$\tilde{g}(1,2)^*$ - HOMEOMORPHISM

A.Thamilisai[1], S.Meenakshi[2]
[1, 2] Department of Mathematics
[1, 2] Kamaraj Engineering College,Virudunagar,, Tamilnadu,India

Abstract- *In this paper introduce two new classes of bitopological function called $\tilde{g}$ (1,2)*and strongly $\tilde{g}$ (1,2)* by using $\tilde{g}$ (1,2)*.Basic properties of theses two functions are studied and the relation between these types and otherexisting ones are established.we will discuss about Every (1,2)*-homeomorphism is $\tilde{g}$ (1,2)*-homeomorphism and we also discuss about the composition of two $\tilde{g}$ (1,2)*-homeomorphisms is not always a $\tilde{g}$ (1,2)*-homeomorphism and $\tilde{g}$ (1,2)*-homeomorphisms and (1,2)*-sg-homeomorphisms are independent of each other.*

Keywords- Bitopological function, hoemomorphism, $\tilde{g}(1,2)^*$-closed set, $\tilde{g}(1,2)^*$-open set (1,2)*-sg-homeomorphism

I. INTRODUCTION

Njastad introduced α-open sets. Maki et al. [3] generalized the concepts of closed sets to α-generalized closed (briefly αg-closed) sets which are strictly weaker than α-closed sets. Veera Kumar [4] defined ĝ-closed sets in topological spaces.Thivagar et al. [5] introduced αĝ-closed sets which lie between α-closed sets and αg-closed sets in topological spaces.

Maki et al introduced the notion of generalized homeomorphisms (briefly g-homeomorphism) which are generalizations of homeomorphisms in topological spaces. Subsequently, Devi et al [6] introduced two class of functions called generalized semi-homeomorphisms (briefly gs-homeomorphism) and semigeneralized homeomorphisms (briefly sg-homeomorphism). Quite recently, Zbigniew Duszynski [5] have introduced αĝ-homeomorphisms in topological spaces.

It is well-known that the above mentioned topological sets and functions have been generalized to bitopological settings due to the efforts of many modern topologists [see 7, 8, 9, 10, 11, 12, 13, 14, 15]. In this present chapter, we introduce two new class of bitopological functions called $\tilde{g}$ (1,2)*-homeomorphisms and strongly $\tilde{g}$ (1,2)*-homeomorphisms by using $\tilde{g}$ (1,2)*-closed sets. Basic properties of these two functions are studied and the relation between these types and other existing ones are established.

II. PRELIMINARIES

Definition 2.1

A subset A of a bitopological space X is called

(1) (1,2)*-semi-open set [13] if
$$A \subseteq \tau_{1,2} - cl(\tau_{1,2} - \text{int}(A))$$
(2) (1,2)*-α-open set [12] if
$A \subseteq \tau_{1,2} - \text{int}(\tau_{1,2} - (cl(\tau_{1,2} - \text{int}(A)))$.
(3) regular (1,2)*-open set [14] if
$$A = \tau_{1,2} - \text{int}(\tau_{1,2} - cl(A)) \quad .$$

Definition 2.2

A subset A of a bitopological space X is called

(i) (1,2)*-generalized closed (briefly, (1,2)*-g-closed) [15] if $\tau_{1,2} - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2} - open$ in X.

(ii) (1,2)*-semi-generalized closed (briefly, (1,2)*-sg-closed) [13] if (1,2)*- $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-semi-open in X.

(iii) ((1,2)*-generalized semi-closed (briefly, (1,2)*-gs-closed) [15] if (1,2)* $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2} - open$ in X.

(iv) (iv) (1,2)*-ĝ-closed [9] if $\tau_{1,2} - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-semiopen in X.

(v) (v) (1,2)*-αg-closed [9] if (1,2)*-αcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2} - open$ in X.

The complements of the above mentioned closed sets are called their respective open sets.

(vi) (1,2)*-αĝ-closed [9] or $\tilde{g}$ (1,2)*-closed if (1,2)*-αcl $(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-ĝ-open in X.

