Fuzzy Almost Resolvable Spaces

S.Rajasekar[1], V.Balakrishnan[2]
[1, 2] Department of Mathematics
[1, 2] CK College of Engineering and Technology, Cuddalore.

Abstract- *This paper attempts to study how the topological spaces can be used in Fuzzy and the relationship between fuzzy resolvable spaces. Also how the fuzzy open sets and closed sets can be used to derive the fuzzy resolvable spaces. We prove that the fuzzy almost irresolvable space is a fuzzy second category space.*

Keywords- Topological Space, fuzzy open and closed sets, fuzzy Baire space.

I. INTRODUCTION

Most of our traditional tools for formal modeling, and computing are crisp, deterministic, and precise in character. By crisp we mean yes or no type rather than more -or-less type. In set theory, an element can either belong to a set or not and in optimization a solution is either feasible or not .To overcome such complex situation, the theory of fuzzy set was **L.A. ZADEH** in his classical paper in the year 1965, describing fuzziness mathematically for the first time.

Among the first field of mathematics to be considered in the content of fuzzy sets was general topology. The concept of fuzzy topology was defined by **C.L.CHANG** in the year 1968. The paper of change paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics. **E.HEWIT** introduced the concepts of resolvability and irresolvablilty in topological spaces.

The concepts of fuzzy almost resolvable spaces and fuzzy almost irresolvable spaces, are introduced by **S.G.THANGARAJ** and **D.VIJAYAN**.

In this dissertation work,

Chapter One: We have the general introduction about the topic.

Chapter two: we have studied preliminaries and results, which are useful for this work.

Chapter three: we have studied some examples, propositions fuzzy almost regarding fuzzy almost resolvable and irresolvable spaces.

II. PRELIMINARIES

By a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X,T).

Definition 1: Let λ and μ be any two fuzzy sets in (X,T),Then we define $\lambda \vee \mu$: $X \to [0,1]$as follows $(\lambda \vee \mu)(x) = Max\ \{\lambda(x),\ \mu(x)\}\quad \forall\, x \in X$
Also we define $\lambda \wedge \mu$: $X \to [0,1]$ as follows $(\lambda \wedge \mu)(x) = Min\ \{\lambda(x),\ \mu(x)\}\quad \forall\, x \in X$

Definition 2: Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define the closure and the interior of λ as follows:
i)$int(\lambda) = \vee\{\mu/\mu \le \lambda,\ \mu \in T\}$,
$ii)cl(\lambda) = \wedge\{\mu/\lambda \le \mu, 1-\mu \in T\}$.

Definition 3: A fuzzy set λ in a fuzzy topological space $(X,T)$is called fuzzy dense if $cl(\lambda) = 1$.

Definition 4: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if $int\ (cl(\lambda)) = 0$.

Definition 5: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy F_σ-Set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, Where $(1-\lambda_i)$ for $i = 1, 2, \dots, \infty$.

Definition 6: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy G_δ-Set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i) = 0$ where $\lambda_i \in T$ for $i = 1, 2 \dots \infty$.

Definition 7: A fuzzy topological space (X,T) is called a fuzzy open hereditarily irresolvable space if $int(\ cl(\lambda)) \neq 0$, then $int(\lambda) \neq 0$, for any non-zero fuzzy set λ in (X,T).

Definition 8: A fuzzy topological space (X,T) is called a fuzzy submaximal space if $cl(\lambda) = 1$ for any non-zero fuzzy set λin (X,T), then $\lambda \in T$.

Definition 9: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category space if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where are $(\lambda_i)'s$ fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of second category space.

Definition 10: fuzzy topological space (X,T) is called fuzzy first category space if $\vee_{i=1}^{\infty}(\lambda_i)$=**1,** where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T).

A topological space which is not of fuzzy first category space is said to be of fuzzy second category space.

Definition 11: Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy σ-nowhere dense set, if λ is a fuzzy F_σ-set in (X,T) such that $int(\lambda) = 0$.

Definition 12: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy σ-first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where the fuzzy sets $(\lambda_i)'s$ are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be fuzzy **σ**-second category

Result 13: If λ is a fuzzy set in a topological space (X,T), then $\lambda \le cl(\lambda)$

Proof: By Definition of $cl(\lambda)$
$cl(\lambda)$ is the smallest closed set containing λ and so…
$\therefore\ \lambda \le cl(\lambda)$

Result 14: If λ is a fuzzy set in a topological space (X,T), then $int(\lambda) \le \lambda$

Proof: By Definition of $int(\lambda)$
$int(\lambda)$ is the largest open set contained in λ and so..
$\therefore\ int(\lambda) \le \lambda$

Result 15: If α and β are fuzzy set in a topological space (X,T), then $\alpha \le \beta \Rightarrow int(\alpha) \le int(\beta)$

Proof: Let, $\alpha \le \beta$ ……………(1)
Also $int(\alpha) \le \alpha$ ………………(2) *(Result 2.14)*
Using (1) & (2) $int(\alpha) \le \alpha \le \beta$
Also $int(\alpha)$ is open and $int(\alpha) \le \beta$
But $int(\beta)$ is largest open set contained in β
$\therefore int(\alpha) \le int(\beta)$

