# **A Goal Programming Model For Measuring The Effectiveness of Quality Control Circles**

**D. Sarath Sen Reddy<sup>1</sup> , Dr. Harish Babu G.A<sup>2</sup>**

 $<sup>1</sup>$  Dept of Mathematics</sup> <sup>2</sup>Associate.Professor, Dept of Mathematics 1 rayalaseema University, Kurnool, Andhra Pradesh. <sup>2</sup>Sri Venkateswara College Of Engineering, Bengaluru, Karnataka.

*Abstract- Goal programming is capable of handling decision problems with single and multiple goals. The distinguishing characteristic of goal programming is itself goals are satisfied in ordinal sequence. That is, solution of goal programming problems achieving some higher goals first, before the lower order goal to the extent defined by the decision maker. Thus decision maker attempts to achieve a satisfactory level of all his by goals rather than optimum solution for a single goal. That is goal programming is a satisfying method rather than optimizing method. In this paper we develop a Mathematical Model to aid Management in measuring the effectiveness of a Quality Control Programme. A Goal Programming Model seems an appropriate technique for this purpose because it is able to take account of the many goals QCC's deal with.*

*Keywords-* Goal programming, Quality, Mathematical, objective function, variables

## **I. INTRODUCTION**

The basic concept of goal programming involves incorporating all goals in one model which can which can be solved simultaneously. In goal programming instead of trying to minimize or maximize the objective function directly as in linear programming, the derivations from established goals within the given set of constraints are minimized. So the simple algorithm of linear programming such deviational variables are called slack or surplus variables. These deviational variables are represented, in two dimensions. i.e., both positive and negative deviations, from each sub goal or goal. The objective functions then becomes the minimization of such of these deviations based on the relative importance within the pre-emptive priority structure assigned to each deviation. The approach to formulate the goal programming model is similar to that of linear programming model. The decision variables x1 ..x2 … xn are first defined. Following this step, all managerial goals are specified and ranked in order of priority.

Thus, a fundamental distinction of goal programming is that it provides for the solution of problems involving multiple conflicting and in common sub-goals or objectives arranged according to the decision maker's structure. Mathematically, the general goal programming model can be stated as follows:

$$
\mathit{Minimize} Z = \sum_{i=1}^m w_i \left(d_i^- - d_i^+\right)
$$

Subject to the constrains:

$$
\sum_{i=1}^{m} a_{ij} + d_i^{\dagger} - d_i^{\dagger} = b_{i\,;\; i = 1,2,3,\ldots\ldots\ldots m \text{ and } x_j, d_i^{\dagger}, d_i^{\dagger} \geq 0 \text{ for all } I \text{ and } j.
$$

 Where is goals are expressed by an m - component column  $b_i$ ,  $x_i$ , represents a decision variable,  $a_{ij}$  represents the  $\rm co$  - efficient for the i<sup>th</sup> constraint,  $x_i$  represent the weight attached to each goal and  $d_i$ ,  $d_i^+$  are deviational variables representing the amount of under and over achievements of the  $i<sup>th</sup>$  goal respectively. If goals are classified in 'K' ranks, the pre-emptive priority factors (symbolized by  $P_1, P_2, \ldots$ .) should be assigned to deviational variables d and  $d_i^+$ according to their order of importance. The p' s are not given actual values, they are simply a convenient way indicating that are goal is more important than another. The priority factors have the relationship of  $pi \>> p_{j+1}$ , (j= 1,2,... ..K) where n is very large. This implies that multiplication by `n' however, large it may be, cannot have  $np_{j+1}$  greater than  $p_j$ . Thus a lower - priority goal will never he achieved at the exposure of a higher priority goal.