Definition 2.3

A function f : (X, τ1, τ2) → (Y, σ1, σ2) is called (1,2)*-g-open [15] (resp.(1,2)*-ĝ-open [9], (1,2)*-open [15], (1,2)*-sg-open [13], (1,2)*-gs-open [15],(1,2)*-α-open [9], (1,2)*-αg-open [9], (1,2)*-αĝ-open [9]) if the image of every

$\tau_{1,2}-open$ set in X is (1,2)*-g-open (resp. (1,2)*-ĝ-open, $\sigma_{1,2}-open$, (1,2)*-sg-open,(1,2)*-gs-open, (1,2)*-α-open, (1,2)*-αg-open, (1,2)*-αĝ-open) in Y.

Definition 2.4

A function f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) is called

(i) (1,2)*-g-continuous [15] if f^{-1}(V) is (1,2)*-g-closed in X, for every $\sigma_{1,2}-closed$ set V of Y.

(ii) (1,2)*-sg-continuous [13] if f^{-1} (V) is (1,2)*-sg-closed in X, for every $\sigma_{1,2}-closed$ set V of Y.

(iii) (1,2)*-gs-continuous [15] if f^{-1} (V) is (1,2)*-gs-closed in X, for every $\sigma_{1,2}-closed$ set V of Y.

(iv) (1,2)*-ĝ-continuous [9] if f^{-1} (V) is (1,2)*-ĝ-closed in X, for every $\sigma_{1,2}-closed$ set V of Y.

(v) (1,2)*-continuous [9] if f^{-1} (V) is $\sigma_{1,2}-closed$ in X, for every $\sigma_{1,2}-closed$ set V of Y.

Definition 2.5

A function f : (X, τ1, τ2) → (Y, σ1, σ2) is called

(i) (1,2)*-g-homeomorphism if f is bijection, (1,2)*-g-open and (1,2)*-gcontinuous.

(ii) (1,2)*-sg-homeomorphism if f is bijection, (1,2)*-sg-open and (1,2)*-sgcontinuous.

(iii) (1,2)*-gs-homeomorphism if f is bijection, (1,2)*-gs-open and (1,2)*-gscontinuous.

(iv) (1,2)*-homeomorphism if f is bijection, (1,2)*-open and (1,2)*-continuous.

Definition 2.6

A function f : (X, τ1, τ2) → (Y, σ1, σ2) is called

(i) (1,2)*- α -continuous if f^{-1} (V) is (1,2)*- $\alpha-open$ in X, for every $\sigma_{1,2}-open$ set V of Y.

(ii) $\tilde{g}$ (1,2)*-continuous if f^{-1} (V) is $\tilde{g}$ (1,2)*-closed in X, for every $\sigma_{1,2}-closed$ set V of Y.

(iii) $\tilde{g}$ (1,2)*-irresolute if f^{-1} (V) is $\tilde{g}$ (1,2)*-closed in X, for every $\tilde{g}$ (1,2)*-closed set V of Y.

Definition 2.7

A function f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) is calledpre-(1,2)*-α-closed (resp. pre (1,2)*-α-open) if the image of every (1,2)*-α-closed (resp. (1,2)*-α-open) in X is (1,2)*-α-closed (resp. (1,2)*-α-open)in Y.

(i) (1,2)*-α-irresolute if f-1(V) is (1,2)*-α-open in X, for every (1,2)*-α-open set V of Y.

(ii) (1,2)*-gc-irresolute if f-1(V) is (1,2)*-g-closed in X, for every (1,2)*-g-closed set V of Y.

a. (iv) (1,2)*-α-homeomorphism if f is bijection, (1,2)*-α-irresolute and pre-(1,2)*-α-closed.

Remark 2.8

(i) Every (1,2)*-α-closed set is $\tilde{g}$ (1,2)*-closed but not conversely.

(ii) Every $\tilde{g}$ (1,2)*-open set is (1,2)*-gs-open but not conversely.

III. $\tilde{g}$ (1,2)*-HOMEOMORPHISMS

Definition 3.1

A bijective function f : (X,τ1, τ $\rightarrow$ (Y, σ1, σ2) is called a strongly $\tilde{g}$ (1,2)*-closed (resp. strongly $\tilde{g}$ (1,2)*-open) if the image of every $\tilde{g}$ (1,2)*-closed (resp. $\tilde{g}$ (1,2)*-open) set in X is $\tilde{g}$ (1,2)*-closed (resp. $\tilde{g}$ (1,2)*-open) of Y.

