A New Algorithm For Solving Intuitionistic Fuzzy Maximal Flow Problems Using Intuitionistic Triangular Fuzzy Numbers

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Abstract- *AmitKumar and ManjotKaur proposed a new algorithm to find the fuzzy maximal flow between source and sink by representing the flow as normal triangular fuzzy numbers. There are several papers in which generalized fuzzy numbers have been used for solving real life problems. In this paper we have tried to introduce a new algorithm to find the intuitionistic fuzzy maximal flow between sources and sink by representing the flow as normal intuitionistic triangular fuzzy numbers. To illustrate the new algorithm a numerical example is solved. If there is no uncertainty about the flow between source and sink then the proposed algorithm gives the same result as in crisp maximal flow problems.*

Keywords- Intuitionstic fuzzy maximal flow problem, ranking function, Intuitionistic triangular fuzzy numbers, and Maximal flow algorithm.

I. INTRODUCTION

The maximal flow problem is one of the classical problems in network optimization. It provides very useful models in a number of practical contexts including electrical power, traffic, communication networks, oil pipelines, logistics and power systems. Due to its wide applicability, designing efficient algorithms for the maximum flow problem has been a popular research topic.

The maximal flow problem was proposed by Fulkerson and Dantzig(1955) originally and solved by specializing the simple method for linear programming. Ford and Fulkerson (1956) solved it by augmenting path algorithm. They developed the fuzzy flow theory, presenting the conditions to obtain an optimal flow by means of definitions on fuzzy matrices. Chanas, S., Kolodziejczyk, W.(1982) studied flow as a real number and the capacities have upper and lower bounds with a satisfaction function. Again they studied (1986) the integer flows in networks with fuzzy capacity constraints. Chanas, S., Delgado, M.,Verdegay, J.L.,(1995) investigated the fuzzy optimal flow on imprecise structures. Kumar et al., proposed a new algorithm to find fuzzy maximal flow between source and sink by using ranking function. Liu, S.T. and Kao ,C.(2004) developed Network flow problems with fuzzy arc lengths. Ji, X., Yang L., and Shao, Zhen (2006) studied chance constrained maximum flow problem with arc capacities. Amit kumar and Manjot Kaur(2010) developed new algorithm for solving fuzzy maximal flow problems using generalized triangular fuzzy numbers.

In this paper an algorithm is introduced for solving the Intuitionistic fuzzy maximal flow problems using triangular intuitionstic fuzzy numbers. This paper is organized as follows; In Section 2, some basic definitions and ranking functions are reviewed. In Section 3, the new algorithm is proposed for solving the intuitionistic fuzzy maximal flow problems. In Section 4, an illustrative example for the proposed algorithm is presented. In section 5 obtained results are discussed. Finally in section 6 some conclusions are drawn.

II. PRELIMINARIES

In this section some basic definitions, arithmetic operations and ranking functions are reviewed.

Definition II.1

A Fuzzy number Ã =(a,b,c) is said to be a triangular fuzzy if its membership function is given by

$$\mu_A(x) = \left\{ \begin{array}{ll} \dfrac{x-a}{b-a}, & a \le x < b \\ 1, & x = b \\ \dfrac{x-c}{b-c}, & b < x \le c \end{array} \right\} \text{ Where } a, b, c \in R$$

Definition II.2

A Triangular fuzzy number Ã =(a,b,c) is said to be a non-negative triangular fuzzy number if and only if a≥0.

Definition II.3

A Triangular fuzzy number $\tilde{A}$ =(a,b,c) is said to be a zero triangular fuzzy number if and only if a=b=c=0.

Triangular Intuitionistic FuzzyNumbers II.4

An Intuitionistic fuzzy number A={<a,b,c;w> <e,b,f;w> is said to be a generalized triangular Intuitionistic fuzzy number if its membership function and non membership function are given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} w & a \le x \le b \\ 1 & x = b \\ \frac{(c-x)}{(c-b)} w & b \le x \le c \end{cases} \text{ where } a, b, c \in R$$

$$\gamma_A(x) = \begin{cases} \frac{(b-x)}{(b-e)} w & e \le x \le b \\ 0 & x = b \\ \frac{(x-b)}{(f-b)} w & b \le x \le f \end{cases} \text{ where } e, b, f \in R$$

Definition II.5

A Triangular intuitionistic fuzzy number $\tilde{A}$ =<(a,b,c) (e,b,f)> is
said to be a zero triangular intuitionistic fuzzy number
if and only if a=b=c=0.