Result 16: If α and β are fuzzy set in a topological space (X,T), then $\alpha \le \beta \Rightarrow cl(\alpha) \le cl(\beta)$

Proof: Let, $\alpha \le \beta$ ……………………………(1)
Also,
$\beta \le cl(\beta)$ ……………………(2) *(By Result 2.13)*
From (1) & (2) $\alpha \le \beta \le cl(\beta)$
That is $cl(\beta)$ contains α and $cl(\beta)$ is closed. Also $cl(\alpha)$ is smallest closed set containing α.
$\therefore cl(\alpha) \le cl(\beta)$

Result 17: If λ is a fuzzy set in a topological space (X,T), then $int(\lambda) = int(cl(\lambda))$

Proof: $\lambda \le cl(\lambda)$(By Result 2.13)
$\alpha \le \beta \Rightarrow int(\alpha) \le int(\beta)$ (By Result 2.15)
$\therefore\ int(\lambda) \le int(cl(\lambda))$

Result 18: If λ is a fuzzy set in a topological space (X,T),then $cl(1-\lambda) = 1 - int(\lambda)$

Proof: $cl(1-\lambda) = \wedge\{\mu / 1-\lambda \le \mu, 1-\mu \in T\}$
$1 - cl(1-\lambda) = 1 - \wedge\{\mu / 1-\lambda \le \mu, 1-\mu \in T\}$
$= \vee\{1-\mu / 1-\lambda \le \mu, 1-\mu \in T\}$
(By De Morgan's Law)
$[1 - \wedge\alpha_i = \vee(1-\alpha_i)]$
Replace $1-\mu$ by β
$= \vee\{\beta / 1-\lambda \le 1-\beta, \text{is open}\}$
$= \vee\{\beta / \beta \le \lambda, \beta \text{ is open}\}$
$1 - cl(1-\lambda) = int(\lambda)$
$\therefore cl(1-\lambda) = 1 - int(\lambda)$

Result 19: If λ is a fuzzy set in a topological space (X,T), then $int(1-\lambda) = 1 - cl(\lambda)$

Proof: $int(1-\lambda) = \vee\{\mu / \mu \le 1-\lambda, \mu \in T\}$
$1 - int(1-\lambda) = 1 - \vee\{\mu / \mu \le 1-\lambda, \mu \in T\}$
$= \wedge\{1-\mu / \mu \le 1-\lambda, \mu \in T\}$
(By De Morgan's Law)
$[1 - \vee\alpha_i = \wedge(1-\alpha_i)]$
Replace $1-\mu$ by β
$= \wedge\{\beta / 1-\beta \le 1-\lambda, \beta \text{ is Closed}\}$
$= \wedge\{\beta / \lambda \le \beta, \beta \text{ is Closed}\}$
$1 - int(1-\lambda) = cl(\lambda)$
$\therefore int(1-\lambda) = 1 - cl(\lambda)$

Result 20:
i)if λ is open then $int(\lambda) = \lambda$
ii)if λ is closed then $cl(\lambda) = \lambda$

Result 21: If λ is a fuzzy set in a topological space (X,T),then $int[cl(1-\lambda)] = 1 - cl[int(\lambda)]$

Proof: $int[cl(1-\lambda)] = int[1 - int(\lambda)]$ **(**By lemma 2.18)
$int[cl(1-\lambda)] = 1 - cl[int(\lambda)]$ **(**By Lemma 2.19)

Result 22: If λ is a fuzzy set in a topological space (X,T), then $cl[int(1-\lambda)] = 1 - int[cl(\lambda)]$

Proof: $cl\,[int\,(1-\lambda)] = cl\,[1 - cl(\lambda)]$ (By Lemma 2.19)$cl\,[int(1-\lambda)] = 1 - int[cl(\lambda)]$ (By Lemma 2.18)

III. FUZZY ALMOST RESOLVABLE SPACES

Definition 1: A fuzzy topological space (X,T) is called a fuzzy almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets $(\lambda_i)'s$ in (X,T) are such that $int(\lambda_i) = 0$. Otherwise $(X,T)$is called a fuzzy almost irresolvable space.

Example 2: Let $X = \{a,b,c\}$the fuzzy sets $\lambda,\mu$and υ are defined on X as follows

$\lambda{:}X \to [0,1]$ is defined as
$\lambda(a) = 1;\ \lambda(b) = 0.3;\lambda(c) = 0.7$
$\mu{:}X \to [0,1]$ is defined as
$\mu(a) = 0.4;\ \mu(b) = 1;\mu(c) = 0.6$
$\upsilon{:}X \to [0,1]$ is defined as
$\upsilon(a) = 0.5;\ \ \upsilon(b) = 0.6;\ \upsilon(c) = 1$
then, Clearly $T = \{0, \lambda, \mu, \upsilon, \lambda\vee\mu,\ \lambda\vee\upsilon,\ \mu\vee\upsilon,\ \lambda\wedge\mu, \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\vee(\mu\wedge\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\wedge(\lambda\vee\upsilon), 1\}$ is a fuzzy topology on X.

