Fitting of Fuzzy Regression Model Based on Distance Criteria

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Abstract- *Fuzzy regression analysis is widely applied in the different field's viz. Agriculture, Economic, industries, Social Sciences. Fuzzy regression analysis is originally developed by H. Tanaka to study the relation between explanatory variables and responses in fuzzy environment. Regression analysis was used by many researchers to estimate significant effect of rainfall on crop production. However, Fuzzy regression works better in the situations of uncertainty there is rainfall having high uncertainty hence fuzzy regression model is fitted to estimate the crop yield of the crops which largely dependence on rainfall. Several methods are available to fit the fuzzy regression model. For the present study parameters are estimated using three different methods viz. Two-stage fuzzy linear regression method, Fuzzy membership function least squares regression method, and fuzzy least square method. The performance of these methods is evaluated using distance criterion to find the most suitable method. Data used for present study is of Kharip Soybean crop from Aurangabad district as it is the major crop in this region and it largely depends on rainfall. It is observed that fuzzy least square method is best fit on the basis of minimum distance criterion than two-stage fuzzy linear regression method and fuzzy membership function least squares regression method.*

Keywords *- Multiple linear Regressions, Defuzzification, Fuzzy least square, fuzzy set,Distance Criterion, fuzzy regression Mode*

I. INTRODUCTION

Multiple linear regression model is one of the widely used statistical tool to explain the variation of a response variable Y interms of the variation of independent variablesX as: Y=f(X). Where f(X) is a linear function to estimate the model parameter. Multiple linear regression model is applied only if the given data are distributed according to a Gaussian assumption and the relation between X and Y is crisp. To eliminate this limitation, the fuzzy regression has been introduced. The relationship between dependent and independent variables are uncertain, qualitative fuzzy way, the fuzzy regression model has been proposed in [1] and [12],The explanatory variables and response variable is a fuzzy function of which the distribution of the parameter is a possibility function proposed in [13].Fuzzy linear regression models can provide an estimated fuzzy number that has a fuzzy membership function. If point has the highest membership value from the estimated fuzzy numbers is not within the support of the observed fuzzy membership function, such type of problem has modification of fuzzy linear regression analysis based on criterion of minimizing the difference between the observed and estimated fuzzy numbers are proposed in[19].The two-stage method to construct the fuzzy linear regression model. In the first stage, fuzzy observation are defuzzify so that the traditional least squares methods is applied to find the crisp regression line as same to linear trends of the data. In second stage, the error terms of fuzzy regression model, which represents the fuzziness of the data is determine to give the regression model the best explanatory power of the data is proposed in[25].The simple least square fitting of fuzzy data, containing triangular fuzzy numbers. The simple algebraic criteria is applied to actual fuzzy numbers the model to be fitted is proposed in[2].Fuzzy regression models are useful to investigate the relationship between explanatory and response variables with fuzzy observation. This mathematical programming methods to construct a fuzzy regression model based on distance criterion is proposed in[18].The fuzzy least square methods and mathematical programming methods ware applied to determine the numeric coefficient based on distance criteria proposed in [17,18].[4] They established a nonlinear programming problem to determine the crisp regression coefficient .After that, the average of individual errors between observed and estimated responses was treated as fuzzy error terms.

II. LITERATURE REVIEW

Lots of research has done on monsoon rainfall effects on total food grain of India [9]. Its effects on different crops studied by various researchers [10]the focuses on the impact of monsoon rainfall on Kharif and Rabi food grains yield over India.Chhaya Sonaret.al[11] they were studied the impact of monsoon and dry spells on oil seeds production.

The possibilistic fuzzy regression model was proposed by H.Tanaka [1], determined the regression coefficient and