The deviational variables at the same priority level may be given differential weights in the objective function so that deviational variables within the same priority have different cardinal weights. Since both under and over achievement of a goal cannot be achieved simultaneously, either one or both of these deviational variables will be equal to zero. The decision maker must analyze each one of the mgoals in the model in terms of whether under or over achievement is capable  $\underline{d_i}^+$ , can be removed from the objective function. On the other hand, if under - achievement is the

accepted  $d_i$ <sup>-</sup> can be removed from the objective function. If exact achievement of the goals is desired, both  $d_i$ , and  $d_i$ must be included in the objective function and ranked according to their pre-emptive priority factor from the most important to the least important. In this way, low order goals are considered only after the higher goals are achieved as defined.

## **II. RELATED WORK**

arbel7 et.al. [1990] gives an application of the ahp to bank strategic planning through the mergers and acquisitions process, atmani9 [1995] explains a production planning model for flexible manufacturing systems with setup cost consideration, arakawa6 et. al. [1998] presents multiobjective optimization using adaptive range genetic algorithms with data envelopment analysis, akash2 et.al.[1999] gives an idea about the multicriteria selection of electric power plants using analytical hierarchy process, aouni14et.al. [2001] illustrates about a goal programming model for a glorious history and a promising future, arenas8 et.al. [2002] presents an analysis via goal programming of the minimum achievable study in surgical waiting lists, aouni5 et. al. [2005] explains the decision-maker's preferences modelling in the stochastic goal programming, abd el-wahed1 et.al. [2006] illustrates an interactive fuzzy goal programming for multi-objective transportation problems, ana barcus3 et.al. [2008] presents a supporting the allocation of software development work in distributed teams with multi-criteria decision analysis. beccali12 et.al. [1998] presents a concept on decision making energy planning for the electro multicriteria analysis approach compared to a fuzzy-sets methodology, ballestero10 [2000] illustrates a project finance using a multi-criteria approach to arbitration, beuthe13 et.al. [2000] gives a practical multicriteria methodology for assessing risky public investments, buyukozkan15 et. al. [2004] explains a fuzzy multi-criteria decision approach for software development strategy selection, bigi giancarlo14 [2006] gives an idea on sufficient second order optimality conditions in multiobjective optimization, blahev11 et. al. [2007] applies a goal programming approach to strategic resource allocation in acute care hospitals. choobineh24 et.al. [1993] presents a ranking fuzzy multi criteria alternatives with respect to a decision maker 's fuzzy goal, chalam18 [1994] gives a fuzzy goal programming approach to a stochastic transportation problem under budgetary constraint, chakraborty17 et.al. [1995] explains a multiobjective transportation problem-a goal programming approach, climaco26 et. al. [1995] illustrates a multiple objective linear programming model for power generation expansion planning, coffin27 et.al. [1996] briefs about multiple criteria r&d project selection and scheduling using fuzzy logic, charnes20 et.al.[1997] presents a notes on

goal programming and multiple objective optimization, clewlow25 et.al. [1998] explains mathematical programming and risk management of derivative securities, choo eu23 et.al. [1999] illustrates interpretation of criteria weights in multicriteria decision making, charles19 et.al. [2006] briefs about extremization of multi-objective stochastic fractional programming problem: an application to assembled printed circuit board problem, cheng t ce et.al. [2006] presents a multiproduct, multicriterion supply-demand network equilibrium model, ching-ter chang22 [2006] gives a notes on mixed binary interval goal programming, carlos16 et.al. [2008] explains about prioritization of bridges and tunnels in earthquake risk mitigation using multicriteria decision analysis: application to lisbon.

#### **III. GOALS AND PRIORITIES**

#### **3.1General Goal Programming Model**

When Charles and Cooper introduced the concept of Goal Programming in 1954 they opened a new door for solving problems hitherto deemed insoluble [7]. Many scholars have attempted to develop Goal Programming further and some have found more efficient methods [15,19].

Goal Programming is an appropriate technique for groups like QCCs which have several, often conflicting, goals. Studies shows that Goal Programming has been applied in many different sciences and that tremendous potential remains for further expansion and application of this technique [20].