Table 1. **OPEN SETS**

	a	b	c
0	0	0	0
λ	1	0.3	0.7
μ	0.4	1	0.6
υ	0.5	0.6	1
$\lambda\vee\mu$	1	1	0.7
$\lambda\vee\upsilon$	1	0.6	1
$\mu\vee\upsilon$	0.5	1	1
$\lambda\wedge\mu$	0.4	0.3	0.6
$\lambda\wedge\upsilon$	0.5	0.3	0.7
$\mu\wedge\upsilon$	0.4	0.6	0.6
$\lambda\vee(\mu\wedge\upsilon)$	1	0.6	0.7
$\mu\vee(\lambda\wedge\upsilon)$	0.5	1	0.7
$\upsilon\wedge(\lambda\vee\upsilon)$	0.5	0.6	0.7
1	1	1	1

Table 2. CLOSED SETS

	a	b	c
0	0	0	0
$1-\lambda$	0	0.7	0.3
$1-\mu$	0.6	0	0.4
$1-\upsilon$	0.5	0.4	0
$1-(\lambda\vee\mu)$	0	0	0.3
$1-(\lambda\vee\upsilon)$	0	0.4	0
$1-(\mu\vee\upsilon)$	0.5	0	0
$1-(\lambda\wedge$	0.6	0.7	.4
$1-(\lambda\wedge$	.5	.7	.3
$1-(\mu\wedge$	.6	.4	.4
$1-(\lambda\vee(\mu\wedge\upsilon))$		.4	.3
$1-(\mu\vee(\lambda\wedge\upsilon))$	.5		.3
$1-(\upsilon\wedge(\lambda\vee\upsilon))$	.5	.4	.3
1			

Table 3. Define α,β,γ as follows

	A	b	c
0	0	0	0
α	1	0.2	0.3
β	0.2	1	0.7
γ	0.3	0.4	1
η	1	0.1	0.5
1	1	1	1

Then $int(\alpha) = 0$ //$int(\beta) = 0$//$int(\gamma) = 0$//$int(\eta) = 0$
$\{(\alpha)\vee(\beta)\vee(\gamma)\vee(\eta)\} = 1$

$\therefore (X,T)$ is a fuzzy almost resolvable space.

Example 3: Let $X = \{a,b,c\}$, The fuzzy sets $\lambda,\ \mu$ and υ are defined on X as follows

- • $\lambda{:}X \to [0,1]$ is defined as $\lambda(a) = 1;\ \lambda(b) = 0.2;\lambda(c) = 0.7$
- • $\mu{:}X \to [0,1]$ is defined as $\mu(a) = 0.3;\ \mu(b) = 1;\mu(c) = 0.2$
- • $\upsilon{:}X \to [0,1]$ is defined as $\upsilon(a) = 0.7;\ \ \upsilon(b) = 0.4;\ \upsilon(c) = 1$

Then,
Clearly
$T = \{0, \lambda,\ \mu, \upsilon, \lambda\vee\mu,\ \lambda\vee\upsilon,\ \mu\vee\upsilon,\ \lambda\wedge\mu,\ \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\vee(\mu\wedge\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\wedge(\lambda\vee\upsilon), 1\}$
is a fuzzy topology on X.

Table 4. OPEN SETS

	A	b	c
0	0	0	0
λ	1	0.2	0.7
μ	0.3	1	0.2
υ	0.7	0.4	1
$\lambda \vee \mu$	1	1	0.7
$\lambda \vee \upsilon$	1	0.4	1
$\mu \vee \upsilon$	0.7	1	1
$\lambda \wedge \mu$	0.3	0.2	.2
$\lambda \wedge \upsilon$	0 .7	0 .2	0 .7
$\mu \wedge \upsilon$	0 .3	0 .4	0 .2
$\lambda \vee (\mu \wedge \upsilon)$	1	0 .4	0 .7
$\mu \vee (\lambda \wedge \upsilon)$	0 .7	1	0 .7
$\upsilon \wedge (\lambda \vee \upsilon)$	0 .7	0 .4	0 .7
1	1	1	1

Table 5. CLOSED SETS

	a	b	c
0	0	0	0
$1 - \lambda$	0	0.8	0.3
$1 - \mu$	0.7	0	0.8
$1 - \upsilon$	0.3	0.6	0
$1 - (\lambda \vee \mu)$	0	0	0.3
$1 - (\lambda \vee \upsilon)$	0.3	0.6	0
$1 - (\mu \vee \upsilon)$	0.3	0	0
$1 - (\lambda\wedge$	0.7	0.8	.8
$1 - (\lambda\wedge$	.3	.8	.3
$1 - (\mu\wedge$	.7	.6	.8
$1 - (\lambda \vee (\mu \wedge \upsilon))$		.6	.3
$1 - (\mu \vee (\lambda \wedge \upsilon))$	.3		.3
$1 - (\upsilon \wedge (\lambda \vee \upsilon))$	.3	.6	.3
1			

Table 6. Define α_n, β_n , γ_n as follows

	a	b	c
0	0	0	0
α_n	1	0.1	0
β_n	0.2	1	0
γ_n	0	0.1	1
1	1	1	1

Then $int(\alpha_n) = 0; int(\beta_n) = 0; \ int(\gamma_n) = 0;$

$$\{(\alpha_n) \vee (\beta_n) \vee (\gamma_n)\} = 1$$

$\therefore (X, T)$ is a fuzzy almost resolvable space.