A bijective function f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) is called an $\tilde{g}$ (1,2)*-homeomorphism if f is both $\tilde{g}$ (1,2)*-open and $\tilde{g}$ (1,2)*-continuous.

Theorem 3.2

Every (1,2)*-homeomorphism is $\tilde{g}$ (1,2)*-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be (1,2)*-homeomorphism. Then f is bijective,(1,2)*-open and (1,2)*-continuous function. Let U be an $\tau_{1,2}-open$ set in X. Since f is (1,2)*-open function, f(U) is an $\sigma_{1,2}-open$ set in Y. Every $\tau_{1,2}-open$ set is $\tilde{g}$ (1,2)*-open and hence f(U) is $\tilde{g}$ (1,2)*-open in Y. This implies f is $\tilde{g}$ (1,2)*-open. Let V be a $\sigma_{1,2}-closed$ set in Y. Since f is (1,2)*-continuous, f^{-1}(V) is $\tau_{1,2}-closed$ in X. Thus f^{-1}(V) is $\tilde{g}$ (1,2)*-closed in X and therefore, f is $\tilde{g}$ (1,2)*-continuous. Hence, f is an $\tilde{g}$ (1,2)*-homeomorphism.

Remark 3.3

The converse of Theorem 3.2 need not be true as shown in the following example.

Example 3.4

Let X = {a, b, c}, τ_1 = {ϕ, X} and □2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {c}} are called $\tau_{1,2}-closed$ .Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in X. Let Y = {a, b, c}, σ1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}} are called $\sigma_{1,2}-open$ and the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y,{a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in Y. Let f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) be the identity function. Then f is a $\tilde{g}$ (1,2)*-homeomorphism but f is not a (1,2)*-homeomorphism.

Proposition 3.5

For any bijective function f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) the following statements are equivalent.

(i) F^{-1} : (Y, σ1, σ2) $\rightarrow$ (X, τ1, τ2) is $\tilde{g}$ (1,2)*-continuous function.
(ii) f is a $\tilde{g}$ (1,2)*-open function.
(iii) f is a $\tilde{g}$ (1,2)*-closed function.

Proof

(i) ⇒ (ii): Let U be an $\tau_{1,2}-open$ set in X. Then X - U is $\tau_{1,2}-closed$ in X. Since f^{-1} is $\tilde{g}$ (1,2)*-continuous, $(f^{-1})^{-1}$(X - U) is $\tilde{g}$ (1,2)*-closed in Y. That is f(X - U) = Y -f(U) is $\tilde{g}$ (1,2)*-closed in Y. This implies f(U) is $\tilde{g}$ (1,2)*-open in Y. Hence f is $\tilde{g}$ (1,2)*-open function.

(ii) ⇒ (iii): Let F be a $\tau_{1,2}-closed$ set in X. Then X - F is $\tau_{1,2}-open$ in X.Since f is $\tilde{g}$ (1,2)*- open, f(X - F) is $\tilde{g}$ (1,2)*-open set in Y. That is Y - f(F) is $\tilde{g}$ (1,2)*-open in Y. This implies that f(F) is $\tilde{g}$ (1,2)*- closed in Y. Hence f is $\tilde{g}$ (1,2)*-closed.

(iii) ⇒ (i): Let V be a $\tau_{1,2}-closed$ set in X. Since f is $\tilde{g}$ (1,2)*-closed function, f(V) is $\tilde{g}$ (1,2)*-closed in Y. That is (f-1)-1(V) is $\tilde{g}$ (1,2)*-closed in Y. Hence f-1 is $\tilde{g}$ (1,2)*-continuous.

Proposition 3.6

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be a bijective and $\tilde{g}$ (1,2)*-continuous function.Then the following statements are equivalent:

(i) f is a $\tilde{g}$ (1,2)*-open function.
(ii) f is a $\tilde{g}$ (1,2)*-homeomorphism.
(iii) f is a $\tilde{g}$ (1,2)*-closed function.

Proof

(i) ⇒ (ii): Let f be a $\tilde{g}$ (1,2)*-open function. By hypothesis, f is bijective and $\tilde{g}$ (1,2)*-continuous. Hence f is a $\tilde{g}$ (1,2)*-homeomorphism.