minimized total fuzziness in the given data.Tanaka et.al [8],used linear systems as a model for analyzing the data. This methods was applied to conventional data obtain fuzzy parameters. Diamond [2] introduced fuzzy regression model to minimize sum of squares of differences for the center of fuzzy numbers and sum of squares of differences for spreads. PierpaoloD'Urso, et al.[3] introduced new approach of fuzzy linear regression analysis. They developed doubly linear adaptive fuzzy regression model, they observed that doubly linear adaptive fuzzy regression analysis was alternative methods for fuzzy linear regression analysis. Chiang Kao et.al [4] proposed classical least square methods to handle fuzzy regression observation in regression analysis. Their methods were based on extension principle they applied classical least square methods in three cases such as crisp input-fuzzy output, fuzzy input-fuzzy output, and non-triangular observation. They observed that classical least square methods were performance better than Sakawa-Yano least square methods.
Ralf Korner, et al.[5],discussed Zadeh's extension principle to classical crisp estimated to the least squares method. They developed linear estimation theory. Unfortunately, since fuzzy sets fail to constitute a linear space w.r.t addition and scalar multiplication and also modified linear estimates often lead to better estimates than by use of the extension principle. Ping-Teng Chang et. al.[6]proposed fuzzy regression analysis was based on the least-squares approach main concept was to estimate the modal value and the spreads separately. After that the interactions between the modal value and the spreads was first analyzed. Then the new fuzzy weighted least-squares regression was introduced finally they concluded that numerical example was provided to show that the proposed method can be an effective computational tool in fuzzy regression.

Georg Peters [7] introduced a new class of fuzzy linear regression models based on Tanaka's approach. He suggested that proposed model was varied to generalization of the Tanaka model and also applied for real-life applications. Byungjoon Kim et. al. [19] proposed, fuzzy regression models to estimate error criterion based on observed and estimated membership function.

III. MATERIAL AND METHODS

To fit Fuzzy regression model the fuzzy numbers are defined as given in section 4.1.

4.1 Fuzzy numbers

A fuzzy number A is fuzzy subsets of the real line R. Its membership function satisfies the following criteria:

(i) α- cut set of $\mu_A(x)$ is closed interval,

(ii) $\exists\, x$ such that $\mu_A(x)=1$ and

(iii) Convexity such that

$$\mu_A(\lambda x_1+(1-\lambda)x_2)\geq \min(\mu_A(x_1),\mu_A(x_2)),\ for\ \lambda\in[0,1]$$

, where α- cut set contains all x elements that have a membership grade $\mu_A(x)\geq\alpha$

A fuzzy number A has the left and right spreads from a real number whose membership is 1.The shape function $L(x)$ of the fuzzy number is defined by [21] as

$$\mu(x)_A = L\left(\frac{x-c}{\sigma}\right)\sigma > 0$$

Where c ,σ are center and spread of the function respectively If L-R fuzzy number has a symmetric continuous membership function the shape function satisfies

(i) $L(x) = L(-x)$ (ii) $L(0) = 1$ (iii) $L(x)$ is strictly decreasing on $[0,\infty]$.

The choice of the L and R function is dependent upon the subjective judgment.

Fuzzy regression model can be used to fit fuzzy dependent and crisp independent variables. Three methods are compared viz, two-stage fuzzy linear regression method, fuzzy membership function least squares regression method, and fuzzy least square method as follows.

4.2 Fuzzy Membership function least-squares regression method

The fuzzy linear regression model is developed in [1,20], when the dependent variable is a fuzzy number and independent variable is crisps fuzzy linear regression model as follows:

$$\tilde{Y} = (y,e)_L = \tilde{A}_0X_0 + \tilde{A}_1X_1 + \cdots + \tilde{A}_NX_N = \tilde{A}X \quad (1)$$

(⟦y,e)⟧_Lis the L-R fuzzy number with center y and spread e ; X is the independent variables vector ; and A represents fuzzy set vector representing parameters of the model, Ais L-R fuzzy number. By applying the extension principle [12], the equation (1) fuzzy numbers $A_i = [\alpha_i, c_i]_L$ as follows:

$$(y,e)_L = (\sum_{i=1}^{m}\alpha_i\, x_i, \sum_{i=1}^{m} c_i\, x_i) \quad (2)$$

Fuzzy linear regression model to estimate parameters using Linear programming problem. Minimizing the total sum of spreads of the fuzzy parameters are introduced in [8] as follows:

$$Min \sum_{i=1}^{m}\sum_{j=0}^{N}\left(c_j|X_{ij}|\right)$$

s. t.