Example 4: Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and υ are defined on X as follow $\alpha: X \to [0,1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.5$; $\alpha(c) = 0.5$
$\beta: X \to [0,1]$ is defined as $\beta(a) = 0.7$; $\beta(b) = 0.8$; $\beta(c) = 0.6$
$\gamma: X \to [0,1]$ is defined as $\gamma(a) = 0.6$; $\gamma(b) = 0.5$; $\gamma(c) = 0.5$
Then, Clearly $T = \{0, \alpha, 1\}$

Table 7. OPEN SETS

	A	b	c
0	0	0	0
α	0.6	0.5	0.5
1	1	1	1

Table 8. CLOSED SETS

	A	b	c
0	0	0	0
$1 - \alpha$	0.4	0.5	0.5
1	1	1	1

$int(\beta) = \alpha$; $int(\gamma) = \alpha$;
$cl(\alpha) = 1$; $cl(\beta) = 1$; $cl(\gamma) = 1$;
$\therefore \alpha$, β and γ are dense sets

Table 9.

	A	B	c
$1 - \alpha$	0.4	0.5	0.5
$1 - \beta$	0.3	0.2	0.4
$1 - \gamma$	0.4	0.5	0.5

$cl(1-\beta)=1-\alpha$
$cl(1-\gamma)=1-\alpha$
$(1-\alpha)\vee(1-\beta)\vee(1-\gamma)\neq 1$
$int(1-\alpha)=1-cl(\alpha)=1-1=0$
$int(1-\beta)=1-cl(\beta)=1-1=0$
$int(1-\gamma)=1-cl(\gamma)=1-1=0$

$\therefore (X,T)$ is fuzzy almost irresolvable space

Proposition 5: If $\wedge_{i=1}^{\infty}(\mu_i)=0$, where the fuzzy set $(\mu_i)'s$ are fuzzy dense set in a fuzzy topological space (X,T) is a fuzzy almost resolvable space.

Proof: Given that $\wedge_{i=1}^{\infty}(\mu_i)=0$......................(1)
Where $(\mu_i)'s$ are fuzzy dense sets.
$\Rightarrow \quad cl(\mu_i)=1 \quad \forall i$
$\Rightarrow \ 1-cl(\mu_i)=1-1=0$ But By Lemma
$\Rightarrow \ 1-cl(\mu_i)=int\,(1-u_i)$

(By Result 2.19)
$\Rightarrow int\,(1-\mu_i)=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$
$1-\wedge_{i=1}^{\infty}(\mu_i)=1-0=1$ (using1)
$\Rightarrow \bigvee_{i=1}^{\infty}(1-\mu_i)=1$
(By Demorgan's Law)

$(1-\wedge_{i=1}^{\infty}(\alpha_i)=\vee_{i=1}^{\infty}(1-\alpha_i))$
Let $(1-\mu_i)=\lambda_i$
Then, we have $\vee_{i=1}^{\infty}(\lambda_i)=1$
Where $int\,(\lambda_i)=int(1-\mu_i)=0$ (Using 2)
$\therefore (X,T)$ is a fuzzy almost resolvable space.

Definition 6: A fuzzy topological space (X,T) is called a fuzzy hyper connected space if every fuzzy open set is fuzzy dense in (X,T), that is $cl(\mu_i)=1$ for all $\mu_i\in T$.

Proposition 7: If $\wedge_{i=1}^{\infty}(\mu_i)=0$, where the fuzzy set $(\mu_i)'s$ are fuzzy open set in a fuzzy hyper connected space (X,T), then (X,T) is a fuzzy almost resolvable space.

Proof: Given that $\wedge_{i=1}^{\infty}(\mu_i)=0$, where $(\mu_i)'s$ are open. Also, given that the fuzzy topological space (X,T) is a fuzzy hyper connected space.
The fuzzy open set μ_i is a fuzzy dense set in (X,T), for each i, $\Rightarrow \quad cl(\mu_i)=1 \quad \forall i$
$\Rightarrow \ 1-cl(\mu_i)=1-1=0 \quad \forall i$
$\Rightarrow \ 1-cl(\mu_i)=int\,(1-\mu_i)$ (By Result 2.19)
$\Rightarrow int(1-\mu_i)=0 \ldots\ldots\ldots (1)$
Given, $\wedge_{i=1}^{\infty}(\mu_i)=0$

$1-\bigwedge_{i=1}^{\infty}(\mu_i)=1-0=1$
$1-\bigwedge_{i=1}^{\infty}(\mu_i)=1$
$\Rightarrow \bigvee_{i=1}^{\infty}(1-\mu_i)=1$
(By Demorgan's Law)
$(1-\wedge_{i=1}^{\infty}(\alpha_i)=\vee_{i=1}^{\infty}(1-\alpha_i))$
Let $(1-\mu_i)=\lambda_i$
$\Rightarrow \bigvee_{i=1}^{\infty}(\lambda_i)=1$
Where $int\,(\lambda_i)=int\,(1-\mu_i)=0$ (from (1))
$\therefore (X,T)$ is a fuzzy almost resolvable space.