The general Model for Goal Programming is described as follows:

Minimize  $Z =$  $(W_{ii}^-d_i^- + W_{ii}^+d_i^+)$ 0  $i=1$  $=$   $J^-$  +  $W^+$   $J^+$  $\sum_{j=0}^{n}$   $\sum_{i=1}^{m}$   $P_j(W_{ij}^-d_i^- + W_{ij}^+d_i^-)$ *i n j*  $P_{i}(W_{ii}^{-}d_{i}^{-} + W_{ii}^{+}d_{i})$ Subjected to:  $\mathbf{u}_i \times \mathbf{v}_j + \mathbf{u}_i - \mathbf{u}_i = \mathbf{v}_i$ *n j*  $A_{ii}X_i + d_i^- - d_i^+ = b$  $\sum_{j=1}$  $(i =$  $1, 2, \ldots, m)$ 

$$
X_j, d_i^-, d_i^+ \ge 0
$$

Where  $P_j$  is the pre-emptive priority and j is the rank of each goal. The variable  $X_i$  represents a decision variable.  $W_{ij}^-$  *and*. $W_{ij}^+$  represents the weights given to each goal and  $J^+$  $d_i^-, d_i^+$  represent the degree of under or overachievement of a goal respectively.  $A_{ij}$  is the technological coefficient while  $b_i$ is the goal level [19].  $W_{ij}^-$  *and*. $W_{ij}^+$  represent differential weights which can be assigned to various sub goals within the same priority level according to their importance to

management. In this paper, however, it is assumed that all sub goals have the same importance to management. Thus, a weight of one has been assigned to all sub goals.

The major objectives of a QCC Programme are usually to meet the organization's Goals of increasing Productivity, Quality, Employee participation and job satisfaction, as suggested by the following list:

- 1. Maximize Utilization of Manpower
- 2. Minimize Cost
- 3. Increase worker participation
- 4. Minimize the Cost savings ratio
- 5. Improve Quality
- 6. Increase Productivity

Since identifying goals and their priorities is management's responsibility, these Goals and their related pre-emptive priorities could be changed and some goals added or eliminated.

# **3.2 Objective Function**

Table 1 shows goals, their properties and the related deviations for this model. The objective function for this model is based on the information provided in Table 1. Therefore, the objective can be stated as minimization of the appropriate deviations regarding the stated goals and their priorities are shown below along with all constraint.





## **3.3 Cost Allocation Constraints**

Equations 2.1 and 2.2 represents the labour hour and cost allocation constraints.

# **Participation and Cost Saving**

There is a relationship between the amount of time spent in QCCs by workers and the number of suggestions they make. For example, studies show that the number of positive suggestions made by QCC members is 17.62 per worker in a twelve month period [12, 14]. This rate can be translated into a ratio of approximately 0.8, which means that, on an average, a member will forward at least one improvement suggestion for every 1.25 hours spent in QCCs – Equation 2.3. In addition, a suggestion for an improvement will save the company on the average about Rs 31.75 – Equation 2.4. Since one of the reasons for introducing QCCs is to save money and

prevent cost increases, management considers a cost or saving ratio of 95% or les to be acceptable for implementing QCCs – Equation 2.5.

Absenteeism is a problem in almost all organizations. The implementation of QCCs results in the reduction of absenteeism. Equations  $2.5 \& 2.6$ , translate this idea into mathematical notations. Equation 2.5 shows the desire of management to have only a five per cent absenteeism rate. Constraint 2.6 shows management's expectation that the implementation of QCCs will be worthwhile means of employee participation if the ratio of absenteeism to total hours spent on QCCs does not exceed 15%.

One of the most important reasons for utilizing the QCCs programme is to increase employee participation, it is unproductive and however, to let employees spend long hours in the QCCs or on related projects. In this case the range used is between 1% and 10% of total man hours – Equation 2.7  $\&$ 2.8.

