# Asymptotic Stability Of Restricted Three Body Problems Using Jacobi Integral

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***Abstract-*** *In this paper, we are using the Jacobi (energy) integral and it is shown that in the restricted three-body problem is not asymptotically stable. One can, in fact, determine, from the initial position and velocity of a particle, whether it is possible for the particle to reach the triangular point in position space. Now, we turn some principle and approaches to dynamics related to the result in order to determine their place in the solution of the general problem.*

## I. INTRODUCTION

In celestial mechanics, Jacobi's integral (also The Jacobi Integral or The Ja-cobi Constant; named after Carl Gustav Jacob Jacobi) is the only known conserved quantity for the circular restricted three-body problem. Unlike in the two-body problem, the energy and momentum of the system are not conserved separately and a general analytical solution is not possible. The integral has been used to derive numerous solutions in special cases.In the dynamics discussed here we study the time behavior of the fundamental inte-gral characteristics of the physical system, i.e. the Jacobi function (moment of inertia) and energy (potential, kinetic and total), which are functions of mass density distribution, and the structure of a system. This approach satis es the requirements of an external observer.

## II. REVIEW OF LITERATURE

In 1842-43, when Jacobi was professor at Konigsberg university he derived a special series of lectures on dynamics. The lecturer was devoted to the dynamics of a system of n mass points whose motion depends only on the distance between them and is independent of velocities. In this connection, by deriving the law of conservative energy from the equation of mass points for conservative systems, where the forces function is a homogenous function of space co-ordinates. Jacobi gave this law an unusual force force and a new content.The location and stability of the ve Lagrangian equilibrium points in the planar, circular restricted three-body problem are investigated when the third body is acted on by a variety of drag forces. The approximate loca-tions of the displaced equilibrium points are calculated for small mass ratios and a simple criterion for their linear stability is derived. If a1 and a3 denote the coe cients of the linear and cubic terms in the characteristic equation derived from a linear stability analysis, then an equilibrium point is asymp-totically stable provided 0 <a1 <a3. In cases where a1 0 or a1 a3 the point is unstable but there is a di erence in the e-folding time scales of the shifted L4 and L5 points such that the L4 point, if it exists, is less unstable than the L5 point. The results are applied to a number of general and speci c drag forces. It is shown that, contrary to intuition, certain drag forces produce asymptotic stability of the displaced triangular equilibrium points, L4 and L5. Therefore, simple energy arguments alone cannot be used to determine stability in the restricted problem. The shifted equilibrium points of all drag forces that havex and y components in the rotating frame evaluated when = =0, the L3 and L4 points move in opposite directions along a circle centered on the primary mass, merge, and disappear; the L5 point moves anticlock-wise along the same circle, meets the displaced L2 point, and disappears; and the inner and outer Lagrangian points, L1 and L2, initially move in opposite directions along separate circles centered on the secondary mass until they reach the primary circle whereupon the L2 point merges with the displaced L5 point and both disappear while the L1 point then moves along the primary circle toward the secondary mass although of the smaller satellites of Saturn.

## Jacobi Constant

Equations yield the Jacobi integral derived first by Jacobi in 1836.

If we define an augmented of effective potential function

$$U = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

These equations have the form

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$
$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y}$$
$$\ddot{z} = \frac{\partial U}{\partial z}$$

For example

$$\frac{\partial U}{\partial x} = x - \frac{(1-\mu)}{r_1^2}\frac{\partial r_1}{\partial x} - \frac{\mu}{r_2^2}\frac{\partial r_2}{\partial x},$$

where

$$r_1^2 = (x+\mu)^2 + y^2 + z^2$$
$$2r_1\frac{\partial r_1}{\partial x} = 2(x+\mu)$$
$$\frac{\partial r_1}{\partial x} = \frac{x+\mu}{r_1}$$

$$r_2^2 = (x-1+\mu)^2 + y^2 + z^2$$
$$\frac{\partial r_2}{\partial x} = \frac{(x-1+\mu)}{r_2}$$

Substituting Equations

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{(x+\mu)}{r_1^3} - \mu\frac{(x-1+\mu)}{r_2^3}$$

By multiplying Equations by $2\dot{x}$, $2\dot{y}$ and $2\dot{z}$, respectively

$$2\dot{x}\ddot{x} - 2\dot{x}\dot{y} = 2\dot{x}\frac{\partial U}{\partial x}$$
$$2\dot{y}\ddot{y} + 2\dot{x}\dot{y} = 2\dot{y}\frac{\partial U}{\partial y}$$
$$2\dot{z}\ddot{z} = 2\dot{z}\frac{\partial U}{\partial z}.$$

Summing these

$$2\dot{x}\ddot{x} + 2\dot{y}\ddot{y} + 2\dot{z}\ddot{z} = 2\dot{x}\frac{\partial U}{\partial x} + 2\dot{y}\frac{\partial U}{\partial y} + 2\dot{z}\frac{\partial U}{\partial z}.$$

This can be integrated to yield

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2U - C.$$

We have used the fact that

$$2\int\left[\frac{dx}{dt}\frac{\partial U}{\partial x} + \frac{dy}{dt}\frac{\partial U}{\partial y} + \frac{dz}{dt}\frac{\partial U}{\partial z}\right] = 2U.$$

This is seen by noting that

$$U = U(x, y, z).$$

Therefore,

$$\frac{dU}{dt} = \frac{\partial U}{\partial x}\frac{dx}{dt} + \frac{\partial U}{\partial y}\frac{dy}{dt} + \frac{\partial U}{\partial z}\frac{dz}{dt}.$$