Proposition 8: If $\vee_{i=1}^{\infty}(\lambda_i)=1$, where the fuzzy sets $(\lambda_i)'s$ are fuzzy σ-nowhere dense set in a fuzzy topological space (X,T), then (X,T) is a fuzzy almost resolvable space.
Proof: Let $(\lambda_i)'s$ $(i=1 \text{ to } \infty)$ be fuzzy σ- nowhere dense sets in (X,T).Then $(\lambda_i)'s$ are fuzzy F_σ- sets and $int(\lambda_i)=0$
Given $\vee_{i=1}^{\infty}(\lambda_i)=1$
Where $int(\lambda_i)=0$
$\therefore (X,T)$ is a fuzzy almost resolvable space

Definition 9: A fuzzy topological space (X,T) is called fuzzy P-space, if countable intersection of fuzzy open sets in (X,T) is fuzzy open, that is every non-zero fuzzy G_δ- set in (X,T) is fuzzy open in (X,T).

Proposition 10: If $\wedge_{i=1}^{\infty}(\mu_i)=0$, where the fuzzy set $(\mu_i)'s$ are fuzzy G_δ-sets in a fuzzy hyper connected space and P- space, then (X,T) is a fuzzy almost resolvable space.

Proof: Let $(\mu_i)'s$ $(i=1 \text{ to } \infty)$ be fuzzy G_δ - Sets in the fuzzy P- space (X,T)
$\Rightarrow (\mu_i)'s$ are G_δ - Sets
$\Rightarrow (\mu_i)'s$ are countable intersection of open sets
$\Rightarrow$ Then $(\mu_i)'s$ are fuzzy open sets in (X,T)
((X,T) is a fuzzy P-space)
Hence, we have $\wedge_{i=1}^{\infty}(\mu_i)=0$
Where, the fuzzy sets $(\mu_i)'s$ are fuzzy open sets in a fuzzy hyper connected space (X,T).By Proposition 3.7

"If $\wedge_{i=1}^{\infty}(\mu_i)=0$, where the fuzzy set $(\mu_i)'s$ are fuzzy open set in a fuzzy hyper connected space (X,T), then (X,T) is a fuzzy almost resolvable space."
$\therefore (X,T)$ is a fuzzy almost resolvable space.

Proposition 11: In a fuzzy almost resolvable space (X,T) if $(\lambda_i)'s$ are fuzzy F_σ-sets, then (X,T) is a fuzzy σ-first category space.

Proof: Let (X,T) be a fuzzy almost resolvable space.
Then we have $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where $int(\lambda_i) = 0$.
Since $(\lambda_i)'s$ are fuzzy F_σ-sets in (X,T) and $int(\lambda_i) = 0$
$\Rightarrow \vee_{i=1}^{\infty}(\lambda_i) = 1$ where $int(\lambda_i) = 0$
We have (X,T) is fuzzy σ-first category space.

Definition 12: A fuzzy topological space (X,T) is called a fuzzy nodec space, if every non-zero fuzzy nowhere dense set in (X,T) is a fuzzy closed set in (X,T).

Proposition 13: If (X,T) is a fuzzy first category space and fuzzy nodec space, then (X,T) is fuzzy almost resolvable space.

Proof: Let (X,T) be a fuzzy first category space.Then we have $\vee_{i=1}^{\infty}(\lambda_i) = 1$ where the fuzzy set $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T).
Since, (X,T) is a fuzzy nodec space, the fuzzy nowhere dense sets are fuzzy closed sets in (X,T).

Hence $(\lambda_i)'s$ are fuzzy closed sets in (X,T). That is $cl(\lambda_i) = \lambda_i$. Now $int\ (cl(\lambda_i)) = 0$ (Because of $\lambda_i's$ are fuzzy nowhere dense sets)
$\Rightarrow int(\lambda_i) = 0$, Hence we have $\vee_{i=1}^{\infty}(\lambda_i) = 1$
Where the fuzzy sets $(\lambda_i)'s$ in (X,T) are such that $int(\lambda_i) = 0$.
Hence (X,T) is a fuzzy almost resolvable space.

Proposition 14: If the fuzzy topological space (X,T) is a second category space, then (X,T) is a fuzzy almost irresolvable space.

Proof: Let (X,T) be a fuzzy second category space,
Then $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$.Where the fuzzy set $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T), That is $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$
Where $int(cl(\lambda_i)) = 0$
$int(\lambda_i) \leq int(cl(\lambda_i))$ (Using Result (2.17))
$\Rightarrow int(\lambda_i) = 0$
Hence $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$
Where, $int(\lambda_i) = 0$
$\therefore (X,T)$ is a fuzzy almost irresolvable space.

Definition 15: A fuzzy topological space (X,T) is called a fuzzy volterra space if (X,T). $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$,Where $(\lambda_i)'s$ are fuzzy dense set and fuzzy G_δ-set in (X,T).

Definition 16: A fuzzy topological space (X,T) is called weakly volterra space if $cl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$,Where $(\lambda_i)'s$ are fuzzy dense set and fuzzy G_δ-sets in (X,T).

Proposition 17: If a fuzzy topological space (X,T) is not a fuzzy weakly volterra space, then (X,T) is a fuzzy almost resolvable space.

