Deterministic Inventory Lot-Size Models with Demand-Dependent on Unit Cost and Varying Holding Cost: A Karush-Kuhn Tucker Conditions Approach

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Abstract- *A multi item inventory model with decreasing holding cost, subject to linear and non-linear constraints is considered in this paper. The varying holding cost is considered to be a continuous function of production quantity. A demand dependent on unit cost has been considered and this unit cost is taken in fuzzy environment. The model is solved using Karush-Kuhn Tucker conditions method. The model is illustrated with a numerical example.*

Keywords- Inventory, Economic production quantity, varying holding cost, number of orders, rate of production, KKT, Membership function, demand dependent on unit cost, Fuzzy unit cost.

I. INTRODUCTION

The constrained multi-item inventory model had been treated by many researchers. Maloney and Klein[9] discussed constrained multi-item inventory systems: an implicit approach using algorithms method. Abou-El-Ata & Kotb [1] developed a crisp inventory model under two restrictions. The pioneer work began by Cheng[3] , who studied an EOQ model with demand- dependent unit cost of single – item using geometric programming approach. Other related studies were written by Juneau and Coates [6] , Jung and Klein [7] , Teng and Yang[17] and Mandal et.al.[10].

Silver [1985] designed the classical inventory problems by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature. But in practical situation unit price and demand rate of an item may be related to each other. When the demand of an item is high, an item is produced in large numbers fixed costs of production are spread over a large number of items. Hence the unit price of an item inversely related to the demand of that item.

Zadeh [1965] first gave the concept of fuzzy set theory. Later on, Bellman and Zadeh [1970] used the fuzzy set theory to the decision-making problem. Tanaka [1974] introduced the objectives as fuzzy goals over the α-cut of a fuzzy constraint set and Zimmermann [1976] gave the concept to solve multi-objective linear programming problem. Park [1987] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Hence we may impose warehouse space, cost parameters, number of orders, production cost etc, in fuzzy environment.

Nirmal Kumar Mandal, et.al.,(2005) formulated Multi-Objective fuzzy inventory model with three constraints and solved by using geometric programming method. Manas Kumar Maiti, Manoranjan Maiti(2006) formulated the fuzzy inventory model with two warehouses one in the heart of the market place and other slightly away from the market place.

The Kuhn-Tucker conditions[14] are necessary conditions for identifying stationary points of a non-linear constrained problem subject to inequality constraints. The development of this method is based on the Lagrangean method. These conditions are also sufficient if the objective function and the solution space satisfy the conditions in the following table 1.1.

Table 1.1

Sense of optimization	Required conditions	
	Objective function	Solution space
Maximization	Concave	Convex Set
Minimization	Convex	Convex Set

The conditions for establishing the sufficiency of the Kuhn-Tucker Conditions [5] are summarized in the following table 1.2

Table 1.2

Problem	Kuhn-Tucker conditions
1.Max z = f(X) subject to $h^i(X) \le 0$ $X \ge 0$, i = 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \le 0, i = 1,2,....m$ $\lambda_i \ge 0, i = 1,2,......m$
2.Min z = f(X) subject to $h^i(X) \ge 0$ $X \ge 0$, i= 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \ge 0, i = 1,2,....m$ $\lambda_i \ge 0, i = 1,2,......m$

In this paper the n-item inventory model with decreasing varying holding cost under available limited storage space and total investment cost are considered. The objective is to minimize the total cost function based on the values of demand rate, rate of production, unit cost, inventory carrying cost and order quantity over a long period of time. The unit cost is considered here in fuzzy environment. The model is illustrated by numerical example.

II. ASSUMPTIONS AND NOTATIONS

A multi-item inventory model without shortages is developed under the following notations and assumptions.

Notations

n = number of items
W = Floor or shelf space available
B = Total investment cost for replenishment
For i^{th} item: (i= 1,2,………..n)
$D_i = D_i(p_i)$ demand rate (function of unit cost price)
Q_i = lot size (decision variable)
S_i = Set-up cost per cycle
$H_i(Q_i)$ = varying holding cost for the i^{th} item of inventory
p_i = price per unit item
d_i = rate of production per unit item
w_i = storage space per item
$TC(Q_i, p_i)$ = expected annual total cost

Assumptions

In addition , the following basic assumption are adopted for developing the mathematical model of the inventory problem.

(1) Replenishment is instantaneous

(2) Shortages are not allowed

(3) The rate of production for each product is finite and constant.

(4) Demand rate is related to the unit price as

$$D_i = A_i\ p_i^{-\beta_i} \quad , (i = 1,2,..n)$$

Where Ai (>0) and β_i ($0<\beta_i<1$) are constants and real numbers selected to provide the best fit of the estimated price function. $A_i>0$ is an obvious condition since both D_i and p_i must be non-negative.