(ii) ⇒ (iii): Let f be a $\tilde{g}$ (1,2)*-homeomorphism. Then f is $\tilde{g}$ (1,2)*-open. By Proposition 3.5, f is $\tilde{g}$ (1,2)*-closed function.

(ii)⇒ (i): It is obtained from Proposition 3.5.

Theorem 3.7

Every (1,2)*-α-homeomorphism is $\tilde{g}$ (1,2)*-homeomorphism

Proof

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be a (1,2)*-α-homeomorphism. Then f is bijective, (1,2)*-α-irresolute and pre-(1,2)*-α-closed. Let F be $\tau_{1,2}-closed$ in X. Then F is (1,2)*-α-closed in X. Since f is pre-(1,2)*-α-closed, f(F) is (1,2)*-α-closed in Y. Every (1,2)*-α-closed set is $\tilde{g}$ (1,2)*-closed and hence f(F) is $\tilde{g}$ (1,2)*-closed in Y. This implies f is $\tilde{g}$ (1,2)*-closed function. Let V be a $\sigma_{1,2}-closed$ set of Y. Thus V is (1,2)*-α-closed in Y. Since f is (1,2)*-α-irresolute $f^{-1}(V)$ is (1,2)*-α-closed in X. Thus $f^{-1}(V)$ is $\tilde{g}$

(1,2)*-closed in X. Therefore f is $\tilde{g}$ (1,2)*-continuous. Hence f is a $\tilde{g}$ (1,2)*-homeomorphism.

Remark 3.8

The following Example shows that the converse of Theorem 3.7 need not be true

Example 3.9

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a}}. Then the sets in {ϕ, X,{a}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}-closed$. Also the sets in {ϕ X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in X and the sets in {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in X. Moreover, the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-α-closed in X and the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-α-open in X. Let Y = {a, b, c}, σ1 = {ϕ, Y} and σ2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called σ1,2-closed. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in {ϕ, Y, {a, b}} are called (1,2)*-α-closed in Y and the sets in {ϕ, Y, {c}} are called (1,2)*-α-open in Y. Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be the identity function. Then f is a $\tilde{g}$ (1,2)*-homeomorphism but f is not a (1,2)*-α-homeomorphism.

Remark 3.10

Next Example shows that the composition of two $\tilde{g}$ (1,2)*-homeomorphisms is not always a $\tilde{g}$ (1,2)*-homeomorphism.

Example 3.11

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, c}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {b}, {b, c}} are called $\tau_{1,2}-closed$. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in X and the sets in {ϕ, X, {a}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in X. Let Y= {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}-open$ and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in Y. Let Z = {a, b,c},η_1={ϕ, Z} and η_2 = {ϕ, Z, {a, b}}. Then the sets in {ϕ, Z, {a, b}} are called $\eta_{1,2}-open$ and the sets in {ϕ, Z, {c}} are called $\eta_{1,2}-closed$. Also the sets in {ϕ, Z, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Z and the sets in {ϕ, Z, {a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in Z. Let f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) and g : (Y, σ_1, σ_2) $\rightarrow$ (Z, η_1, η_2) be two identity functions. Then both f and g are $\tilde{g}$ (1,2)*-homeomorphisms. The set {a, c} is τ1,2-open in X, but (g o f)({a, c}) = {a, c} is not $\tilde{g}$ (1,2)*-open in Z. This implies that g o f is not $\tilde{g}$ (1,2)*-open and hence g o f is not $\tilde{g}$ (1,2)*-homeomorphism.

Theorem 3.12

Every $\tilde{g}$ (1,2)*-homeomorphism is (1,2)*-gs-homeomorphism but not conversely.

Proof

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be a $\tilde{g}$ (1,2)*-homeomorphism. Then f is a bijective, $\tilde{g}$ (1,2)*-open and $\tilde{g}$ (1,2)*-continuous function. Let U be an $\tau_{1,2}-open$ set in X. Then f(U) is $\tilde{g}$ (1,2)*-open in Y. Every $\tilde{g}$ (1,2)*-open set is (1,2)*-gs-open and hence, f(U) is (1,2)*-gs-open in Y. This implies f is (1,2)*-gs-open function. Let V be $\sigma_{1,2}-closed$ set in Y. Then $f^{-1}(V)$ is $\tilde{g}$ (1,2)*-closed in X. Hence $f^{-1}(V)$ is (1,2)*-gs closed in X. This implies f is (1,2)*-gs-continuous. Hence f is (1,2)*-gs homeomorphism.

