Multi Item Multi Objective Demand Dependent Unit Cost Under Fuzzy Environment Using Karush Kuhn Tucker Conditions

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Abstract- *In this paper a mathematical model of inventory control problem with shortages for determining the minimum total cost of multi-item multi-objective fuzzy inventory model has been formulated with three constraints. The three constraints are warehouse space constraint, investment amount constraint and the third one is the percentage of utilization of volume of the warehouse space. Warehouse maintenance is one of the essential part of service operation. The warehouse space in the selling stores plays an important role in stocking the goods. In this paper, the warehouse space in the selling store is considered in volume. The unit cost dependent demand is assumed and Karush-Kuhn-Tucker condition is used to solve the model. The cost parameters are imposed here in fuzzy environment and illustrated with numerical example.*

Keywords- Inventory, Membership Function, Karush -Kuhn-Tucker Condition, volume of the warehouse.

I. INTRODUCTION

The literal meaning of inventory is the stock of goods for future use(production/sales). The control of inventories of physical goods is a problem common to all enterprises in any sector of an economy. The basic objectives of inventory control is to reduce investment in inventories and ensuring that production process does not suffer at the same time. The different types of cost (purchasing cost, set up cost, holding cost, etc) involved in inventory problems affect the efficiency of an inventory control problem.

Warehouse space available in the selling store plays an important role in inventory model. Warehouse space can be considered in terms of area and /or volume. Here the warehouse space in the selling store is considered in volume.

In general, the demand rate of an item is considered to be constant and deterministic and the unit price of an item is considered to be constant and independent in nature, in the case of a classical inventory problems. In practical, unit price and demand rate of an item may be related to each other. When the demand is high, it is produced in large number so that the unit cost of the item decreases (i.e) the unit price of an item inversely relates to the demand of that item. So demand rate of an item may be considered as a decision variable.

Zadeh [1] first gave the concept of fuzzy set theory. Later on, Bellman and Zadeh [2] used the fuzzy set theory to the decision-making problem. The concept of solving a multi objective linear programming problem is introduced by Zimmerman[3].Now the fuzzy set theory has made an entry into the inventory control systems. Sommer [4] applied the fuzzy concept to an inventory and production scheduling problem. Park[5] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Nirmal Kumar Mandal[6] formulated a multi objective fuzzy inventory model with three constraints and solved by using Geometric programming method. In all the above mentioned articles, the warehouse space available in the selling store is taken in terms of area. If the warehouse space is taken in terms of volume then less percentage of volume of the warehouse space will be consumed. consequently, the maximum of the volume of the warehouse space can be utilized effectively.

In this paper a multi-item multi objective fuzzy inventory model with shortages is developed under three constraints. Here the volume of the unit items are taken for calculations. The unit cost is considered here in fuzzy environment. The demand, lot size and shortage level are considered here as decision variables.

The Karush -Kuhn-Tucker Condition[7]are necessary conditions for identifying stationary points of a non linear constrained problem subject to constraints. The development of this method is based on the Lagrangean method. These conditions are also sufficient if the objective function and solution space satisfy the conditions in the following table 1.1

Table 1.1

Sense of optimization	Required conditions	
	Objective function	Solution space
Maximization	Concave	Convex Set
Minimization	Convex	Convex Set

The conditions for establishing the sufficiency of the Kuhn-Tucker Conditions[8] are summarized in the following table 1.2

Table 1.2

Problem	Kuhn-Tucker conditions
1.Max z = f(X) subject to $h^i(X) \leq 0$ $X \geq 0$, i = 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \leq 0, i = 1,2,\ldots m$ $\lambda_i \geq 0, i = 1,2,\ldots\ldots m$
2.Min z = f(X) subject to $h^i(X) \geq 0$ $X \geq 0$, i= 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \geq 0, i = 1,2,\ldots m$ $\lambda_i \geq 0, i = 1,2,\ldots\ldots m$

Kuhn-Tucker Conditions also known as Karush Kuhn-Tucker (KKT) Conditions was first developed by W.Karush in 1939 as part of his M.S thesis at the university of chicago. The same conditions were developed independently in 1951 by W.Kuhn and A.Tucker.

II. ASSUMPTIONS AND NOTATIONS

A multi-item, multi-objective inventory model with shortages is developed under the following notations and assumptions.