$$\sum_{j=0}^{N}\alpha_j X_{ij} + (1-h)\sum_{j=0}^{N} c_j|X_{ij}| \geq Y_i + (1-h)e_i$$
$$\sum_{j=0}^{N}\alpha_j X_{ij} - (1-h)\sum_{j=0}^{N} c_j|X_{ij}| \leq Y_i - (1-h)e_i$$
$$i = 1, 2, \dots, M \qquad (3)$$
$$c_j \geq 0, a \in R, X_{i0} = 1, (0 \leq h \leq 1)$$

By using equation (3) fuzzy membership function least square regression model is proposed in[19] as follows:
The membership function $\mu_A(x)$ is calculated as

$$\mu_A(x) = \begin{cases} L\left(\frac{c-x}{c-l}\right) & l \leq x \leq c, \\ R\left(\frac{x-c}{r-c}\right) & c \leq x \leq r \end{cases} \qquad (4)$$

Where c is the center l and r are left and right end points of fuzzy numbers A.

L-R are continuous strictly decreasing function on [0,1] and $L(x) = R(x) = 1\, if$

$x = 0, L(x) = R(x) = 0$ if $x = 1$ then applying the extension principal introduced in [12],then the sum of n fuzzy numbers $is\ (l_i, c_i, r_i; i = 1, 2, \dots, n)$.

By using equation (4) fuzzy membership function least square model is as follows:

$$Y = (y - |L^{-1}(\alpha)| \times e, y + |L^{-1}(\alpha)| \times e)$$
$$= (l_1, c_1, r_1)x_1 + (l_2, c_2, r_2)x_2 + \dots\dots + (l_m, c_m, r_m)x_m$$
$$= (\sum_{i=1}^{m} l_i x_i, \sum_{i=1}^{m} c_i x_i, \sum_{i=1}^{m} r_i x_i) \qquad (5)$$

The fuzzy regression model (5) is decomposed into three least square regression models:

$$y - |L^{-1}(\alpha)| \times e = \sum_{i=1}^{m} l_i x_i$$
$$y = \sum_{i=1}^{m} c_i x_i \qquad (6)$$
$$y + |L^{-1}(\alpha)| \times e = \sum_{i=1}^{m} r_i x_i$$

By solving the least –square normal equations (6) to obtained , and then to estimate observed and estimated fuzzy numbers.

4.3Two-stage fuzzy linear regression method

The classical regression model is given as follows:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots \dots + \beta_k X_{ik} + \varepsilon_i\ ; \quad i = 1, 2, \dots\dots n \qquad (7)$$

The regression parameter β_j must usually be estimated from sample data.
If observations are fuzzy numbers, since crisp values can be represented by degenerated fuzzy numbers then the fuzzy linear regression model is introduced in [25] as follows:

$$\tilde{Y}_i = \beta_0 + \beta_1 \tilde{X}_i + \cdots + \tilde{\varepsilon}_i\ ; i = 1, 2, \dots\dots n \qquad (8)$$

Where $\tilde{X}_i$, $\tilde{Y}_i$, $\tilde{\varepsilon}_i$ are fuzzy numbers.
The $\tilde{Y}_i = (Y_{il}, Y_{im}, Y_{iu})$ membership function $\mu_{\tilde{Y}_i}$ isas fallows

$$\mu_{\tilde{Y}_i}(y) = \begin{cases} (y - y_{il})/(y_{im} - y_{il}), & y_{il} \leq y \leq y_{im} \\ (y_{iu} - y)/(y_{iu} - y_{im}), & y_{im} \leq y \leq y_{iu} \end{cases}$$

To estimate for β_0 and β_i, using defuzzified the fuzzy observations $\tilde{X}_i$ and $\tilde{Y}_i$ to the crisp value and the conventional least square method is applied. The formulated fuzzy regression model is given as:

$$\hat{\tilde{Y}}_i = b_0 + b_1 \tilde{X}_i + \tilde{E} \qquad (9)$$
$$\tilde{E} = (-l, 0, r)$$
$$(\hat{Y}_{im} - \hat{Y}_{il}) \geq l_{min}, (\hat{Y}_{iu} - \hat{Y}_{im}) \geq r_{min},$$

Response $\hat{\tilde{Y}}_i$ is estimated using equation (9) as follows:

$$\hat{\tilde{Y}}_i = (\hat{Y}_{il}, \hat{Y}_{im}, \hat{Y}_{iu})$$
$$= (b_0 + b_1 x_{i1} - l, b_0 + b_1 x_{im}, b_0 + b_1 x_{iu} + r)$$

