Non Linear stability of Equilibrium points in the Generalized phogravitional Restricted Three Body Problems through Kolmogonov -Arnold Moser Theory

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Abstract*- In this Paper we are presented a key points of criticism of KAM (Kolmogorov-Arnold- Moser) theory case of Generalized photo gravitional restricted three-body problem when small primaries is oblate spheroid and bigger primaries is radiated.we should especially note that there is no analogue of Jacobian-type integral of motion in the case of photo gravitational restricted three-bodt problem if we take into consideration small Yarkovsky e ect. The KAM-theory is that an appropriate Hamilton formalism should be valid for the KAM thoery , but Hamilton formalism could not be applied for restricted three-body problem (which is proved to have only the Jacobian type integral of motion, but the integrals of energy, momentum are not invariants).*

Keywords- KAM theory, Hamilton formalism, Yarkovsky ef-fect, photo gravitational restricted three-body problem, Jacobian - type integral of motion

I. INTRODUCTION

KAM theory is known to be applied for research of stability of solar system in term of R3BP If we consider R3BP with additional in uence of Yarkovsky e ect of non-gravitional nature on equation of motion. The Yarkovsky e ect is a force acting on a rotating body in space caused by the an isotropic emission of thermal photons, which carry momentum. It is usually considered in relation to meteoroids or small asteroids as its in uence is most signi cant for these bodies. Such a force is produced by the way an asteroid absorbs energy from the sun and re-radiates it into space as heat by an isotropic way. In fact, there exists a disbalance of momentum when asteroid at rst absorbs the light, radiating from the sun, but then asteroid re-radiates the heat. Such a disbalance is caused by the rotating of asteroid during period of warming as well as it is caused by the isotropic cooling of surface inner layers; the processes above depend on an isotropic heat transfer in the inner layers of asteroid. Thus,Yarkovsky e ect is small but very important e ect in celestial mechanics as well as in calculating of a proper orbits of asteroids other small bodies. Besides, Yarkovsky e ect is not predictable (it could be only observed measured by astronomical methods); the main reason is un-predictable character of the rotating of small bodies in the case when there is no any collision between them.

Modi ed Equations of Motion (YORP-E ect)

Modi ed equations of motion for the three dimensional restricted 3-bodies problem, with an oblate primary m2, both primaries radiating, and the in-nitesimal mass m under the in uence of YORP-e ect, should be presented in barycentric rotating co-ordinate system in the form below:

These equations have the form

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} + Yx(t)$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} + Yy(t)$$

$$\frac{\partial \Omega}{\partial z} + Yz(t)$$

where Yx(t), Yy(t), Yz(t) are the projecting of YORP-e ect acceleration Y(t) on the appropriate axis Ox, Oy, Oz.

Location of Equilibrium Points

The location of equilibrium points for system (in general is given by con-ditions $\ddot{x} = \ddot{y} = \ddot{z} = \dot{x} = \dot{y} = \dot{z} = 0$

$$\frac{\partial \Omega}{\partial x} = -Yx(t)$$

$$\frac{\partial \Omega}{\partial y} = -Yy(t)$$

$$\frac{\partial \Omega}{\partial z} = -Yz(t)$$

which determine the location of equilibrium points

we should note that a case of Yarkovsky e ect is negligible determines the existence of quasi-planar equilibrium points in which conditions Yz aproaches to 0, z approches to 0 are valid simultaneously. The strongest simplifying of system is possible when Yarkovsky e ect is zero, Yx = Yy = Yz = 0. In such a case, it has been proved the existence equilibrium points in photogravita-tional restricted 3-bodies problem.

STABILITY

Now we apply Mosers modi ed form of Arnolds theorem to discuss the non linear stability that equations of restricted three-body problem are proved to describe the system with non-Hamilton formalism

$$= \frac{x^2+y^2}{2} + \frac{1}{r1} + \frac{}{r2}$$

These equations have the form

$$x\bullet - 2 y = @_{@x}$$

$$y\bullet + 2 x = @_{@y}$$

$$z\bullet = @_{@z}$$

For example

$$\frac{@}{@x} = x - \frac{(1\ )}{r} \frac{@r1}{@x} - \frac{@r2}{r} \frac{}{@x}$$

where

$$r1^2 = (x +)^2 + y^2 + z^2$$

$$2\ r1\ @r_{@x}1 = 2\ (x + \ )$$

$$\frac{@r1}{@x} = \frac{x+}{r_1}$$

$$r2^2 = (x \quad 1 +)^2 + y^2 + z^2$$

$$\frac{@r2}{@x} = \frac{x \quad 1+}{r_2}$$

Substituting Equations and into Equation yields the rst of equations

$$x\bullet - 2 y = x - (1 \quad) \frac{(x+)}{r1} - \frac{(x+)}{r1}(1 \quad) \frac{(x \ 1+)}{r2}$$