Ranking of intuitionstic triangular fuzzy number II.6

An Triangular intuitionistic fuzzy number $\tilde{A}' = \{(a, b, c; w_1); (e, b, f; w_2)\}$

is completely defined by its membership and non-membership function as follows:

$$L_\mu(x) = \frac{x-a}{b-a} w \ ; a \le x \le b \ \& \ R_\mu(x) = \frac{c-x}{c-b} w \ ; b \le x \le c$$

$$L_\gamma(x) = \frac{b-x}{b-e} w \ ; e \le x \le b \ \& \ R_\gamma(x) = \frac{x-b}{f-b} w \ ; b \le x \le f$$

Then L^{-1} and R^{-1} are inverse functions of functions L and R respectively,

$$L_\mu^{-1}(h) = a + (b-a)\, h/w \ \& \ R_\mu^{-1}(h) = c - (c-b)\, h/w$$

$$L_\gamma^{-1}(h) = b - (b-e)\, h/w \ \& \ R_\gamma^{-1}(h) = b + (f-b)\, h/w$$

Then the Graded Mean Integration Representation of membership function and non-membership function are

$$P_\mu(A) = \frac{a+4b+c}{6} \quad \text{and} \quad P_\gamma(A) = \frac{b+e+f}{3}$$

Let $\tilde{A}' = (< a, b, c; w_1 >< e, b, f; w_2 >)$ and

$\tilde{B}' = (< a', b, c'; w_1' >< e', b, f'; w_2' >)$ be any two

Intuitionistic triangularfuzzy numbers then

(i) $P_\mu^\alpha(\tilde{A}') < P_\mu^\alpha(\tilde{B}')$ and $P_\gamma^\beta(\tilde{A}') < P_\gamma^\beta(\tilde{B}')$ $\therefore \tilde{A}' < \tilde{B}'$.

(ii) $_\mu^\alpha(\tilde{A}') > P_\mu^\alpha(\tilde{B}')$ and $P_\gamma^\beta(\tilde{A}') > P_\gamma^\beta(\tilde{B}')$ $\therefore \tilde{A}' > \tilde{B}'$.

(iii) $_\mu^\alpha(\tilde{A}') = P_\mu^\alpha(\tilde{B}')$ and $P_\gamma^\beta(\tilde{A}') P_\gamma^\beta(\tilde{B}')$ $\therefore \tilde{A}' \approx \tilde{B}'$.

Arithmetic operations of Triangular Intuitionistic fuzzy number II.7

If $\tilde{A}^I = \{(a_1, a_2, a_3; w_1); (a'1, a_2, a'3; w_2)\}$ and
$\tilde{B}^I = \{(b_1, b_2, b_3'; w_1'); (b'1, b_2, b'3; w_2')\}$

are ttwo intuitionistic fuzzy numbers we define, Addition:
Aaa

$$\tilde{A}^I + \tilde{B}^I = \left\{ \begin{pmatrix} a_1 + b_1, a_2 + b_2, a_3 + b_3 ; \min(w_1, w_1'); \\ (a'1 + b'1, a_2 + b_2, a'3 + b'3; \max(w_2, w_2') \end{pmatrix} \right\}$$

SubSubtraction:

$$\tilde{A}^I - \tilde{B}^I = \left\{ \begin{pmatrix} a_1 - b_3, a_2 - b_2, a_3 - b_1; \min(w_1, w_1'); \\ (a'1 - b'3, a_2 - b_2, a'3 - b'1; \max(w_2, w_2') \end{pmatrix} \right\}$$

III. NEW ALGORITHM

In this section a new algorithm is proposed for solving the intuitionistic fuzzy maximal-flow problems. It is very easy to understand and apply for solving intuitionistic fuzzy maximal-flow problems in real life situations. The intuitionistic fuzzy maximal flow algorithm is based on finding breakthrough paths with net positive flow, between the source and sink nodes Consider an arc (*i*, *j*) with initial intuitionistic fuzzy capacities

$$(<\widetilde{If\bar{c}}_{\mu_{ij}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ij}};\max(w_2,w_2')>$$

$$<\widetilde{If\bar{c}}_{\mu_{ji}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ji}};\max(w_2,w_2')>)$$

to represent these intuitionistic fuzzy residuals. As portions of these intuitionstic fuzzy capacities are committed to the flow in the arc, the intuitionistic fuzzy residuals (or remaining intuitionsitic fuzzy capacities)of the arc are updated. We use the notation

$$(<\widetilde{If\bar{c}}_{\mu_{ij}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ij}};\max(w_2,w_2')>$$

$$<\widetilde{If\bar{c}}_{\mu_{ji}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ji}};\max(w_2,w_2')>)$$

to represent these intuitionistic fuzzy residuals. For a node *j* that receives flow from node *i* we attach a label $[<\widetilde{Ifa}_{\mu_j},\widetilde{Ifa}_{\gamma_j}>i]$ where $<\widetilde{fa}_{\mu_j},\widetilde{Ifa}_{\gamma_j}>$ is the intuitionistic fuzzy flow from node i to j. The steps of the algorithm are summarized as follows:

Step 1.