4.4 Diamond least square method:

The fuzzy least squares (FLS) method was proposed in [2] to determine fuzzy parameters. He has considered crisp independent variables and fuzzy dependent variables the fuzzy least square model is as follows:

$$Y = A + BX\ ; \quad X \in R\ , \quad A \in T(R), \quad B \in P(R)$$

$$Minimization \;\; r\,(A\,,B) = \sum_{i=1}^{n} d(a + x_i\beta_{i\,,}Y_i)^2$$

The parameters of $A = (a, \underline{\alpha}, \overline{\alpha})_T$ *and*

$$B = (b, \underline{\beta}, \overline{\beta})_T \qquad (10)$$

Satisfy the system

$$a = \hat{y} - b\,\bar{x}\alpha = \hat{\eta} - \beta\,\bar{x}$$
$$b = {k}/{T^2}\,\beta = {s}/{T^2}$$

Where $k = \sum_{i=1}^{n}(x_i - \bar{x})\,(y_i - \bar{y})$,

$s = \sum_{i=1}^{n}(x_i - \bar{x})\;(\eta_i - \hat{\eta})$

$T^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$

4.5 Goodness of fit criterion

After estimating the fuzzy regression parameters to minimum sum of absolute values of differences between the estimated and observed fuzzy numbers, which is introduced in[18],formulated as follows:

$$Min\sum_{i=1}^{n}\left|\hat{\tilde{Y}}_i - \tilde{Y}_i\right| \qquad (11)$$

In equation (11), the fuzzy arithmetic operation is used introduced in [24], then the lower and upper bounds of estimated fuzzy response are$(\hat{\tilde{Y}}_i)^L$ and $(\hat{\tilde{Y}}_i)^U$ respectively.

Where

$$(\hat{\tilde{Y}}_i)^L = b_0 + b_1(X_{i1})^L + b_2(X_{i2})^L + \cdots + b_p(X_{ip})^L + (\delta)^L \qquad (12)$$

$$(\hat{\tilde{Y}}_i)^U = b_0 + b_1(X_{i1})^U + b_2(X_{i2})^U + \cdots + b_p(X_{ip})^U + (\delta)^U \qquad (13)$$

Equation (12) and (13) solved by using conventional regression model when all fuzzy observation converted to crisp. Then equation (11) based on the operations of fuzzy equationsin [24], the absolute deviations D_i be the average error as formulated :

$$D_i = \frac{1}{2m}\sum_{k=1}^{m}\left(\left|\left(\hat{\tilde{Y}}_i'\right)_{ak}^{L} - (Y_i)_{ak}^{L}\right| + \left|\left(\hat{\tilde{Y}}_i'\right)_{ak}^{U} - (Y_i)_{ak}^{U}\right|\right) \qquad (14)$$

Where,

For a certain α- level$\left(\hat{\tilde{Y}}_i'\right)_{ak}^{L}$, $\left(\hat{\tilde{Y}}_i'\right)_{ak}^{U}$, are the lower and upper bounds of the i^{th} estimated fuzzy response, and $(Y_i)_{ak}^{L}$, $(Y_i)_{ak}^{U}$ are the lower and upper bounds i^{th} fuzzy response. Then, the average mean deviation index(MD, as a measure of goodness fit of the model.

$$MD = \frac{1}{n}\sum_{i=1}^{n} D_i \qquad (15)$$

Equation (15) is equivalent to the absolute difference of two fuzzy numbers, if both the estimated and observed fuzzy numbers converted to crisp.

IV. RESULTS AND ANALYSIS

The average rainfall for the month of June, July, August, September, October,in millimeter. Soybean crop production in quintal per hector is taken from the websites www.mahaagri.gov.in for the year 2000 to 2013. The variables are defined as in table 1 and as listed in Colum one as follows:

X_1: Average monthly rainfall in month of June

X_2: Average monthly rainfall in month of July

X_3: Average monthly rainfall in month of August

X_4: Average monthly rainfall in month of September

X_5:Average monthly rainfall in month of August

And response Y is defined as production of Soybean crop in quintal per hector. For the fitting of fuzzy regression model the Y is the central value of crop production and also left and right spread of crop production Y is $(12\%\ or\ 14\%, 8\%\ or 10\%)$. The resulting fuzzy observations are nonsymmetrical triangular fuzzy numbers. The fuzzified variables are defined in table 1 and listed in Colum two as follows.