(5) lead time is zero

(6) The holding cost for i^{th} item is a decreasing continuous function of the production quantity (Q_i) and takes the form:

$$H_i(Q_i) = a + bQ_i^{-1} \quad , \qquad i = 1,2,3,…,n$$

Where a (>0) and b (>0) are real constants selected to provide the best fit of the estimated cost function. Since $H_i(Q_i)$ must be non-negative.

III. FORMULATION OF INVENTORY MODEL

The objective of the inventory model is to minimize the annual relevant total cost (i.e the sum of production, setup and inventory carrying costs) which according to the basic inventory lot-size model is,

Total cost = Production cost + Set up cost + Holding cost
Total average cost of the i^{th} item is

$$MinTC(p_i, Q_i) = p_i D_i + \frac{S_i D_i}{Q_i} + \frac{1}{2}\left[1 - \frac{D_i}{d_i}\right] Q_i H_i(Q_i)$$

--------------(3.1)

Substituting D_i & $H_i(Q_i)$ in (3.1) yields:

$$MinTC(p_i, Q_i) = A_i p_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{1}{2}\left[1 - \frac{A_i p_i^{-\beta_i}}{d_i}\right] Q_i [a + bQ_i^{-1}]$$

for i=1,2,3,………n

$$MinTC(p_i, Q_i) = \sum_{i=1}^{n}\left[A_i p_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{1}{2}\left[1 - \frac{A_i p_i^{-\beta_i}}{d_i}\right] Q_i [a + bQ_i^{-1}]\right]$$

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

(i) There is a limitation on the available warehouse floor space where the items are to be stored

$$\sum_{i=1}^{n} w_i Q_i \le W$$

(ii) Investment amount on total production cost cannot be infinite, it may have an upper limit on the maximum investment

$$\sum_{i=1}^{n} p_i Q_i \le B \qquad \text{where } p_i, Q_i > 0 \ (i=1,2\ldots.n)$$

Membership function

Here the unit price p_i is taken under fuzzy environment.

The membership function for the triangular fuzzy variable p_i is defined as follows

$$\mu_{p_i}(X) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

Numerical example

The model is illustrated for one item (i=1) and also the common parametric values assumed for the given model are

For i = 1,
A_1 = 10000, S_1 = \$10000, w_1 = 200sq.ft, W = 60000sq.ft , B = \$120000,
d_1 = 30000 units and p_1 = (220,260,300) , $\alpha_1 = 1$

The proposed model is solved by Karush-Kuhn-Tucker conditions and the optimal results are presented in the table 5.1

Table 5.1
The optimum solution table:

b	β_1	p_1	Q_1	D_1	H_1	μ_{p1} value	Expected Total cost
0.10	0.87	223.04	300	90.55	1.003	0.076	23365.19
	0.88	244.41	300	79.15	1.003	0.61	22133.26
	0.89	269.66	300	68.64	1.003	0.758	20946.98
	0.90	299.95	300	58.97	1.003	.0013	19804.60

From the above table it follows that as the parametric value β increases, the unit cost price also increases where as the lot size and the holding cost remains constant. But the value of the min total cost decreases. By fuzzifying the unit cost it follows that the optimal solution corresponds to the membership function value 0.758. Hence the optimal solution is
Q1 = 300 , p1 = 269.66, D1 = 68.64, H1 = 1.003,Min TC=20946.98

Table 5.2
The optimum solution table:

β_1	b	p_1	Q_1	D_1	H_1	μ_{P1} value	Expected Total cost
0.87	0.10	223.04	300	90.55	1.003	0.076	23365.19
	1.0	223.04	300	90.55	1.0033	0.076	23365.64
	12.0	223.04	300	90.55	1.04	0.076	23371.13
	50.0	223.04	300	90.55	1.1667	0.076	23390.07

From the above table it follows that as the parameteric value b increases, the holding cost alone increases whereas all other values namely the lot size, unit price and membership function remains constant. Also the minimum total cost increases and the optimum solution is given by

Q_1 = 300 , p_1 = 223.04, D_1 = 90.55, H_1 = 1.003,Min TC=23365.19

IV. CONCLUSION

This paper is dedicated to solve a multi-item economic production run size inventory model with decreasing varying holding cost. The varying holding cost is considered to be a continuous function of production quantity. The model is solved using Karush-Kuhn-Tucker conditions method. The optimal order quantity and the unit price is evaluated and the minimum annual total cost is deduced. The model is solved by varying the parametes β and b and the optimal solution is deduced by fuzzifying the unit cost. This work can be extended by varying the constraints like limited set up cost, budgetary etc.

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