Proof: Let (X,T) be a fuzzy non-weakly volterra space. Then we have $cl(\wedge_{i=1}^{N}(\lambda_i)) = 0$
Where $(\lambda_i)'s$ are fuzzy dense set and fuzzy G_δ-sets in (X,T).
Now $cl(\wedge_{i=1}^{N}(\lambda_i)) = 0$
$\Rightarrow int(\vee_{i=1}^{N}(1-\lambda_i)) = 1$ and $cl(\lambda_i) = 1$
$1 - cl(\lambda_i) = 1 - 1 = 0$
$\Rightarrow int(1-\lambda_i) =$ (By Lemma 2.19)
Let $(\mu_j)'s$ $(j = 1$ to $\infty)$ be fuzzy sets in (X,T)
such that $\Rightarrow int(\mu_j) = 0$
Take the first N $(\mu_j)'s$ as $(1-\lambda_i)'s$
Now $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{j=1}^{\infty}(\mu_j)$

$$\Rightarrow int\left(\bigvee_{i=1}^{N}(1-\lambda)\right) \leq int\left(\bigvee_{j=1}^{\infty}(\mu_j)\right) \leq \bigvee_{j=1}^{\infty}(\mu_j)$$

Then we have $1 \leq (\vee_{j=1}^{\infty}(\mu_j)) \Rightarrow (\vee_{j=1}^{\infty}(\mu_j)) = 1$
Where the fuzzy sets $(\mu_j)'s$ in (X,T) are such that $int(\mu_j) = 0$.
$\therefore (X,T)$ is a fuzzy almost resolvable space.

Proposition 18: If the fuzzy almost resolvable space (X,T) is a fuzzy submaximal space, then (X,T) is a fuzzy first category space.

Proof: Let (X,T) be a almost resolvable space.
Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, Where the fuzzy sets $(\lambda_i)'s$ in (X,T) are such that $int(\lambda_i) = 0$, $\vee_{i=1}^{\infty}\lambda_i = 1$, $int(\lambda_i) = 0$
$1 - \vee_{i=1}^{\infty}\lambda_i = 1 - 1 = 0$, $1 - int(\lambda_i) = 1$
Then we have $\wedge_{i=1}^{\infty}(1-\lambda_i) = 0$, where $cl(1-\lambda_i) = 1$ (By Lemma 2.19)
Since, the space (X,T) is a fuzzy submaximal space, the fuzzy dense sets.$(1-\lambda_i)'s$ are fuzzy open sets in (X,T).Then $(\lambda_i)'s$ are fuzzy closed sets in (X,T) and hence $cl(\lambda_i) = \lambda_i$.
Now $int(\ cl(\lambda_i)) = \lambda_i = int(\lambda_i) = 0$.Then $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T)
Hence $\vee_{i=1}^{\infty}(\lambda_i) = 1$.
Where the fuzzy sets $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T)
$\therefore (X,T)$ is a fuzzy first category space.

Proposition 19: If the fuzzy almost irresolvable space (X,T) is a fuzzy submaximal space, then (X,T) is a fuzzy second category space.

Proof: Let (X,T) be a fuzzy almost irresolvable space then $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$.Where the fuzzy sets $(\lambda_i)'s$ are such that $int(\lambda_i) = 0$

Now $int(\lambda_i) = 0$

$1 - int(\lambda_i) = 1 - 0 = 1$

$\Rightarrow cl(1 - \lambda_i) = 1$ (using result 2.19)

ie., $(1 - \lambda_i)'s$ are fuzzy dense sets in (X, T).

Since (X, T) is a fuzzy submaximal space, the fuzzy dense sets $(1 - \lambda_i)'s$ are fuzzy open sets in (X, T).

Then $(\lambda_i)'s$ are fuzzy closed sets in (X, T)

$cl(\lambda_i) = \lambda_i$,

Now $int(\lambda_i) = 0$

$\Rightarrow int(\lambda_i) = 0$

Then $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T)

Hence we have $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$,where the fuzzy sets $(\lambda_i)'s$ are fuzzy nowhere dense sets in(X, T).

$\therefore (X, T)$ is a fuzzy second category space.

Definition 20: A fuzzy topological space (X, T) is called a fuzzy Baire space if $int\, [\vee_{i=1}^{\infty}(\lambda_i)] = 0$,Where$(\lambda_i)'s$ are nowhere dense sets in (X, T).

Proposition 21: If the fuzzy almost resolvable space (X, T) is a fuzzy submaximal space, then (X, T) is not a fuzzy Baire space.

Proof: Let the fuzzy almost resolvable space (X, T) be a fuzzy submaximal space. Then By Proposition 3.18"(X, T) is a fuzzy first category space and hence $\vee_{i=1}^{\infty}(\lambda_i) = 1$ ",Where the fuzzy sets $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T).

Now $int[\vee_{i=1}^{\infty}(\lambda_i)] = int\,(1) = 1 \neq 0$

$\therefore (X, T)$ is not a fuzzy Baire space.

Proposition 22: If the fuzzy almost resolvable space (X, T) is a fuzzy open here ditarily irresolvable space, then (X, T) is not a fuzzy baire space.

Proof: Let (X, T) be a fuzzy almost resolvable space then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, Where the fuzzy sets $(\lambda_i)'s$ in (X, T) are such that $int(\lambda_i) = 0$.

Since (X, T) is a fuzzy open hereditarily irresolvable space,

$\Rightarrow int(\lambda_i) = 0$

$\Rightarrow int\, cl(\lambda_i) = 0$

Now, $int[\vee_{i=1}^{\infty}(\lambda_i)] = int(1) = 1 \neq 0$.