$\tilde{X}_1, \tilde{X}_2, \tilde{X}_3, \tilde{X}_4, \tilde{X}_5$					$\tilde{Y} = (l\ m\ u)$			Distance criterion		
								KB	KC	DM
181.8	74.4	101	41.4	435.1	659.8	754	861.7	89.92089	65.47023	88.39864
187.6	135.1	85.4	57.7	524.7	437.5	500	571.4	158.6142	134.1123	156.1855
214.3	139.5	139	43.3	578	1156.8	1322	1510.9	23.11546	9.859286	20.37414
98.9	66.2	209.1	107.7	567.2	901.3	1030	1177.1	7.795063	17.68071	6.843578
151.2	46.8	227.8	114.7	734.8	673.8	770	880.0	78.00933	53.58546	77.10535
143.5	17.7	239.6	76.4	675.1	937.1	1071	1224.0	19.68621	16.5825	18.94101
234.6	55	204.1	83.2	628.6	1341.4	1533	1752.0	42.61665	67.13964	43.78768
266.6	54.8	315.7	130.4	863.7	1435.0	1640	1874.3	37.95527	62.55514	39.07823
242.4	27.8	177.1	101.3	615.4	1190.9	1361	1555.4	29.17906	53.63907	29.7389
138.5	174.7	137.8	90.9	590.5	1179.5	1348	1540.6	12.60601	37.1067	15.40038
137.5	218.2	147.2	69	651.9	1103.4	1261	1441.1	33.63165	11.49321	29.85574
91.5	155.2	153.7	119.1	646.5	889.9	1017	1162.3	26.273	18.02893	23.96385
92.1	183.2	103.7	188.7	625.7	834.8	954	1090.3	9.729179	19.71643	7.739357
91.4	213.4	79.8	204.8	671.7	823.4	941	1075.4	13.03404	20.06464	10.67194
113.7	180.4	46.5	158	671.7	763.9	873	997.7	32.69649	21.88607	30.35186
37.3	186.4	100.6	163.4	533.9	1051.8	1202	1373.7	51.22107	75.51916	53.33624
36.8	353.7	93	178.6	745.5	1034.3	1182	1350.9	14.05987	13.60929	9.214358
57.4	345.3	98.3	233.7	840.9	961.6	1099	1256.0	17.51451	15.8325	13.17325
165.8	182	268.7	202	924	1365.0	1560	1782.9	45.00187	69.53153	47.39297
132.5	110.8	298	219.3	883.3	1052.6	1203	1374.9	11.52617	35.94166	12.59738
100.8	48	221.6	230.8	729.6	716.6	819	936.0	4.713898	23.3325	4.981546
147.2	161.5	105.5	163.3	592.5	1406.1	1607	1836.6	102.7564	127.1441	104.6297
163	143.5	131.9	140.3	600.6	1140.1	1303	1489.1	27.38827	51.81783	29.24202
174.4	80.9	116.7	320	714.2	395.5	452	516.6	55.01818	33.16286	55.70345
48.5	89.8	131.4	331.6	652	717.5	820	937.1	60.66926	84.70934	59.86065
70.1	116.7	116.4	241.2	602.5	538.1	615	702.9	27.94658	28.79679	27.52681
64.3	82.4	169.2	283.7	650.5	176.8	202	230.9	94.03113	69.90166	94.46731
33.8	180	123.3	156.7	691.9	973.9	1113	1272.0	23.05265	47.38116	25.42318
45.9	185.2	174.4	123	633.1	1015.9	1161	1326.9	3.964744	28.37865	6.649239
67.8	131.1	169.4	211.1	762.6	450.6	515	588.6	82.64456	58.36096	81.4186
176.4	234.3	255.5	97.9	887.6	1859.4	2125	2428.6	99.75507	124.4613	103.9136
92.7	295.1	291.8	113.8	894.2	1419.3	1622	1853.7	1.005332	25.70273	5.90674
145.2	275.9	258.9	155.3	962.4	1200.5	1372	1568.0	38.16545	13.5017	33.83485
109.4	354.9	315.1	167.5	1052.3	1942.5	2220	2537.1	104.2152	128.9633	109.7587
140.2	326.2	266.7	167.6	1035.7	1494.5	1708	1952.0	13.16734	37.87942	18.25736
83.5	376.1	299	219.3	1125.8	1295.9	1481	1692.6	22.78923	5.600357	17.32368
71.3	359.2	307.3	183.4	1013.3	1339.6	1531	1749.7	14.41568	10.2717	9.046597
119	526.7	380.7	163.5	1246.9	1400.9	1601	1829.7	110.7309	85.71726	102.2248
56.1	179.2	209.4	114.5	589.2	1242.5	1420	1622.9	45.27559	69.7219	47.87915
38	209.7	192.9	102	560.7	1306.4	1493	1706.3	54.68609	79.14819	57.8165
68.4	302.4	243.5	69.1	740.2	1352.8	1546	1766.9	9.824442	14.86023	4.844589
34	318.5	255.1	88.4	710.4	1442.0	1648	1883.4	22.13828	46.78881	27.30175
51.6	176.3	182.2	56.5	540.2	1323.9	1513	1729.1	51.82503	76.30841	54.88008

