An Iterative Algorithm for Image Denoising Based on Tuning

Ms. Vinaya Chandran[1], Ms. Resmi E[2]
[1, 2] Department of Electronics and Communication Engineering
[1, 2] Government Engineering College Wayanad

Abstract- *Compressed sensing is recent novel method for the effective acquisition and reconstruction of a signal. Compressed sensing says that certain images and signals can be recreated from lesser number of samples and measurements than the traditional methods. The aim is to enhance over existing compressed imaging techniques in terms of reconstruction error. Here, proposed a compressive imaging algorithm that employs an iterative denoising algorithm framework. The proposed method gives a high PSNR for colour images than that of the existing methods.*

Keywords- Compressed sensing, PSNR.

I. INTRODUCTION

A. Motivation

Most of the devices in the fields of medical imaging, radars, communication receivers and audio-visual electronics employ Shannon's sampling theorem for signal acquisition. Shannon's sampling theorem expresses that the Nyquist rate for bandlimited signals might be greater than or equal to two times the maximum frequency content in the signal. For signals which are not band limited, the Nyquist rate is decided by the spatial or temporal resolution and the signals are subjected to anti-aliasing lowpass filter to make them bandlimited prior to sampling. In many of the imaging systems, Nyquist rate of sampling results in large number of samples resulting in large storage requirements and reduction in the speed of transmission. In lot of the methods used for photograph compression today, turn out to be of the input photo is taken and a gigantic fraction of the change into coefficients are discarded, at the same time nonetheless attaining correct photograph reconstruction. This strategy is observed to be tedious and costly in numerous applications.

CS is a recently emerged method that enables us to sample and compress the signals simultaneously [1].Compressed sensing[2],[3] is being utilized as a part of compressed imaging, the input information is an image, and the aim is that to acquire the image using less samples. Getting images in a compressed manner requires less examining time than imaging techniques that are conventional. Uses of compressive imaging is in numerous areas such as medical imaging[14], seismic imaging[15], and hyper spectral imaging[16],[17]. Many algorithms were proposed for the efficient reconstruction of images from their compressively sampled form. Major constraints in doing so were found to be computation time, image quality and memory requirements.

B. Related Works

So many works have been done in the CS recovery algorithms taking advantage of the signal sparsity all around the globe in the objective of minimizing the runtime as well as the number of samples required. Those methods have its own pros and cones. Researches have been going on inorder to overcome those issues and for getting better performances. Overviews of some methods developed by researchers are discussed below.

Kostadin Dabov[4] et.al presented an image denoising technique taking into account on an upgraded sparse representation in 3D-transform domain which employs the collaborative filtering. The sparsity is accomplished by gathering comparable 2D pieces of the picture into 3D information clusters which is the gatherings. Outcome is the 3D appraisal of the gathering which comprises of a cluster of mutually sifted 2D parts. Because of the closeness between the gathered hinders, the change can accomplish a very inadequate representation of the genuine flag so that the artifacts can be all around isolated by shrinkage. It is a computationally scalable algorithm. Kivan et.al[5] introduced a spatially versatile measurable model for wavelet image coefficients using the estimation quantization coder and apply it to image denoising. They displayed wavelet image coefficients as Gaussian random variables with zero mean of high local correlation. It expects a known distribution on wavelet coefficients variances and ascertaining them using an approximate extreme a posteriori likelihood rule. At that point apply an estimated least MSE estimation system to reestablish the noisy wavelet image coefficients. This work does not endeavor to explore the theoretical properties of the proposed models and calculations in general settings.

Michael Elad et.al[6] addressed the denoising problem, where independent AWGN noise is to be expelled from a given image. The methodology taken is based on sparse and excess representations over prepared references that is the trained dictionary. It uses the K-SVD algorithm, and get a dictionary that portrays the image effectively. In this work they considered two training options, they are training the dictionary using patches from the ruined image itself and training on a set of patches taken from a high-quality set of images. Dictionary learning is restricted in handling small image patches, so trouble occurs. Iteration enhances the denoising results. It cannot be conveyed on larger blocks because the dictionary learning is limited in handling small image patches, a natural difficulty emerges.