Remark 3.13

The following Example shows that the converse of Theorem 3.12 need not be true.

Example 3.14

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}-closed$. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*- closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in X. Moreover, the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-gs-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} are called (1,2)*-gs-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Moreover, the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}-open$ and $\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {a}, {b, c}} are called $\tilde{g}$ (1,2)*-closed and $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-g-closed and (1,2)*-gopen in Y. Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*- gs-homeomorphism but f is not a $\tilde{g}$ (1,2)*-homeomorphism.

Remark 3.15

The following Examples show that the concepts of $\tilde{g}$ (1,2)*-homeomorphisms and (1,2)*-g-homeomorphisms are independent of each other.

Example 3.16

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {b}, {c}, {b, c}} are called $\tau_{1,2}-closed$. Also the sets in {ϕ, X, {b}, {c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed and (1,2)*-g-closed in X. Moreover, the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open and (1,2)*-ĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {b}, {a, b}} are called $\sigma_{1,2}-open$ and the sets in {ϕ, Y, {c}, {a, c}} are called $\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {a}, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {b, c}} are called $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-g-open in Y. Define a function f : (X, τ_1, τ_2) $\longrightarrow$ (Y, σ_1, σ_2) by f(a) = b, f(b) = a and f(c) = c. Then f is a $\tilde{g}$ (1,2)*-homeomorphism but f is not a (1,2)*-g-homeomorphism.

Example 3.17

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X,{a}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}-closed$. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed and (1,2)*-gclosed in X. Moreover, the sets in {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open and (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}-open$ and $\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {a}, {b, c}} are called $\tilde{g}$ (1,2)*-closed and $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-gs-closed and (1,2)*-gs-open in Y. Define a function f : (X, τ_1, τ_2) $\longrightarrow$ (Y, σ_1,σ_2) by f(a) = b, f(b) = c, f(c) = a. Then f is a (1,2)*-g-homeomorphism but f is not a $\tilde{g}$ (1,2)*-homeomorphism.

Remark 3.18

$\tilde{g}$ (1,2)*-homeomorphisms and (1,2)*-sg-homeomorphisms are independent of each other as shown below.

Example 3.19

The function f defined in Example 3.16 is $\tilde{g}$ (1,2)*-homeomorphism but not (1,2)*-sg-homeomorphism.

Example 3.20

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2= {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}-open$ and $\tilde{g}$ (1,2)*-open in X; the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}-closed$ and $\tilde{g}$ (1,2)*-closed in X. Also, the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-sg-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} are called (1,2)*-sg-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}-open$ and

$\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {a}, {b, c}} are called $\tilde{g}$ (1,2)*-closed and $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b},{c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-sg-closed and (1,2)*-sg-open in Y. Define a function f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) by f(a) = b, f(b) = a and f(c) = c. Then f is (1,2)*-sg-homeomorphism but not $\tilde{g}$ (1,2)*-homeomorphism.

IV. STRONGLY $\tilde{g}$ (1,2)*-HOMEOMORPHISMS

Definition 4.1

A bijection f : (X, τ_1, $\tau_2 \rightarrow$ (Y, σ_1,σ_2) is said to be strongly $\tilde{g}$ (1,2)*-homeomorphism if f is $\tilde{g}$ (1,2)*-irresolute and its inverse f^{-1} is also $\tilde{g}$ (1,2)*-irresolute.

Theorem 4.2

Every strongly $\tilde{g}$ (1,2)*-homeomorphism is $\tilde{g}$ (1,2)*-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be strongly $\tilde{g}$ (1,2)*-homeomorphism. Let U be $\tau_{1,2}-open$ in X. Then U is $\tilde{g}$ (1,2)*-open in X. Since f^{-1} is $\tilde{g}$ (1,2)*-irresolute, $(f^{-1})^{-1}$(U) is $\tilde{g}$ (1,2)*-open in Y. That is f(U) is $\tilde{g}$ (1,2)*-open in Y. This implies f is $\tilde{g}$ (1,2)*-open function. Let F be a $\sigma_{1,2}-closed$ in Y. Then F is $\tilde{g}$ (1,2)*-closed in Y. Since f is $\tilde{g}$ (1,2)*-irresolute, f-1(F) is $\tilde{g}$ (1,2)*-closed in X. This implies f is $\tilde{g}$ (1,2)*-continuous function. Hence f is $\tilde{g}$ (1,2)*-homeomorphism.

