \cdots & ^{k}n(n-1) & 0 & 0 \end{pmatrix}.$$



Large network consists of 1000nodes.

V. CONCLUSION

We have developed a theoretical framework for the topology design problems from structural controllability perspective is explained in this paper.In a large network system it is more efficient to inject the input signals into a limited number of nodes in the network system.Finding minimum number of driver nodes is related to the topology of the network system.Therefore a proper topology design enablesthe control of the network system.It is also shown that multiple identical nodes may lead to the uncontrollability in the network system.

VI. ACKNOWLEDGEMENTS

We would like to thank, Almighty God at first, without his grace and blessings this work would not have been possible. We would also like to thank the faculty members of Mount Zion College of Engineering, for their great support towards this work.

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