For all arcs (*i*, *j*) , set the residual intuitionistic fuzzy capacity equal to the initial intuitionistic fuzzy capacity i.e.,

$$(<\widetilde{If\bar{c}}_{\mu_{ij}};\min(w_1,w_1'),\widetilde{Ifc}_{\gamma_{ij}};\max(w_2,w_2')>$$

$$<\widetilde{Ifc}_{\mu_{ji}};\min(w_1,w_1'),\widetilde{Ifc}_{\gamma_{ji}};\max(w_2,w_2')>) =$$

$$(<\widetilde{If\bar{c}}_{\mu_{ij}};\min(w_1,w_1')\widetilde{If\bar{c}}_{\gamma_{ij}};\max(w_2,w_2')>$$

$$<\widetilde{If\bar{c}}_{\mu_{ji}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ji}};\max(w_2,w_2')>)$$

. Let $\widetilde{Ifa}_1=(<\propto,\propto,\propto;1><\propto,\propto,\propto;0>)$ and label source 1 with $[(<\propto,\propto,\propto;1><\propto,\propto,\propto;0>)$ -] set i =1 go to step 2.

Step 2.

Determine S_i , the set of unlabelled nodes *j* that can be reached directly from node *i* by arcs with positive residuals (i.e. $(<\widetilde{Ifc}_{\mu_{ij}};\min(w_1,w_1'),\widetilde{Ifc}_{\gamma_{ij}};\max(w_2,w_2')>)$ if it is a non-negative fuzzy number for all $j\in s_i$). If $s_i\neq\varphi$, go to step 3. otherwise, go to step 5.

Step 3.

Determine $k\in s_i$ such that maximum

$$R(<\widetilde{Ifc}_{\mu_{ij}};\min(w_1,w_1'),\widetilde{Ifc}_{\gamma_{ij}};\max(w_2,w_2')>)$$

$$=R(<\widetilde{Ifc}_{\mu_{ik}};\min(w_1,w_1'),\widetilde{Ifc}_{\gamma_{ik}};\max(w_2,w_2')>)$$

. Set $(<\widetilde{Ifa}_{\mu_k},\widetilde{Ifa}_{\gamma_k}>)=(<\widetilde{Ifc}_{\mu_{ik}},\widetilde{Ifc}_{\gamma_{ik}}>)$ and label node k with $[<\widetilde{Ifa}_{\mu_k},\widetilde{Ifa}_{\gamma_k}>,i]$If $k = n$, the sink node has been labeled, and a breakthrough path is found, go to step 5. Otherwise, set $i = k$, and go to step 2.

Step 4.(Back tracking).

If $i = 1$, no breakthrough is possible; go to step 6. Otherwise, let *r* be the node that has been labeled immediately before the current node *i* and remove *i* from the set of nodes adjacent to *r*. Set $i = r$, and go to step 2.

Step 5. (Determination of Residuals)

Let $N_p=(1,k_1,k_2\ldots,k_n)$ define the nodes of the p^{th} breakthrough path from source node 1 to sink node *n*. Then the maximal flow along the path is computed as

$$\widetilde{If}_p=\text{minimum}(<\widetilde{Ifa}_{\mu_1},\widetilde{Ifa}_{\gamma_1}><\widetilde{Ifa}_{\mu_{k_1}},\widetilde{Ifa}_{\gamma_{k_1}}>$$

$$<\widetilde{Ifa}_{\mu_{k2}},\widetilde{Ifa}_{\gamma_{k2}}>\ldots\ldots<\widetilde{Ifa}_{\mu_n},\widetilde{Ifa}_{\gamma_n}>/\ \min>0)$$

The residual capacity of each arc along the breakthrough path is decreased by $\widetilde{If}_p$ in the direction of the flow and increased in the reverse direction i.e., for nodes *i* and *j* on the path, the residual flow is changed from the current

$$(<\widetilde{If\bar{c}}_{\mu_{ij}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ij}};\max(w_2,w_{2,}')>$$

$$<\widetilde{If\bar{c}}_{\mu_{ji}};\min(w_1,w_1'),\widetilde{If\bar{c}}_{\gamma_{ji}};\max(w_2,w_{2,}')>)$$

to

$$1.(<\widetilde{If\bar{c}}_{\mu_{ij}};\min(w_1,w1'),\widetilde{If\bar{c}}_{\gamma_{ij}};\max(w_2,w_2')>\Theta\widetilde{If}_p)$$

if the flow is from i toj.

$$2.(<\widetilde{If\bar{c}}_{\mu_{ji}};\min(w_1,w1'),\widetilde{If\bar{c}}_{\gamma_{ji}};\max(w_2,w_2')>\oplus\widetilde{If}_p)$$

if the flow is from j to i .

To restore the previous condition any nodes that were removed in step 4. Set i = 1, and return to step 2 to attempt a new breakthrough path.

Step 6. (Solution)
Given that m breakthrough paths have been determined, the intuitionistic fuzzy maximal flow in the network is $\tilde{F} = (I\tilde{f}_1 \oplus I\tilde{f}_2 \oplus ... I\tilde{f}_m)$ where m is the number of iterations that get no breakthrough.

IV. ILLUSTRATIVE EXAMPLE

In this section a new algorithm for solving intuitionistic fuzzy maximal flow problem is illustrated by solving a numerical example.

Consider the network shown in Figure 1. The bidirectional intuitionistic fuzzy capacities are shown on the respective arcs.