By multiplying Equations 2x ,2y and 2 z respectively

$$2\ xx\bullet - 22\ xy = 2x @U_{@x}$$

$$2\ yy\bullet + 22\ xy = 2y\ @_{@y}\ 2\ zz\bullet = 2z\ @_{@z}$$

Summing these

$$2\ x\bullet x\bullet + 2\ y\bullet y\bullet + 2\ z\bullet z\bullet = 2x @_{@x} + 2y\ @_{@y} + 2z\ @_{@z}$$

This can be integrated to yield

$$x^2 + y^2 + z^2 = 2 \qquad C$$

We have used the fact that

$$2 \int \left[\frac{dx}{dt}\frac{@}{@} + \frac{dx}{dt}\frac{@}{@y} + \frac{dz}{dt}\frac{@}{@z}\right]$$

We represent a di usion mechanism for the time - dependent perturbation of autonomous Hamiltorian system introduced unlike other approach does not rely on the KAM theory. According to above constant, we could obtain from the equations of system a Jacobian-type integral of motion

Besides, the restricted three-body problem is proved to have a new, the only stable invariant = Jacobian-type integral of motion.

Hamilton systems are assumed to be the systems without di usion.It means that such a dynamical systems should have a weak Arnold-di usion.

- where C is so-called Jacobian constant. As it was proved such a Ja-cobian type integral of motion should not be depending on time for large time-period. Additionally, we should especially note obvious fact: in the case of photogravitational restricted three-body problem with Yarkovsky ef-fect there is no analogue of Jacobian-type integral for ODE system of motion.

II. CONCLUSION

KAM theory tried to predict the stability in generalization photogravi-tional restricted three body problem. We consider solar system as Arnold di usion there is no analogue when we consider very small Yarkovsky e ect as we discuss above that stable invariant behave as Jacobian constant.It proved that the existence of liberation points in R3BP when we take into consider even a small Yarkovsky e ect.

REFERENCES

[1] Arnold V. (1978). Mathematical Methods of Classical Mechanics. Springer, New York.

[2] Arnold V. (1963). Small Divisor Problems in Classical and Celestial Me-chanics Russ. Math. Surveys 18, 85-191.

[3] Szebehely V. (1967). Theory of Orbits. The Restricted Problem of Three Bodies. Yale University, New Haven, Connecticut. Academic Press New-York and London.

[4] Shankaran, Sharma J.P., and Ishwar B. (2011). Equilibrium points in the generalized photogravitational non-planar restricted three body problem. In-ternational Journal of Engineering, Science and Technology, Vol. 3 (2), pp. 63-67.

[5] V.V. Radzievskii, A mechanism for the disintegration of asteroids and mete-orites, Doklady Akademii Nauk SSSR, 97 (1954), 49-52.

[6] David P. Rubincam, Radiative spin-up and spin-down of small asteroids, Icarus, 148 (2000), 2-11.

[7] V.V. Radzievskii, The restricted problem of three bodies taking account of light pressure, Akad. Nauk. USSR, Astron. Journal, 27 (1950), 250.

[8] Shankaran, J.P. Sharma, B. Ishwar, Equilibrium points in the generalized photogravitational non-planar restricted three body problem, International Journal of Engineering, Science and Technology, 3, No. 2 (2011), 63-67.

[9] C.N. Douskos, V.V. Markellos, Out-of-plane equilibrium points in the re-stricted three body problem with oblateness, AA, 446 (2006), 357-360.

[10] Y.A. Chernikov, The photogravitational restricted three-body problem, So-viet Astronomy, 14 (1970), 176.

[11] Singh Jagadish, Oni Leke, Stability of the photogravitational restricted three-body problem with variable masses, Astrophys Space Sci., 326 (2010), 305-314.