Som and Schniter[7], proposed an algorithm that adventures the sparsity and perseverance over 2D wavelet transform coefficients of image. They modeled wavelet structure by Hidden Markov Tree (HMT). They adopted a “turbo” passing [13] algorithm that change the exploitation of HMT structure and compressive estimation structure. Turbo messaging algorithm utitlized here emphasize the exploitation of HMT structure and perception structure. Baron et.al [8] proposed a denoising algorithm. AMP is a signal reconstruction algorithm that perform scalre denoising at every iteration. AMP to recreate the input image well, agood denoiser must be used. The challenge in applying denoiser in AMP is that it might be difficult to calculate the “Onsager reaction term”. AMP offers better MSE than existing compression techniques.

II. PROPOSED ALGORITHM

A. Methodology

The proposed method is shown in Fig.1. We have an input image. We are rearranging the input image **x** containing **N** pixels, as a column vector which is of length **N**, for example for an image of **128*128** size, the value of **N** will be $\mathbf{128^2}$. We then multiply the image **x** with a known measurement matrix **A** and add i.i.d Gaussian noise **z** with the product. Thus the image will be like:

$$\mathbf{y = Ax + z} \quad (1)$$

The measurement matrix $\mathbf{A} \in \boldsymbol{R^{M*N}}$ and the noise that added for corrupting the image will be independent and identically distributed Gaussian noise. Now we want to estimate and reconstruct the original image **x** from the observation vector **y**.

Since input image is noisy, we require denoising of the recovered image.

The proposed method uses message passing algorithm[12] for denoising the image.

III.TUNED MESSAGE PASSING

TMP is an iterative signal reconstruction technique in matrix channels. Consider a matrix channel which is expressed as (1)
where the distribution of signal follows $x_i \sim f_i$ and the noise added is i.i.d WGN. The entries of the measurement matrix are i.i.d distributed. The iteration proceeds as per

$$x^{t+1} = \eta_t(A^T r^t + x^t) \quad (2)$$

$$r^t = y - A x^t + \frac{1}{R} r^{t-1} \langle \eta'_{t-1}(A^T r^{t-1} + x^{t-1}) \rangle \quad (3)$$

here

A^T is the transpose of A
$R = \frac{M}{N}$ is the measurement rate
η_t is the denoising function at t^{th} iteration
$\eta'_t = \frac{\partial}{\partial x}\eta(s)$
$\langle u \rangle = \frac{1}{N}\sum_{i=1}^{N} \mu_i$ for some vector $\mu = (\mu_1, \mu_2\ \mu_3 \ldots \mu_n)$
The noise variance can be calculated as

$$\sigma_t^2 = \frac{1}{M} \sum_{i=1}^{M} {r_i^t}^2 \quad (4)$$

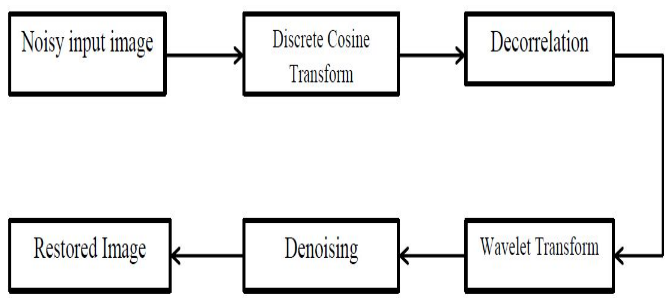

Fig. 1.Block diagram for the proposed method

IV. DENOISING USING TMP

In this section, the description of wavelet-based image denoising is done using TMP.

A. Wavelet Transform in TMP

In image processing, always computing the wavelet coefficients of images and then applying some signal processing techniques to them and at last the inverse of wavelet transform is applied to obtain the processed images. The wavelet-based image denoiser proceeds as follows:

- Apply a wavelet transform to the image and obtain wavelet coefficients
- Denoise the wavelet coefficients
- Apply an inverse wavelet transform to the denoised wavelet coefficients, yielding a denoised image.

Here we are computing the wavelet coefficients of the signal. The image denoising is done in TMP in the wavelet domain.