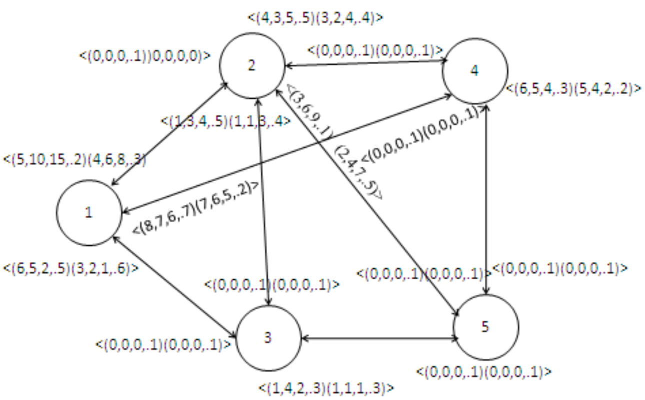

Fig 1

The algorithm is applied in the following manner.

Iteration 1. Set the residuals intuitionistic fuzzy capacity

$$(< I\tilde{f}c_{\mu_{ij}}; \min(w_1, w_1^{'}), I\tilde{f}c_{\gamma_{ij}}; \max(w_2, w_2^{'}) >$$

$$< I\tilde{f}c_{\mu_{ji}}; \min(w_1, w_1^{'}), I\tilde{f}c_{\gamma_{ji}}; \max(w_2, w_2^{'}) >)$$

= initial capacities

Step 1.
Set $I\tilde{f}a_1 = (< \infty, \infty, \infty; 1 >< \infty, \infty, \infty; 0 >)$ and label node 1 with $[(< \infty, \infty, \infty; 1 >< \infty, \infty, \infty; 0 >) -]$ Set i = 1.

Step 2.
(i) S_1={2,3,4}(≠φ)

Step 3.
k=2 , because maximum $\{R(I\tilde{f}c_{12}), R(I\tilde{f}c_{13}), R(I\tilde{f}c_{14})\} = R(I\tilde{f}c_{12})$
That is maximum of (<10;.2> <6;.3>, <4.5;.5><3;.6> <(7; .7)(6; .2)>)= (<10;.2><6;.3>)
Set $I\tilde{f}a_2 = I\tilde{f}c_{12} = (< 5,10,15;.2 >< 4,6,8;.3 >)$,and label node 2 with $[(< 5,10,15;.2 >< 4,6,8;.3 >) -]$ Set i =2, and repeat step 2.

Step 4:
(ii) S_2={3,4,5}(≠φ) k=5 , because maximum $\{R(I\tilde{f}c_{23}), R(I\tilde{f}c_{24}), R(I\tilde{f}c_{25})\} = R(I\tilde{f}c_{25})$
That is maximum (<2.75;.5> <1.5;.4)>, <3.75;.5> <2.75;.4>, <6;.1> <4.25; .5>)= (<6;.1> <4.25;.5>)
Set $I\tilde{f}a_5 = I\tilde{f}c_{25} = (< 3,6,9;.6 >< 2,4,7;.5 >)$,and label node 5 with $[(< 3,6,9;.6 >< 2,4,7;.5 >) -]$.

Step 5.
The break through path is determined from the labels starting at node 5 and moving backward to node 1 and the breakthrough path is 1→2→5.Thus, N_1={1,2,5} and

$$I\tilde{f}_{p_1} = \min(< I\tilde{f}a_{\mu_1}, I\tilde{f}a_{\gamma_1} >, < I\tilde{f}a_{\mu_2}, I\tilde{f}a_{\gamma_2} >, < I\tilde{f}a_{\mu_5}, I\tilde{f}a_{\gamma_5} >)$$

$$= \min(< \infty, \infty, \infty; 1 >< \infty, \infty, \infty; 0 >, < 5,10,15; 0.2 >$$

$$< 4,6,8; 0.3 >, < 3,6,9; 0.6 >< 2,4,7; 0.5 >)$$

$$= (< 3,6,9; 0.6 >< 2,4,7; 0.5 >)$$

The intuitionistic fuzzy residual capacities along path N_i are

$$(I\tilde{f}c_{12}, I\tilde{f}c_{21}) = (< (5,10,15; 0.2 >< 4,6,8; 0.3 > \Theta$$