#### **Productivity**

By implementing QCCs, management anticipates higher productivity – the more hours spent on QCCs, the higher the productivity. One way to assure productivity is by using the ratio of total output to total input – Equation 2.9, assuming a 90% or higher productivity rate. The second method for measuring the effect of QCCs on productivity is to compare the hours spent on the QCCs programme to the time spent producing good units (outputs). The ratio should fall within a pre specified range if QCCs is to affect productivity. Formation of Equations 2.10  $& 2.11$  shows this ratio with range from 10% to 30%. If the ratio exceeds the upper limit of the range, time is being spent on QCCs which should be used for production of good units. A value exceeding the lower limit indicates that the QCCs programme is not influencing productivity as it should.

#### **Quality Improvement**

Formulation of Equations 2.12 & 2.13, evaluates quality increase or decrease. Management may be expecting that the percentage of scrap and reworkable units should be less than 1% and 2% respectively. On the other hand, the idea of quality improvement can be demonstrated by the relationship between time spent on QCCs and on a reworkable and scrap unit respectively. The more hours spent on QCCs, the fewer are spent on rework and scrap. Management indicates that a ratio of 3% and 5% or less for time spent on scrap and reworkable units will satisfy its goal of improving quality are given by Equations 2.14 & 2.15.

## **3.4Mathematically the Model is Developed as Follows:** Minimise Z

=



 $X_1$  = Number of hours spent on scrap by QCC members  $X_2$  = Number of hours spent on reworkable units by QCC members

 $X_3$  = Number of hours spent on production of good units by all members

 $X_4$  = Number of hours spent in QCC by worker – including the implementation of QCC suggestions

 $X_5$  = Number of hours of absenteeism

 $X_6$  = Number of improvement suggestions made by QCC members

#### **IV. RESULTS AND DISCUSSION**

The solution will be obtained by using the  $QSB^+$ computer software. This table shows the significant results of the computer run, which are: amount of time to be set aside for each decision variable in order to achieve a set of desired goals, the degree of goals achievement with regards to their priority, and the priority structure of goals.

The original run was set up to priorities man power and cost, since many US industries see these two variables as the main reasons for implementing QCCs. The solutions for the original run indicate the goals one through four is achieved and that two other goals- quality improvements  $(P_5)$  and an increase in productivity  $P_6$ -cannot be achieved. Thus, QCC cannot be very effective with this priority structure, since the two important objectives of QCC, improving quality and increasing productivity, sacrificed in order to achieve the priority goals set up by management.

In this model the man hours wasted (spent on scrap) is far more than that expected by management - 54 percent (950 hours of 1,760 hours available) versus one percent. Although the ration of QCC time to production time, i.e., time spent on good units, falls out of the expected range by one percent, the productivity rate is 50 percent less than minimum desired level - 41 percent versus 90 percent. The results indicate that the allocation of resources should be altered if management would like to improve quality and productivity. To accomplish this, the priority structure of the model has been changed several times in order to find the best set of resource allocation for the decision variable. Then, the resource allocation for the decision variables. Then, the results of the best set of allocation times are considered as criteria for measuring the effectiveness of the QCCs.





## **4.1 First Run**

Minimize  $Z =$  $P_1$ (*d* +d<sub>0</sub> +d<sub>1</sub>)+ $P_2$ ( $d_2$  +d<sub>3</sub> +d<sub>4</sub> +d<sub>4</sub>)+ $P_3$ ( $d_3$  +d<sub>3</sub> +d<sub>5</sub> +d<sub>3</sub> +d<sub>3</sub>)+ $P_4$  $d_4$  + $P_5$  $d_2$  + $P_6$  $d_1$ 