93.1	210.5	293.9	90.3	708.1	1117.4	1277	1459.4	40.59866	15.98356	37.06103
58.1	289.1	187.5	89.8	667.5	1042.1	1191	1361.1	49.44588	24.85606	44.80822
154.1	176.1	290.2	142.7	764.1	1218.0	1392	1590.9	5.327884	19.25105	2.718291
49.1	118.6	67.6	84.7	375.5	650.1	743	849.1	37.93376	25.36821	36.32283
43.4	95.3	62.8	78.8	324.6	611.6	699	798.9	35.74052	26.54679	34.51159
81.1	108.6	82.7	107.3	461.1	1454.3	1662	1899.4	142.9541	167.2567	144.349
159.3	161.7	112.6	149.8	736.4	947.6	1083	1237.7	19.27363	16.26107	16.99966
171.3	252.8	122.4	148.6	844.1	1449.9	1657	1893.7	52.50922	77.07541	56.35454
130.2	210.1	86.2	151.8	729.9	1630.1	1863	2129.1	137.0228	161.462	139.9825
					Total estimation error			**45.099**	**50.487**	**44.810**

From equation (6) estimated parameters by using MATLAB software the Fuzzy membership function least squares regression method as follows:

$$Y_{KB} = (573.2615, 655.1253, 784.7012) + (1.4200, 1.6228, 1.8547)X_1 + (1.5902, 1.8174, 2.0770)X_2 + (1.0013, 1.1443, 1.3079)X_3 + (-1.7077, -1.9519, -2.2303)X_4 + (0.1598, 0.1826, 0.2087)X_5$$

From equation (8) and (9) estimated the parameters by using MATLAB software the Two-stage fuzzy linear regression method as follows:

$$Y_{KC} = (659) + (1.6325)X_1 + (1.8282)X_2 + (1.1512)X_3 + (-1.9631)X_4 + (0.1837)X_5 + (-345, 0, 107.7)$$

Also from equation (10) estimated the parameters by using MATLAB software the fuzzy least regression methods as follows:

$$Y_{DM} = (573.2312, 653.1252, 784.7013) + (1.4100, 1.5327, 1.8625)X_1 + (1.4402, 1.543, 2.0671)X_2 + (1.0012, 1.8162, 1.3080)X_3 + (-1.6072, -1.9426, -2.2301)X_4 + (0.1497, 0.1812, 0.2008)X_5$$

V. CONCLUSION

There is large variation in yield of soyabean and it depends on rainfall hence fuzzy regression models are applied for estimation of the parameters. We compared three different fuzzy regression methods that are Two-stage fuzzy linear regression method, Fuzzy membership function least squares regression method, and fuzzy least square method on the basis of distance criteria, for estimating soyabean yield in Aurangabad district. The average error of mean absolute deviation by fuzzy least squares method is minimum than the other two methods hence Diamond fuzzy least square method gives better fit as compared to Two-stage fuzzy linear regression method, Fuzzy membership function least squares regression method.

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