Remark 4.3

The following Example shows that the converse of Theorem 4.2 need not be true.

Example 4.4

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, c}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {b}, {b, c}} are called $\tau_{1,2}-closed$. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in X and the sets in {ϕ, X, {a}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in X. Let Y = {a, b, c}, σ1 = {ϕ, Y, {a}} and σ2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}-open$ and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}-closed$. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in Y. Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2)be the identity function. Then f is a $\tilde{g}$ (1,2)*-homeomorphism but f is not a strongly $\tilde{g}$ (1,2)*-homeomorphism.

Theorem 4.5

The composition of two strongly $\tilde{g}$ (1,2)*-homeomorphisms is a strongly $\tilde{g}$ (1,2)*-Homeomorphism.

Proof

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) and g : (Y, σ1, σ2) $\rightarrow$ (Z, η1, η2) be two strongly $\tilde{g}$ (1,2)*-homeomorphisms. Let F be a $\tilde{g}$ (1,2)*-closed set in Z. Since g is $\tilde{g}$ (1,2)*-irresolute, g^{-1}(F) is $\tilde{g}$ (1,2)*-closed in Y. Since f is a $\tilde{g}$ (1,2)*-irresolute, $f^{-1}(g^{-1}(F))$ is $\tilde{g}$ (1,2)*-closed in X. That is (g o f)-1(F) is $\tilde{g}$ (1,2)*-closed in X. This implies that g of : (X, τ1, τ2) $\rightarrow$ (Z, η1, η2) is $\tilde{g}$ (1,2)*-irresolute. Let V be a $\tilde{g}$ (1,2)*-closed in X.Since f-1 is a $\tilde{g}$ (1,2)*-irresolute, $(f^{-1})^{-1}$(V) is $\tilde{g}$ (1,2)*-closed in Y. That is f(V) is $\tilde{g}$ (1,2)*-closed in Y. Since g^{-1} is a $\tilde{g}$ (1,2)*-irresolute, (g-1)-1(f(V)) is $\tilde{g}$ (1,2)*-closed in Z. That is g(f(V)) is $\tilde{g}$ (1,2)*-closed in Z. So, (g o f)(V) is $\tilde{g}$ (1,2)*-c losed in Z.This implies that ((g o f)-1)-1(V) is $\tilde{g}$ (1,2)*-closed in Z. This shows th at $(g \circ f)^{-1}$: (Z, η1, η2) $\rightarrow$ (X, τ1, τ2) is $\tilde{g}$ (1,2)*-irresolute. Hence g o f is a strongly $\tilde{g}$ (1,2)*- homeomorphism.We denote the family of all strongly $\tilde{g}$ (1,2)*-homeomorphisms from a bitopological space (X, τ1, τ2) onto itself by $s\tilde{g}$ (1,2)* $\rightarrow$h(X).

Theorem 4.6

The set $s\tilde{g}$ (1,2)*-h(X) is a group under composition of functions.

Proof

By Theorem 4.5, g o fε$s\tilde{g}$ (1,2)*-h(X) for all f, g ε $s\tilde{g}$ (1,2)*-h(X). We know that the composition of functions is associative. The identity function belonging to $s\tilde{g}$ (1,2)*-h(X) serves as the identity element. If f ε $s\tilde{g}$ (1,2)*-h(X), then f-1 ε $s\tilde{g}$ (1,2)*-h(X) such that f o f^{-1} = f^{-1} o f = I and so inverse exists for each element of $s\tilde{g}$ (1,2)*-h(X). Hence $s\tilde{g}$ (1,2)*-h(X) is a group under the composition of functions.

Theorem 4.7

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be a strongly $\tilde{g}$ (1,2)*-homeomorphism. Then f induces an (1,2)*-isomorphism from the group $s\tilde{g}$ (1,2)*-h(X) onto the group $s\tilde{g}$ (1,2)*-h(Y).