The first run Table 2 reveals that by interchange priorities one and two with six and five, respectively, the result can be changed drastically. In this run, an increase in productivity is introduced as the first goal and the available resource (time) appears as the last priority. The solution reveals that all priorities are achieved. The result shows, however, that to achieve all goals, management must spend Rs 15,438 more than the allocated amount. The over spending of Rs 15,438 is the result of allocating more than 1,543 hours on producing good units  $(X_3)$  with a cost of Rs 20 per hour, as shown in Equation 2.2.The ratio of time spent on scrap and time spent on rework to total time show improvement. In fact, they are better than management experts – 8% and 1.4% versus 1% and 2%, respectively. Productivity is 9% more than the desired level – 99% versus 90%. Although time spent on QCCs and time lost by absenteeism reach their maximum (10% and 15% respectively), the participant Goal is achieved and all allocated man hours are utilized.

## **4.2 Second Run**

Minimize  $Z =$  $P_1$ ( $d_5$  + $d_1$ <sub>0</sub>+ $d_1$ <sup>+</sup>) + $P_2$ ( $d_2$  + $d_3$ <sup>+</sup> $d_4$ <sup>+</sup> $d_5$ <sup>+</sup> $P_3$  $d_4$ <sup>+</sup> $P_4$  $d_2$ <sup>+</sup> $P_5$ ( $d_3$ <sup>+</sup> $d_5$ <sup>+</sup> $d_6$ <sup>+</sup> $d_7$ <sup>+</sup> $d_8$ <sup>+</sup> $P_6$  $d_4$ <sup>+</sup>

The second run in Table 2 shows the quality improvement and increasing productivity are introduced as the second and first goals respectively. At the same time, cost minimization and man hour maximization switch to priorities three and four  $(P_3 \text{ and } P_4)$  respectively. Participation and cost saving goals are introduced as goals five and six respectively. The solution for this run in Table 2.2 indicates that not all goals can be achieved  $(P_5 \text{ and } P_6)$ . Although the ratio of Absenteeism to QCC is at its peak (15%), time spent in QCCs reaches a very high level. As a result, the ratio of QCCs to total utilized time shows an intolerable ratio 22%, about 7% more than expected. Productivity rate shows an overachievement of more than 17% - 97% verses 90%.

Moreover, this set of priorities shows that time spent on production of good units is reduced by approximately 258 hours and absenteeism is increased by 31.5 hours, compared to the previous run. In all three runs available man hours are fully utilized, but time allocated for each activity is very sensitivity to the changes in priorities. The second resource (cost) is fully achieved in the original ranking of priorities. Changing priorities shows that management needs more resources, about Rs 15,000 and Rs 12,000 for the first and second run respectively.

The result of these runs indicates to management whether the priorities used will meet the QCC objectives. If, for example, the results obtained from a given set of priorities are not satisfactory, then management can reprioritize the goals and run the model again. The results also measure each QCC's activities so that any time, management can make an assessment of where the QCC stands in meeting its goals, such as use of resources or quality and productivity. This assessment can be achieved by substituting the values of the decision variables into the goal constraint equations. The results will indicate a positive or negative deviation, i.e. over or underachievement of the goal. For example, the results from the original run in Table 2.2, if substituted in the Equation 2.1, yield 1,760, which indicate the goal is achieved exactly.

Using the above process, management will know when it chooses a set of prioritized goals what the estimated allocation of available resources should be. Thus management can prepare to exercise other means to reduce undesired allocated time for waste, reworkable units and absenteeism.

## **V. CONCLUSION**

That is goal programming is a satisfying method rather than optimizing method. In this paper we develop a Mathematical Model to aid Management in measuring the effectiveness of a Quality Control Programme. A Goal Programming Model seems an appropriate technique for this purpose because it is able to take account of the many goals QCC's deal with. Results shows the goal programming model produces better results.