Also, for example

$$\frac{d\dot{x}^2}{dt} = 2\dot{x}\ddot{x}.$$

Hence,

$$C = 2U - v^2 = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - v^2.$$

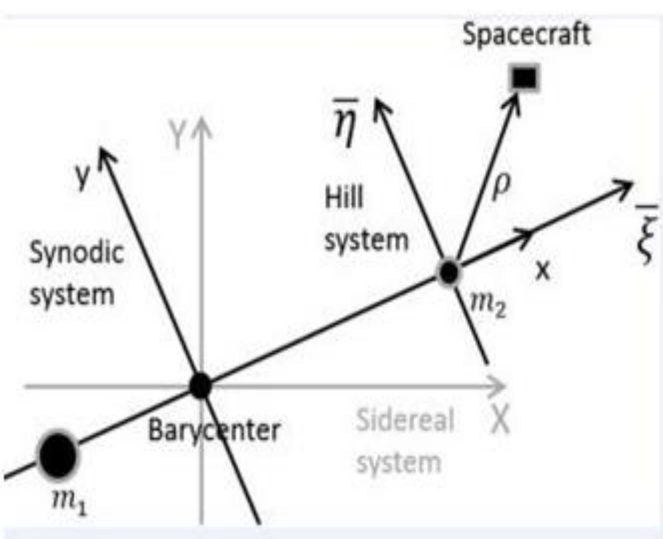
**Jacobi Constant , C**

As a particle moves through the CRTBP, the value of the Jacobi constant does not vary, assuming the energy does not change by a non-conservative maneuver. For a given Jacobi constant, the motion of a particle is limited to certain regions of space due to the constraint that the velocity can not have imaginary components. This leads to Forbidden Regions in space. The forbidden regions can be found in the CRTBP by generating zero-velocity curves. These zero-velocity curves are computed by setting the velocity in Equation 20 equal to zero and mapping the resultant curves for positions in the vicinity of the primaries. The motion of an object with a speci ed Jacobi constant is bound within its zero-velocity curve and can only cross the boundaries under some non-conservative force, such as thrusting. Figure 2 shows a contour plot of the zero-velocity curves in the x-y plane of the Earth-Moon system.

**Asymptotic stability**

The restricted three body problem is asymptotic stable if it satisfy Lyapunov stability.So for this we have nd the Lyapunov condi-tion of Hill stability. An important in solving the R3BP was made by Jacobi(1836) who found a new constant of motion we is now called the Ja-cobi constant or the Jacobi Intergal. The Jacobi Constant is used to nd out the stability of the problem. Equation (14) determines relationship between postion and velocity of body with m3.

"C" is a solution to the R3BP which stil remain non-integrable.

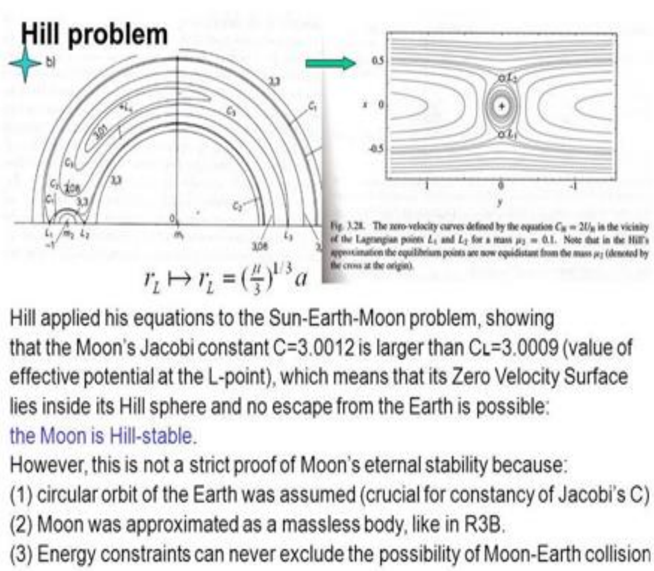



Energy criterion guarantees that a particle cannot cross the zero velocity Curve and therefore is table in the Jacobi sense(energetically).In our case we substituting V=0 into Jacobi constant

**Zero-Relative Velocity Surfaces**

Analyzing the non-dimensional Jacobi integral it is possible to set the velocity equal to zero and nd a three dimensional surface for which there is a constant value for C. It is bene cial to do visualize these surfaces because for any satellite with that value of C in the Jacobi integral, it cannot pass through the zero-relative-velocity surface. The value of C for a satellite is found by plugging the velocity and position into the Jacobi integral and solving for C. If the value of C for a given satellite is greater than the value of C for a satellite positioned at L2 and not moving in the rotating frame, then the satellite can never transfer from the Earth to the Moon. similarly, satellites with a greater value of C than the C for a satellite at L3 could never leave the Earth-Moon system by Plotting of the three dimensional surfaces.The equation for the surface is: The concept of Zero velocity curve may be used to formally de ne the hill stability:- If a given co ordinate system the motion of one mass is possible only in the region bounded by a closed zero velocity curve then the motion of this body is still stable.

**Hill Stability of R3BP**




## III. CONCLUSION

The notion of the hill stability can be extended very naturally from the CR BP to the general three body problem and the distance curve presentation gives a close idea of the phenomenon.The hill stability can even be extended to some three body system of postive energy.

Most triple system are hill stable;their close binary can't approach by the third body and its major axis has only one small pertubation.The Sun-Planet-Satellite system are hill stable.However hill stability doesnot prevent an escape of that third body.So, it is shown that in the restricted three-body problem is not asymptotically stable.

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