The wavelet transform is denoted by $\boldsymbol{\omega}$ and its inverse is given by $\boldsymbol{\omega}^{-1}$. Then the wavelet transform is applied to the vectorised image $\mathbf{x}$ and thus obtaining the coefficient vector as $\boldsymbol{\theta}_x = \boldsymbol{\omega}\mathbf{x}$. So that the matrix channel in (1) can be written as

$$\mathbf{y} = \mathbf{A}\,\boldsymbol{\omega}^{-1}\boldsymbol{\theta}_x + \mathbf{z} \qquad (5)$$

Now $\mathbf{A}\,\boldsymbol{\omega}^{-1}$ is the new matrix in the equation (5) and θ_x is the input signal.
Now the equation (2) and (3) can be rewritten as

$$\theta_x^{t+1} = \eta_t((A\omega^{-1})^T r^t + \theta_x^t) \qquad (6)$$

$$r^t = y - A\omega^{-1}\theta_x^t + \frac{1}{R} r^{t-1} \langle \eta'_{t-1}((A\omega^{-1})^T r^{t-1} + \theta_x^{t-1}) \rangle \qquad (7)$$

$$r^t = y - Ax^t + \frac{1}{R} r^{t-1} \langle \eta'_{t-1}((A\omega^{-1})^T r^{t-1} + \theta_x^{t-1}) \rangle \qquad (8)$$

At first the $\boldsymbol{r^t}$ and $\boldsymbol{x^t}$ are set to zero and the iteration takes place. The steps will be

- Calculating $\boldsymbol{r^t}$, the residual term.
- Calculating the noisy image $\boldsymbol{q^t} = \boldsymbol{A^t}\, \boldsymbol{r^t} + \boldsymbol{x^t}$ and the applying the wavelet transform to the noisy image for obtaining the wavelet coefficients. Those coefficients are the input of the scalar denoiser.
- Obtaining the denoised coefficient by applying the denoiser to the coefficients.
- Apply IDWT on the coefficients inorder to get the estimated image. This is used to calculate the $\boldsymbol{r^t}$ for the forthcoming iterations.

B. Image Denoisers

Here we are using the denoisers presented by Figueiredo and Nowak [18] TMP based Weiner denoiser and Mhcak et al. [19], TMP based Amplitude scale Invariant Bayes Estimator. These denoisers are used why because they are simple to implement. The variance is obtained by (4). As we are using the orthonormal wavelet transform the noise variance will be same for both the wavelet as well as image domain.

Amplitude Scale Invariant Bayes Estimator: For each noisy coefficient the estimate for the forthcoming iteration will be

$$\theta_{x,i}^{t+1} = \eta_t(\theta_{q,i}^t) = \frac{({\theta_{q,i}^t}^2 - 3\sigma_t^2)_+}{\theta_{q,i}^t} \qquad (9)$$

where, σ_t^2 is the noise variance of the t^{th} iteration of AMP,$(.)_+$ is a function such that $(\mu)_+ = \mu$ if $\mu > 0$ and $(\mu)_+ = 0$ if $\mu < 0$. This function is continuous and differentiable except for the point $\theta_{q,i}^t = \pm\sqrt{3\sigma_t}$ Taking the derivative of the above function to get the 'Onsager reaction term'[21].

Adaptive Weiner Filter This filter is designed in order to evaluate the variances of the wavelet coefficients, and at that point applies the Wiener filtering. The variance of those coefficients is evaluated from its neighboring coefficients. The denoising fuction is given as,

$$\theta_{x,i}^{t+1} = \eta_t(\theta_{q,i}^t) = \frac{\hat{\sigma}_i^2 - \sigma_t^2}{\hat{\sigma}_i^2}\,\theta_{q,i}^t$$

The denoising function derivative is the scaling factor of the Weiner filter and thus the Onsager reaction term can be obtained. The adaptive Wiener filter in (10) is not a entirely separable denoising function, and TMP-Wiener experiences convergence issues. Luckily, a method called 'damping' [20] solves for the convergence issue of TMP-Wiener.

C. Tuning

In the acquisition and transmission phases images will be corrupted. The noise removal techniques that are based on the wavelet domain should use some threshold value in order to remove the small coefficients of the sub bands while preserving the large coefficients. Why it is necessary to remove the small coefficients and keeping the large one is because those large coefficients contains the main features of the image where as the other contain noise. So it is necessary to have a thresholding function for the thresholding.