$$< 3,6,9 ; 0.6 >< 2,4,7; 0.5 >; < 0,0,0; 1 >< 0,0,0; 0 > \oplus$$

$$< 3,6,9; 0.6 >< 2,4,7; 0.5 >)$$

$$= (< 5-9, 10-6, 15-3; \min(0.2,0.6) >$$

$$< 4-7, 6-4, 8-2; \max(0.3,0.5) >; < 0+3, 0+6, 0+9; \min(1,0.6) >$$

$$< 0+2, 0+4, 0+7; \max(0,0.5) >)$$

$$= (< -4,4,12; 0.2 >< -3,2,6; 0.5 >, < 3,6,9; 0.6 >< 2,4,7; 0.5 >)$$

$$(I\tilde{f}c_{25}, I\tilde{f}c_{52}) = (< 3,6,9; 0.6 >< 2,4,7 ; 0.5 > \Theta$$

$$< 3,6,9; 0.6 >< 2,4,7; 0.5 >; < 0,0,0; 1 >< 0,0,0; 0 > \oplus$$

$$< 3,6,9; 0.6 >< 2,4,7; 0.5 >)$$

$$= (< -6,0,-6; 0.6 >< -5,0,5; 0.5 >; < 3,6,9; 0.6 >< 2,4,7; 0.5 >)$$

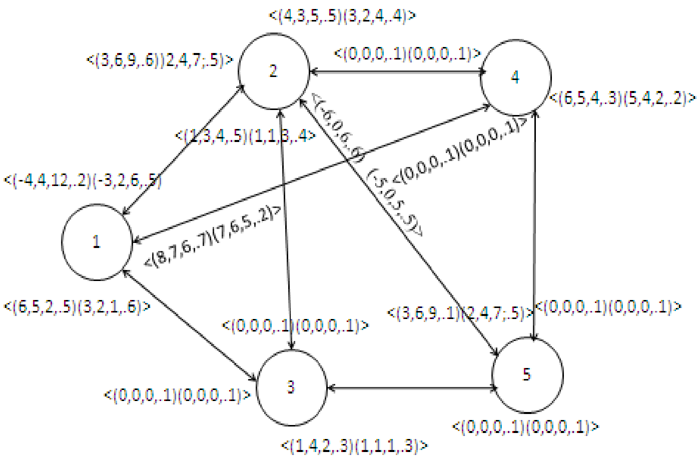

Fig 2 Network obtained after iteration 1

Iteration 2.

Repeating the procedure described in the previous iteration, at the starting node 1, the break through obtained is 1→4→5 and $\tilde{If}_2 = (<6,5,4;0.3><5,4,2\ ;0.2>)$

Step 1.

Set $\tilde{fa}_1 =<(\infty,\infty,\infty,1)(\infty,\infty,\infty,0)>$ and label node with $[<(\infty,\infty,\infty,1)(\infty,\infty,\infty,0)>-]$ Set i = 1.

Step 2.

$S_1=\{2,3,4\}(\neq\varphi)$

Step 3.

k=4, because maximum $\{R(\tilde{Ifc}_{12}), R(\tilde{Ifc}_{13}), R(\tilde{Ifc}_{14})\} = R(\tilde{Ifc}_{14})$

That is maximum{<(4,.1)(1.75,.5)> <(4.5,.5)(3,.6)> <(7,.7)(6,.2)>}

= <(7,.7)(6,.2).

Set

$\tilde{Ifa}_4 = \tilde{Ifc}_{14} =<(8,7,6;.7)(7,6,5;.2)>$ and label node 4 with [(8,7,6;.7)(7,6,5;.2)–]

K=5 here we have only one point. Therefore obviously the only one point is our maximum point that is max

$R(\tilde{Ifc}_{45}) = R(\tilde{Ifc}_{45})$

Set $\tilde{Ifa}_5 = \tilde{Ifc}_{45} =<(6,5,4;.3)(5,4,2;.2)>$ and label node 5 with [< (6,5,4;.3)(5,4,2;.2)–]

Step 4.

The break through path is determined from the labels starting at node 5 and moving backward to node 1 and the breakthrough path1→4→5.Thus, $N_2=\{1,4,5\}$ and

$$\tilde{If}_2 = \min\{\tilde{Ifa}_1, \tilde{Ifa}_4, \tilde{Ifa}_5\} = \min\{<(\infty,\infty,\infty;0.1)(\infty,\infty,\infty;0)>$$
$$<(8,7,6;0.7)(7,6,5;0.2)><(6,5,4;0.3)(5,4,2;0.2)\}$$
$$=(6,5,4;0.3)(5,4,2;0.2)$$

.The fuzzy residual capacities along path N_i are

$$(\tilde{Ifc}_{14,}\tilde{Ifc}_{41}) = \{<(8,7,6;0.7)(7,6,5;0.2)>\Theta$$
$$<(6,5,4;0.3)(5,4,2;0.2)>, <(0,0,0;0.1)(0,0,0;0)>\oplus$$
$$<(6,5,4;0.3)(5,4,2;0.2)>\}$$
$$=<(4,2,0;0.3)(5,2,0;0.2)>;<(6,5,4;0.1)(5,4,2;0.2)>$$
$$(\tilde{Ifc}_{45,}\tilde{Ifc}_{54}) = \{<6,5,4;0.3)(5,4,2;0.2>\Theta$$
$$<6,5,4;0.3)(5,4,2;0.2>, <(0,0,0;0.1)(0,0,0;0)>\oplus$$
$$<(6,5,4;0.3)(5,4,2;0.2)>\}$$
$$=<(2,0,-2;0.3)(3,0,-3;0.2)>;<(6,5,4;0.1)(5,4,2;0.2)>$$