Proof

Using the function f, we define a function f : $s\tilde{g}$ (1,2)*-h(X) $\rightarrow$ $s\tilde{g}$ (1,2)*-h(Y) by θ_f(k) = f o k o f^{-1} for every k ε $s\tilde{g}$ (1,2)*-h(X). Then θ_F is a bijection. Further, for all k1, k2 ε $s\tilde{g}$ (1,2)*-h(X), $\rightarrow$f(k1 o k2) = f o (k1 o k2) o f^{-1} = (f o k1 o f^{-1}) o(f o k2 o f-1) = θ_f (k1) o θ_f (k2). Therefore θ_f is an (1,2)*-isomorphism induced by f.

Remark 4.8

The concepts of strongly $\tilde{g}$ (1,2)*-homeomorphisms and (1,2)*-α-homeomorphisms are independent notions as shown in the following examples.

Example 4.9

Let X = {a, b, c}, τ_1 = {ϕ, X, } and τ_2 = {ϕ, X, {a, b}} are called $\tau_{1,2}-open$ and the sets in {ϕ, X, {a, b},} are called $\tau_{1,2}-closed$. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*- closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in X. Let Y = {a, b, c}, σ1 = {ϕ, Y, {a}} and σ2 = {ϕ, Y, {b}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}} are called σ1,2-open and (1,2)*-α-open; and the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed and (1,2)*-α-closed in Y. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y,{a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open in Y. Let f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) be the identity function. Then f is a strongly $\tilde{g}$ (1,2)*-homeomorphism but f is not (1,2)*-α-homeomorphism.

Example 4.10

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Then the sets in {ϕ,X, {a}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in X. Moreover, the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-α-closed in X and then sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-α-open in X. Let Y = {a, b, c}, σ1 = {ϕ, Y} and σ2 = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called σ1,2-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in {ϕ, Y, {b}, {c},{b, c}} are called (1,2)*-α-closed in Y and the sets in{ϕ ,Y,{a},{a, b}, {a, c}} are called (1,2)*-α-open in Y. Let f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) be the identity function. Then f is a (1,2)*-α-homeomorphism but not strongly $\tilde{g}$ (1,2)*-homeomorphism.

Definition 4.11

A bijective function f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) is called (1,2)*-gchomeomorphism if f is (1,2)*-gc-irresolute and f-1 is (1,2)*-gc-irresolute.

Remark 4.12

The concepts of strongly $\tilde{g}$ (1,2)*-homeomorphisms and (1,2)*-gchomeomorphisms are independent of each other as the following examples show.

Example 4.13

Let X = {a, b, c}, τ_1 = { ϕ, X, {a}} and τ_2 = { ϕ, X, {a, b}}. Then the sets in { ϕ, X, {a}, {a, b}} are called $\tau_{1,2}$-open and the sets in { ϕ, X, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in { ϕ, X, {b}, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in X and the sets in { ϕ, X, {a}, {b}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in X. Moreover, the sets in { ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in X and the sets in { ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-g-open in X. Let Y = {a, b, c},σ1 = { ϕ, Y, {b}, {a, b}} and σ2 = { ϕ, Y, {a}, {a, c}}. Then the sets in { ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called $\sigma_{1,2}$-open and the sets in { ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called σ1,2-closed. Also the sets in { ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*- closed and (1,2)*-g-closed in Y and the sets in { ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open and (1,2)*-g-open in Y. Let f : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) be the identity function. Then f is a strongly $\tilde{g}$ (1,2)*-homeomorphism but not (1,2)*-gchomeomorphism.

Example 4.14

Let X = {a, b, c}, τ_1 = { ϕ, X, {a}} and τ_2 = { ϕ, X, {b}}. Then the sets in { ϕ,X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in { ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in { ϕ, X, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*- closed and (1,2)*-g-closed in X, and the sets in { ϕ , X, {a}, {b}, {a, b}} are called $\tilde{g}$ (1,2)*-open and (1,2)*-g-open in X. Let Y = {a, b, c}, σ1 = { ϕ, Y, {a}} and σ2 = { ϕ, Y, {a, b}}. Then the sets in { ϕ, Y, {a}, {a, b}} are called σ1,2-open and the sets in { ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in { ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\tilde{g}$ (1,2)*-closed in Y and the sets in { ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called $\tilde{g}$ (1,2)*-open in Y. Moreover, the sets in { ϕ, Y, {c}, {a, c}, {b,c}} are called (1,2)*-g-closed in Y and the sets in { ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-g-open in Y. Letf : (X, τ1, τ2) $\rightarrow$ (Y, σ1, σ2) be the identity function. Then f is a (1,2)*-gc-homeomorphism but not strongly $\tilde{g}$ (1,2)*-homeomorphism.