The most regularly used thresholding are soft and hard thresholding[9]. Hard thresholding functions are not continuous and they are not differentiable where as soft differentiable techniques are continues but they don't have first order derivative. So these functions are not suitable for gradient based learning tools.

So another thresholding technique based on the Thresholding Neural Network[10],[11] is used in this work. These is both differentiable as well as continues and have higher order derivatives. The function is tuned by Least Mean Square algorithm. Here the thresholding value can be adjusted in the learning phase

The TNN based tuning function is given by:

$$\eta(x,t)= x+\frac{1}{2}\left\{\left|x-\frac{t}{e^{\left(\frac{|x|-t}{\lambda}\right)}}\right|-\left|x+\frac{t}{e^{\left(\frac{|x|-t}{\lambda}\right)}}\right|\right\} \quad (11)$$

where,

t is the threshold

$\lambda > 0$ which is a user defined parameter.

V. RESULTS AND DISCUSSION

A method is proposed to reconstruct the noisy image using compressed sensing is done. The result obtained is shown in Fig. 2. The numerical results showed a better result compared to any other compressed sensing techniques. The thresholding for the reconstruction is done by using the TNN based thresholding techniques.

The experiments based on the proposed method were carried out on a PC with a 3.20 GHz Intel Core i5 CPU and 8 GB of memory. All algorithms were implemented using MATLAB R2014a. In this method, an input image of size 256*256 issued to test the proposed algorithm.

The proposed method is compared with the AMP based denoising algorithm. Numerical results showed that this method achieves the lowest reconstruction error among all competing algorithms in all simulation settings. The performance of the proposed technique is measured using the parameters peak signal to noise ratio (PSNR) and structural similarity index (SSIM) which were widely used to assess the performance of reconstruction and denoising algorithms.

PSNR for an image X and its degraded version Y is given by,

$$\text{PSNR}=20\log_{10}\left\{\frac{MaxX}{\sqrt{MSE}}\right\} \quad (12)$$

where, mean squared error (MSE) is given by,

$$\text{MSE} = \frac{1}{mn}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\|X(i,j)-Y(i,j)\|^2 \quad (13)$$

(a) Input Image (b) Noisy Image (c) Output Image

SSIM is a measure of likeness between two images. For 2 image X and Y , SSIM is given by,

$$\text{SSIM(X,Y)} = \frac{(2\mu_x\mu_y+C_1)(2\sigma_{xy}+c_2)}{(\mu_X^2+\mu_Y^2+c_1)(\sigma_X^2+\sigma_Y^2+c_2)} \quad (14)$$

where, μ_x is the mean of X, $\mu_y$is the mean of Y, $\sigma_x^2$is the variance of X, σ_y^2 is the variance of Y, σ_{xy} is the covariance of X and Y , c1 = $(0.01L)^2$ and c2 = $(0.03L)^2$are variables for a dynamic range L of pixel values. SSIM obtained is above 0.8453.

Value of PSNR obtained in the proposed method is compared with that of AMP based method for different variance is given in Table I.

TABLE I: Comparison of PSNR values for different variance

Variance	PSNR for proposed method	PSNR for AMP based method
10^{-4}	88.06	33.79
10^{-3}	82.15	33.67
10^{-2}	74.60	32.57
10^{-1}	67.55	25.09

Value of SSIM obtained in the proposed method is compared with that of AMP based method for different variance is given in Table II.

TABLE II: Comparison of SSIM values for different variance

Variance	SSIM for proposed method	SSIM for AMP based method
10^{-4}	88.06	33.79
10^{-3}	82.15	33.67
10^{-2}	74.60	32.57
10^{-1}	67.55	25.09

The proposed method is found to achieve PSNR and SSIM more than the existing methods that uses AMP-Weiner. The quality of the reconstructed image is found to be good as we obtained the PSNR of 67.55 and SSIM of 0.8453 for a variance of 10^{-1}.

VI. CONCLUSION

A unique iteration based algorithm is proposed for denoising of images based on compressed sensing. This method uses TNN based tuning method for better denoising. The proposed method gives a high PSNR of about 88.06 for a variance of 10^{-4} and 67.55 for a variance of 10^{-1}. It gives a better structural similarity index measure. It is a better a algorithm for image reconstruction than existing algorithm. Future exploration could be stretched out to processing of MRI images, hyper spectral images etc.

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