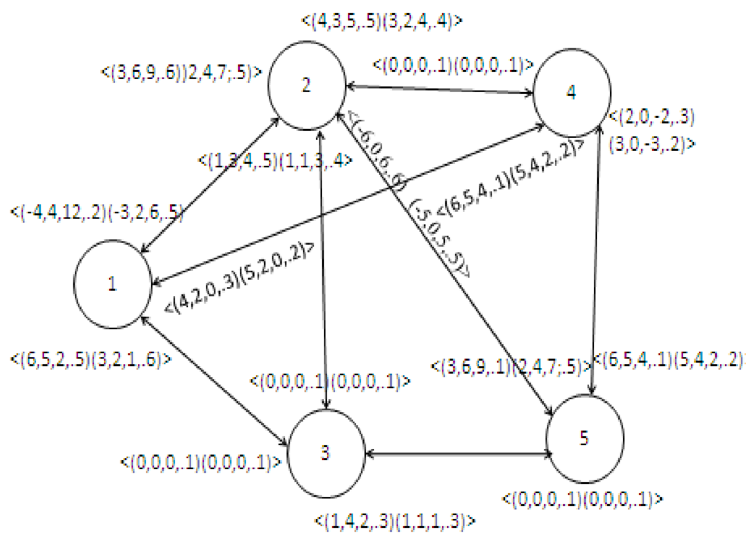

Fig 3 Network obtained after iteration 2

Iteration 3.

Repeating the procedure as described in the previous iteration, at the starting node 1, the break through obtained is 1→3→5.

And $\tilde{If}_3 =<(1,4,2;0.3)(1,1,1;0.3)>$

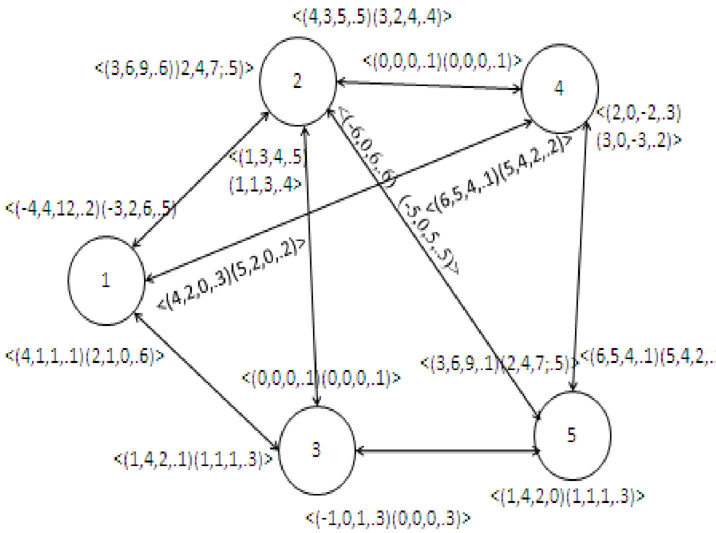

Fig 4 Network obtained after iteration 3

Iteration 4.

Repeating the procedure described in the previous iteration, at the starting node 1, the break through obtained is 1→2→4→5.

And $\tilde{\text{If}}_4 =< (-4,4,12\,;0.2)(-3,2,6\,;0.5) >$

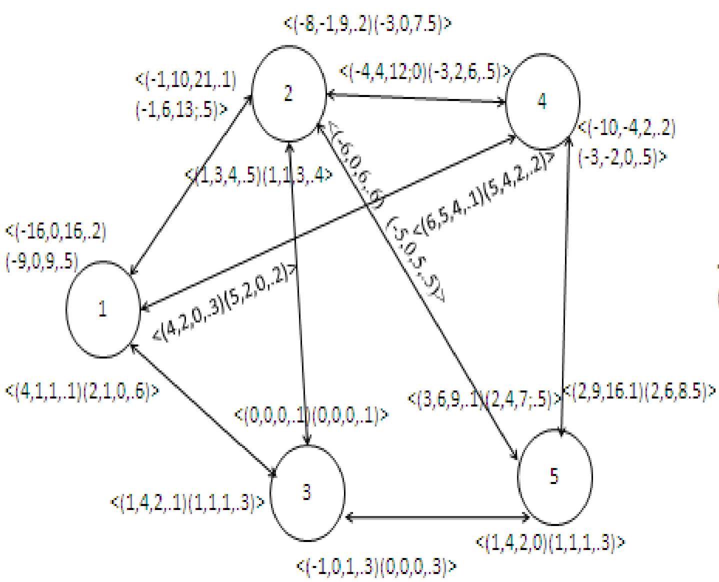

Fig 5 Network obtained after iteration 4

Iteration 5.

Repeating the procedure described in the previous iteration, at the starting node 1, the obtained break through is 1→4→5.