#### **REFERENCES**

- [1] ABD EL-WAHED, WAIEL F, LEE AND SANG M [2006]: Interactive fuzzy goal programming for multiobjective transportation problems, Omega, 34(2), 158- 166.
- [2] AKASH BA, MAMLOOK R, MOHSEN M.S [1999]: Multicriteria selection of electric power plants using analytical hierarchy process, Electric Power Systems Research, 52 (1), 29-35.
- [3] ANA BARCUS AND GILBERTO MONTIBELLER[2008]: Supporting the allocation of software development work in distributed teams with multi-criteria decision analysis. Omega, 36(3):464 – 475.
- [4] AOUNI B AND KETTANI O [2001]: Goal programming model: a glorious history and a promising future, European Journal of Operational Research 133,225-231.
- [5] AOUNI B, BEN ABDELAZIZ F AND MARTEL 3 M [2005]: Decision-maker's preferences modelling in the

stochastic goal programming, European Journal of Operational Research, 162(3), 610-618.

- [6] 6. ARAKAWA M, NAKAYAMA H, HAGIWARA I AND YAMAKAWA H [1998]: Multiobjective Optimization Using Adaptive Range Genetic Algorithms with Data Envelopment Analysis, A Collection of technical Papers on 7'h Symposium on Multidisplinary Analysis and Optimization (TP98-4970), AIAA, 3, 2074-2082.
- [7] ARBEL A AND Y E ORGLER [1990]: An Application of the AHP to Bank StrategicPlanning: The Mergers and Acquisitions Process European Journal of Operational Research, 48(1), 27-37.
- [8] ARENAS M ET AL [2002]: Analysis via goal programming of the minimum achievable study in surgical waiting lists, Journal of the operational research society, 53, 387- 396.
- [9] ATMANI A [1995]: A production planning model for flexible manufacturing systems with setup cost consideration, Computers and Industrial Engineering Journal 29[l-4], 723 -727.
- [10]BALLESTERO E [2000]: Project Finance: A Multi-Criteria Approach to Arbitration Journal of the Operational Research Society, 51(2), 183-197.
- [11]BLAHE, JOHN T., CARTER, MICHEAL W [2007]: A goal programming Approach to strategic resource allocation in acute care hospitals. European Journal of operational Research. 541-561.
- [12]BECCALI M, CELLURA M, AND ARDENTE D [1998]: Decision making energy planning: the Eleclremullicriteria analysis approach compared to a fuzzy-sets methodology Energy Conversion and Management 39 (16/18), 1869-1881.
- [13]BEUTHE M, L. EECKHOUDT, AND G. SCANNELLA [2000]: A Practical Multicriteria Methodology for Assessing Risky Public Investments Socio-Economic Planning Sciences, 34(2), 121-139.
- [14] BIGI GIANCARLO [2006]: On sufficient second order optimality conditions in multiobjective optimization, Mathematical Methods of Operations Research, 63(1), 77- 85.
- [15]BUYUKOZKAN G, KAHRAMAN C AND RUAN D [2004]: A fuzzy multi-criteria decision approach for software development strategy selection, International Journal of General Systems.
- [16]CARLOS A., BANA E COSTA, CARLOS S. OLIVEIRA AND VICTOR VIEIRA [2008]: Prioritization of bridges and tunnels in earthquake risk mitigation using multicriteria decision analysis: Application to Lisbon. Omega,  $36(3)$ ,  $442 - 450$ .
- [17]CHAKRABORTY M AND SINHA A [1995]: Multiobjective Transportation problem-A Goal

Programming Approach, Industrial Engineering journal 14[7], 17-22.

- [18]CHALAM GA [1994]: Fuzzy goal programming approach to a stochastic transportation problem under budgetary constraint, Fuzzy Sets and Systems 66, 293- 299.
- [19]CHARLES V, AND DUTTA D [2006]: Extremization of multi-objective stochastic fractional programming problem: An application to assembled printed circuit board problem, Annals of Operations Research, 143(1), 297-304.
- [20]CHARNES A AND COOPER WW [1997]: Goal programming and multiple objective optimization, Part I, European Journal of Operational Research 1, 39-54.