$\therefore (X, T)$ is not a fuzzy Baire space

Proposition 23: If the fuzzy almost irresolvable space (X, T) is a fuzzy open hereditarily irresolvable space, then (X, T) is a fuzzy second category space.

Proof: Given (X, T) is fuzzy almost irresolvable space. Then $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, Where the fuzzy set $(\lambda_i)'s$ in (X, T) are such that $int(\lambda_i) = 0$

Since (X, T) is a fuzzy open hereditarily irresolvable space, $int(\lambda_i) = 0$

$int\, cl(\lambda_i) = 0$

Then$(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T).Hence $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$,Where the fuzzy sets $(\lambda_i)'s$ fuzzy nowhere dense sets in (X, T).

$\therefore (X, T)$ is a fuzzy second category space.

Proposition 24: If λ is a fuzzy nowhere dense set in (X, T) then $int(\lambda) = 0$.

Proof: Let λ be a fuzzy nowhere dense set in (X, T)

$int\,(cl(\lambda)) = 0$……………………..(1)

Now $\lambda \leq cl(\lambda)$ (By Result 2.13)

$int\,(\lambda) \leq int\,(cl(\lambda)) = 0$ (using 1)

Hence, we have $int\,(\lambda) = 0$

Proposition 25: If λ is a fuzzy nowhere dense set in (X, T), then $1 - \lambda$ is a fuzzy dense set in (X, T)

Proof: Let λ be a fuzzy nowhere dense set in (X, T)

$int\,(\lambda) = 0$ (By Proposition 3.24)

Now $cl(1 - \lambda) = 1 - int(\lambda)$ (By result 2.18)

$= 1 - 0$

$= 1$

$\therefore 1 - \lambda$ is a fuzzy dense set in (X, T).

Proposition 26: If $\vee_{i=1}^{N}(\lambda_i) = 1$, where the fuzzy sets$(\lambda_i)'s$ are fuzzy nowhere dense sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy almost resolvable space.

Proof: Given $\vee_{i=1}^{N}(\lambda_i) = 1$

Where the fuzzy sets $(\lambda_i)'s$ are fuzzy nowhere dense sets in a fuzzy topological space (X, T).

Then $int(\vee_{i=1}^{N}(\lambda_i)) = int(1) = 1$

$$\Rightarrow 1 - int\left(\bigvee_{i=1}^{N}(\lambda_i)\right) = 0$$

Then we have $cl(\wedge_{i=1}^{N}(1 - \lambda_i)) = 0$…………..(1)

Since $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T)

By Proposition 3.25 "$(1 - \lambda_i)'s$ are fuzzy dense sets in (X, T)"

Let $(\mu_i)'s$ $(i = 1,2,3 \ldots \infty)$ be fuzzy dense sets in (X, T)

In which the first N fuzzy sets be $1 - \lambda_i$

Then $\wedge_{i=1}^{\infty}(\mu_i) \leq \wedge_{i=1}^{N}(1 - \lambda_i) \leq cl(\wedge_{i=1}^{N}(1 - \lambda_i))$.

From (1)$\Rightarrow \wedge_{i=1}^{\infty}(\mu_i) \leq 0$

That is $\wedge_{i=1}^{\infty}(\mu_i) = 0$

$\Rightarrow 1 - \bigwedge_{i=1}^{\infty}(\mu_i) = 1 - 0 = 1$

$\Rightarrow 1 - \wedge_{i=1}^{\infty}(\mu_i) =$ (By Demorgan's Law)

Then $\vee_{i=1}^{\infty}(1 - \mu_i) = 1$

Now $cl(\mu_i) = 1$

$1 - cl(\mu_i) = 1 - 1 = 0$

$\Rightarrow int(1 - \mu_i) = 0$ (By Lemma 2.19)

Let $\delta_i = 1 - \mu_i$

Then, we have

$\bigvee_{i=1}^{\infty}(\delta_i) = 1$

Where, $int(\delta_i) = 0$

$\therefore$ (X, T) is a fuzzy almost resolvable space.

Proposition 27: If each fuzzy set λ_i is a fuzzy F_σ-set in a fuzzy almost resolvable space (X, T) then $\wedge_{i=1}^{\infty}(1 - \lambda_i) = 0$, where $(1 - \lambda_i)'s$ are fuzzy dense sets in (X, T).

Proof: Let (X, T) be a fuzzy almost resolvable space.
Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets $(\lambda_i)'s$ in (X, T) are such that $int(\lambda_i) = 0$

$$\Rightarrow 1 - \left(\bigvee_{i=1}^{\infty}(\lambda_i)\right) = 1 - 1 = 0$$

$$\Rightarrow 1 - int(\lambda_i) = 1 - 0 = 1$$

Then $\wedge_{i=1}^{\infty}(1 - \lambda_i) = 0$ (By Demorgan's Law)
and $cl(1 - \lambda_i) = 1$ (By Lemma 2.18)
Since the fuzzy sets $(\lambda_i)'s$ in (X, T) are fuzzy F_σ-sets $(1 - \lambda_i)'s$ are fuzzy G_δ – sets in (X, T)
We have $\wedge_{i=1}^{\infty}(1 - \lambda_i) = 0$
where $(1 - \lambda_i)'s$ are fuzzy dense and fuzzy G_δ - sets in (X, T).