And $\tilde{\text{If}}_5 =< (4,2,0\,;0.3)(5,2,0\,;.0.2) >$

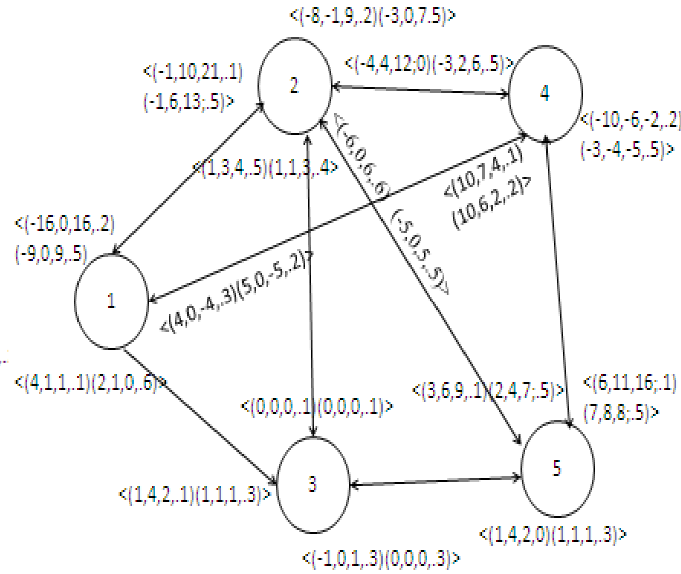

Fig 6 Network obtained after iteration 5

Iteration 6.

Repeating the procedure described in the previous iteration, at the starting node 1, the break through obtained is 1→3→5.

And $\tilde{\text{If}}_6 =< (4,1,1;0.1)(2,1,0\,;0.6) >$

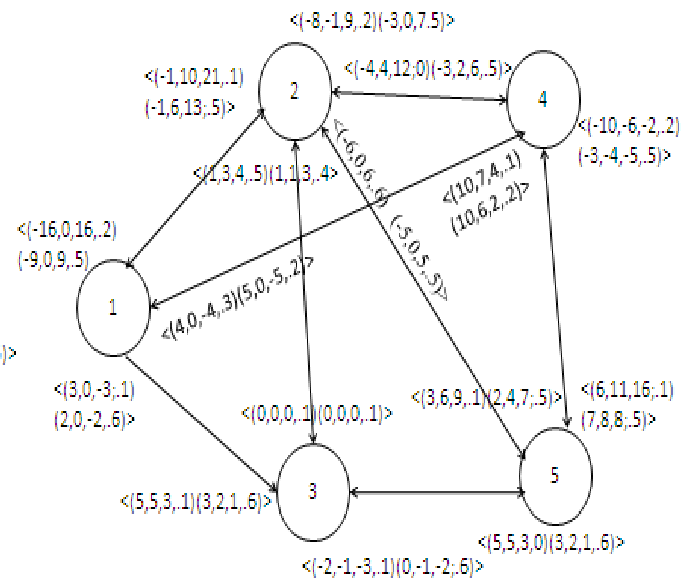

Fig 7 Network obtained after iteration 6

More iterations are not possible after 6th iteration because there is no way out to reach the sink from source. The fuzzy maximal flow is:

$$\tilde{IF} = \tilde{If}_1 + \tilde{If}_2 + \tilde{If}_3 + \tilde{If}_4 + \tilde{If}_5 + \tilde{If}_6 = \{< (3,6,9;.6)(2,4,7;0.5) >$$
$$\oplus < (6,5,4;0.3)(5,4,2;0.2) > \oplus < (1,4,2;0.3)(1,1,1;0.3) > \oplus$$
$$< (-4,4,12;0.2)(-3,2,6;0.5) > \oplus < (4,2,0;0.3)(5,2,0;0.2) >$$
$$\oplus < (4,1,1,;0.1)(2,1,0\ ;0.6) >\}$$
$$= \{< (14,22,28;0.1)(12,14,16;0.6) >\}$$

The intuitionistic fuzzy flow in different arcs is computed by subtracting the last intuitinistic fuzzy residuals

$$(< \tilde{If}\bar{c}_{\mu_{ij}}; \min(w_1, w_1'), \tilde{If}\bar{c}_{\gamma_{ij}}; \min(w_2, w_2') >$$
$$< \tilde{If}\bar{c}_{\mu_{ji}}; \min(w_1, w_1'), \tilde{If}\bar{c}_{\gamma_{ji}}; \min(w_2, w_2') >) --(1)$$

from the initial intuitionistic fuzzy capacities

$$(< \tilde{If}\bar{c}_{\mu_{ij}}; \min(w_1, w_1'), \tilde{If}\bar{c}_{\gamma_{ij}}; \min(w_2, w_2') >$$
$$< \tilde{If}\bar{c}_{\mu_{ji}}; \min(w_1, w_1'), \tilde{If}\bar{c}_{\gamma_{ji}}; \min(w_2, w_2') >) --(2)$$

and is shown in the following table.