V. CONCLUSION

Topology as a branch of mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformation or homeomorphisms. Topology operates with more general concepts that anlaysis. Differential properties of a given transformation are non essential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of pproblems to which anlaysis cannot give the answer. In this paper I introduced $\tilde{g}(1,2)*$ -closed maps, $\tilde{g}(1,2)*$-open maps, $\tilde{g}(1,2)**$-closed maps and $\tilde{g}(1,2)**$ –open maps in bitopological spaces and obtain certain characterization of these classes of maps.

REFERENCES

[1] Dharmalingam.K.M,Thamilisai.A,Ravi.O, $\tilde{g}(1,2)^*$ -Closed and $\tilde{g}(1,2)^*$ -Open sets in Bitopological spaces,

[2] Thamilisai.A, $\tilde{g}(1,2)^*$ -Closed and $\tilde{g}(1,2)^*$ -Open maps in Bitopological spaces,

[3] Maki, H., Devi, R. and Balachandran, K.: Associated topologies of generalized α -closed sets and α -generalized closed sets, Mem. Fac. Sci.Kochi. Univ. Ser. A. Math., 15 (1994), 51-15.

[4] Veera kumar, M. K. R. S.: ĝ-closed sets in topological spaces, Bull. Allah. Math. Soc., 18 (2003), 99-112.

[5] Zbigniew Duszynski, Rose Mary, S. and Lellis Thivagar, M.: Remarks on αĝhomeomorphisms,
   a. Math. Maced, 7 (2009), 13-20.

[6] Devi, R., Balachandran, K. and Maki, H.: Semi-generalized closed maps and generalized semi-closed maps, Mem. Fac. Kochi Univ. Ser. A. Math., 14 (1993), 41-54.

[7] Antony Rex Rodrigo, J., Ravi, O., Pandi, A. and Santhana, C. M.: On (1,2)*-s-normal spaces and pre-(1, 2)*-gs-closed functions, International Journal of Algorithms, Computing and Mathematics, 4(1) (2011), 29-42.

[8] Duszynski, Z., Jeyaraman, M., Sajan Joseph, M., Lellis Thivagar, M. and Ravi, O.: A new generalization of closed sets in bitopology, South Asian Journal of Mathematics, 4 (5) (2014), 215-224.

[9] Jafari, S., Lellis Thivagar, M. and Nirmala Mariappan.: On (1,2)*-αĝ-closed sets, J. Adv. Math. Studies, 2(2) (2009), 25-34.

[10] Kayathri, K., Ravi, O., Thivagar, M. L. and Joseph Israel, M.: Decompositions of (1,2)*-rg-continuous maps in bitopological spaces, Antarctica J. Math., 6(1) (2009), 13-23.

[11] Lellis Thivagar, M., Ravi, O. and Abd El-Monsef, M. E.: Remarks on bitopological (1,2)*-quotient mappings, J. Egypt Math. Soc., 16(1) (2008),17-25.

[12] Ravi, O., Thivagar, M. L. and Hatir, E.: Decomposition of (1,2)*-continuity and (1,2)*- α -continuity, Miskolc Mathematical Notes., 10(2) (2009),115-171.

[13] Ravi, O., Kamaraj, M., Pandi, A. and Kumaresan, K.: (1,2)*- g -closed and (1,2)*- g -open maps in bitopological spaces, International Journal of Mathematical Archive, 3(2) (2012), 586-594.

[14] Ravi, O., Ekici, E. and Lellis Thivagar, M.: On (1,2)*-sets and decompositions of bitopological (1,2)*-continuous mappings, Kochi J. Math., 3 (2008),181-189.

[15] Ravi, O., Kayathri, K., Thivagar, M. L. nd Joseph Israel, M.: Mildly (1,2)*-normal spaces and some bitopological functions, Mathematical Bohemica,135(1) (2010), 1-1