Definition 28: A fuzzy set λ is called a residual set if $1 - \lambda$ is a fuzzy first category set.

Proposition 29: Let (X, T) be a fuzzy topological space. Then the following are equivalent:
$(i)(X, T)$ is a fuzzy baire space
$(ii) int(\lambda) = 0$ for every fuzzy first category set λ in (X, T)
$(iii) cl(\mu) = 1$ for every fuzzy residual set μ in (X, T)

Proof: (i)⇒(ii)

Let λ be a fuzzy first category set in (X, T)

$\Rightarrow \lambda = \vee_{i=1}^{\infty}(\lambda_i)$,

where $\lambda_i's$ are fuzzy nowhere dense set in (X, T).

Now, $int(\lambda) = int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$

((X, T) is a fuzzy Baire space)

$\therefore int(\lambda) = 0$

(ii)⇒(iii)

Let μ be a fuzzy residual set in (X, T)

$\Rightarrow 1 - \mu$ is a fuzzy first category set in (X, T).

By hypothesis, $int(1 - \mu) = 0$

$\Rightarrow 1 - cl(\mu) = 0$ (By lemma 2.19)

$\therefore cl(\mu) = 1$

(iii)⇒(i)

Let λ be a fuzzy first category set in (X, T)

$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$…………………(1)

Where $\lambda_i's$ are fuzzy nowhere dense set in (X, T)

$\Rightarrow (1 - \lambda)$ is a fuzzy residual set in (X, T)

By hypothesis, we have $cl(1 - \lambda) = 1$

$\Rightarrow 1 - int(\lambda) = 0$ (By lemma 2.18)

$int(\lambda) = 0$

$int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ (Using 1)

Where $\lambda_i's$ are fuzzy nowhere dense set in (X, T)

$\therefore (X, T)$ is a fuzzy Barie space.

$1 - \bigwedge_{i=1}^{\infty}(\lambda_i) = 1 - 0 = 1$

$\Rightarrow \vee_{i=1}^{\infty}(1 - \lambda_i) = 1$ (By Demorgan's Law)

Let $1 - \lambda_i = \mu_i$

Hence $\vee_{i=1}^{\infty}(\mu_i) = 1$

Where $int(\mu_i) = 0$

$\therefore (X, T)$ is a fuzzy almost resolvable space. (X, T) has a dense fuzzy set λ_j such that $cl(1 - \lambda_j) = 1$

$\therefore (X, T)$ is a fuzzy resolvable space

But this is a contradiction to (X, T) being a fuzzy irresolvable space.
$\therefore (X, T)$ is not a fuzzy almost resolvable space.

IV. CONCLUSION

We already study about the fuzzy topology, fuzzy vector space with their properties. Based on the above result I trying to prove fuzzy is almost resolvable space of fuzzy baire space. Also it is proved that a fuzzy first category space and fuzzy nodec space is fuzzy almost resolvable space.

REFERENCES

[1] K.K. Azad, On Fuzzy semi continuity, Fuzzy almost continuity and Fuzzy Weakly continuity, J. Math. Anal, Appl, 82 (1981), 14-32.

[2] G. Balasubramanian, Maximal fuzzy topologies, KYBERNETIKA No.5, 31(1991),459-464.

[3] C.L. Chang, Fuzzy Topological Spaces, J. Math.Anal. Appl. 24, (1968), 182-190.

[4] A.G. EL'KIN, Ultrafilters and Undecomposable Spaces, VestnikMosk. Univ. Mat., 24 (5), (1965), 51.

[5] E.Hewitt, A Problem in Set Theoretic Topology, Duke Math. Jour., 10, (1943) 309-333.

[6] M.S.E. Naschie, On the Certification ofhetericstrings, M theory and e^∞Theory, chaos, solitions and Fractals (2000), 2397-2408.

[7] P.V.Ramakrishnan and Laksmana Gomathi Nayagam, Nearly Fuzzy Hausdroff Spaces, Indian J. Pure Appl. Math., 31 (5) (June, 2000) 695-712.

[8] Ricard Bolstein, Sets of Points of Discontiunity., Proc. Amer. Math. Society Vol. 38. No.1, (March 1973), 193-197.

[9] G. Thangaraj and G. Balasubramanian, On Somewhat fuzzy continuous functions, J. Fuzzy Math., Vol.11, No.2, (2003) 1-12.

[10] G. Thangaraj and G. Balasubramanian, On Fuzzy Resolvable and Fuzzy Irresolvable Spaces, Fuzzy Sets, Rough Sets and Multi valued Operations and Applications, Vol. 1, No.2, 173-180, 2009.

[11] G. Thangaraj and S. Anjalmose, On Fuzzy Baire Spaces, (Communicated to J. Fuzzy Math.)

[12] L.A. Zadeh, Fuzzy Sets, information and Control, Vol. 8, 338-358, 1965.

[13] Thangaraj .G, Vijayan.D, on fuzzy almost resolvable and fuzzy almost irresolvable spaces ,inter,J.Statistika and Mathmatika, Vol.9, No.2, (2014),61-65.