Intuitionistic fuzzy optimal flow in different arc

Arc	Initial intuitionistic fuzzy capacity(1)- Last intuitionstic fuzzy residuals(2)	R(Initial intuitionistic fuzzy capacity)- R(Last intuitionstic fuzzy residuals)	Intuitionstic fuzzy flow amount	Direction
(1,2)	<(-11,10,31;.2) (-5,6,17;.5)> <(-21,-10,1;1) (-13,-6,1;.5>	<(10,6); (-10,-6)>	<(-11,10,31;.2) (-5,6,17;.5)>	1-2
(1,3)	<(9,5,-1;.1) (5,2,-1;.6)> <(6,-5,-11;1) (2,-4,-6;.6>	<(4.5,2); (-4.5,-3)	<(9,5,-1;.1) (5,2,-1;.6)>	1-3
(1,4)	<(12,7,2;.3) (12,6,0;.2)> <(-4,-7,-10;1) (-2,-6,-10;.2>	<(7,6); (-7,-6)>	<(12,7,2;.3) (12,6,0;.2)>	1-4
(2,3)	<(-3,0,3;.5) (-2,0,2;.4)> <(0,0,0;.1) (0,0,0;0>	<(0,0); (0,0)>	--	---
(2,4)	<(-5,4,13;.2) (-4,2,7;.5)> <(-12,-4,4;.1) (-6,-2,3;.5>	<(4,1.75); (-4,-1.7)>	<(-5,4,13;.2) (-4,2,7;.5)>	2-4
(2,5)	<(-3,6,15;.6) (-3,4,12;.5)> <(-9,-6,-3;.1) (-7,-4,-2;.5>	<(6,4.2); (-6,-4.2)>	<(-3,6,15;.6) (-3,4,12;.5)>	2-5
(3,5)	<(4,5,4;.1) (3,2,1;.6)> <(-3,-5,-5;0) (-1,-3,-3;.6)>	<(4.67,2); (-4.67,-2.33)>	<(4,5,4;.1) (3,2,1;.6)>	3-5
(4,5)	<(8,11,14;.2) (10,8,8;.5)> <(-16,-11,-6;.1) (-8,-8,-7;.5)>	<(11,8.5); (-11,-7.7)>	<(8,11,14;.2) (10,8,8;.5)>	4-5

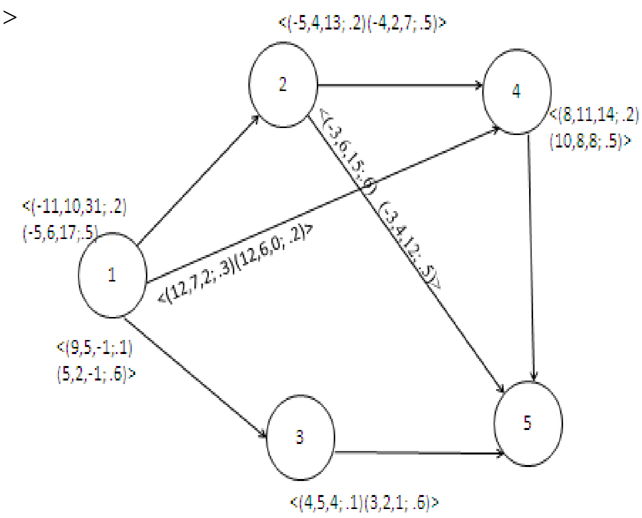

Fig 8 No breakthrough path

V. RESULT AND DISCUSSION

The obtained result can be explained as follow:

1. The amount of flow between source and sink for membership function is greater than 14 and less than 28 for nonmembership functions greater than 12 and less than 16 units.
2. Maximum number of persons are in favor that amount of flow will be 22 units for membership functions and 14 for non-membership functions.
3. The percentage of favorness for remaining flow can be

obtained as follow:

Let *x* represents the amount of flow, then the overall level of satisfaction of the decision maker for membership functions

1) the percentage of the favourness for

$$\mu_A(x) = \begin{cases} \left(\frac{x-14}{7}\right).1 & 14 < x \le 22 \\ 1 & x = 22 \\ \left(\frac{28-x}{6}\right).1 & 22 \le x < 28 \end{cases}$$

$$\gamma_A(x) = \begin{cases} \left(\dfrac{14-x}{2}\right).6 & 12 < x \le 14 \\ 0 & x = 14 \\ \left(\dfrac{x-14}{2}\right).6 & 14 \le x < 16 \end{cases}$$

and

VI. CONCLUSION

The maximum flow problem arises in a wide variety of situations. It occurs directly in problems as diverse as the flow of commodities in pipeline net-works, parallel machine scheduling, distributed computing on multi-processor computers, matrix rounding problems, the baseball elimination problem, and the statistical security of data. In this paper an algorithm is proposed for finding the intuitionistic fuzzy optimal flow of intuitionistic fuzzy maximal flow problems in which the flows are represented by generalized triangular fuzzy numbers. A numerical example is solved to illustrate the proposed algorithm, The proposed algorithm is very easy to realize and to apply for solving intuitionistic fuzzy maximal flow problems occurring in real life situations. The idea of finding the intuitionistic fuzzy best possible solution for intuitionistic fuzzy maximal flow problems accessible in this paper is applicable in other network flow problems like shortest path problems, critical path method etc. The concept of intuitionistic fuzzy sets can also be used to develop new algorithms for solving network flow problems

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