Notations

n = number of items
W = Floor space available
B = Totel investment cost for replenishment
l = inside length of the warehouse
b = inside breadth of the warehouse
h = maximum height of the shelf
V = Volume of the warehouse space
For i^{th} item: (i= 1,2,………..n)
$D_i = D_i(p_i)$ demand rate [function of unit cost price]
Q_i = lot size (decision variable)
M_i = Shortage level (decision variable)
S_i = Set-up cost per cycle
H_i = Inventory holding cost per unit item
m_i = Shortage cost per unit item
p_i = price per unit item (fuzzy decision variable)
w_i = storage space per item
l_i = Length of the unit item i
b_i = breadth of the unit item i
h_i = height of the unit item i
v_i = Volume of the unit item i
V_w = Percentage of utilization of volume of the warehouse.
TC(p,Q,M) = expected annual total cost

Assumptions

(i) Replenishment is instantaneous
(ii) lead time is zero
(iii) Unit price is related to the demand as

$$p_i = A_i^{\beta_i} D_i^{-\beta_i}$$

Where Ai(>0) and β_i(β_i>1) are constants and real numbers selected to provide the best fit of the estimated price function. A_i>0 is an obvious condition since both D_i and p_i must be non-negative.

Volume of the unit item is defined by $v_i = l_i \times b_i \times h_i$

To calculate the volume of the warehouse space, multiply the lengths of the dimensions of the inside of the warehouse, that is, multiply the inside length, inside breadth and maximum shelf height.

i.e., Volume = l x b x h

III. FORMULATION OF INVENTORY MODEL WITH SHORTAGES

The Total cost = Production cost + Set up cost + Holding cost + Shortage cost

$$TC(p_i, Q_i, M_i) = p_i D_i + \frac{S_i D_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i}$$

$$TC(D_i, Q_i, M_i) = A_i^{\beta_i} D_i^{1-\beta_i} + \frac{S_i D_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i}$$

for i=1,2,3,………n

$$MinTC(D_i, Q_i, M_i) = \sum_{i=1}^{n} \left[A_i^{\beta_i} D_i^{1-\beta_i} + \frac{S_i D_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i} \right]$$

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

(i) The limitation on the available warehouse space in the store $\sum_{i=1}^{n} v_i Q_i \leq V$

(ii) The upper limit of the total amount investment

$$\sum_{i=1}^{n} A_i^{\beta_i} D_i^{-\beta_i} Q_i \leq B$$

where p_i, D_i, Q_i, M_i >0 (i=1,2….n)

(iii) Percentage of utilization of volume of the warehouse

$$\frac{VXV_W}{\left(\sum_{i=1}^{n} v_i Q_i\right)100} = 1, 0 \leq V_W \leq 100$$

IV. FUZZY INVENTORY MODEL

When p_i's are fuzzy decision variables, the above crisp model is transformed in fuzzy environment.

$$MinTC\left(\tilde{p}, D, Q, M\right) = \sum_{i=1}^{n}\left[A_i^{\beta_i} D_i^{1-\beta_i} + \frac{S_i D_i}{Q_i} + \frac{H_i\left(Q_i - M_i\right)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i}\right]$$

Subject to the constraints

$$\sum_{i=1}^{n} v_i Q_i \leq V$$

$$\sum_{i=1}^{n} A_i^{\beta_i} D_i^{-\beta_i} Q_i \leq B$$

$$\frac{VXV_W}{\left(\sum_{i=1}^{n} v_i Q_i\right)100} = 1$$

and $\tilde{p}_i, D_i, Q_i, M_i > 0$ (i=1,2,3… n), $0 \leq V_W \leq 100$

[Here cap ' ~ ' denotes the fuzzification of the parameters]

V. MEMBERSHIP FUNCTION

The membership function for the fuzzy variable p_i is defined as follows

$$\mu_{p_i}(X) = \begin{cases} 1, p_i \leq L_{L_i} \\ \dfrac{U_{L_i} - p_i}{U_{L_i} - L_{L_i}}, L_{L_i} \leq p_i \leq U_{L_i} \\ 0, p_i \geq U_{L_i} \end{cases}$$

Here U_{Li} and L_{Li} are upper limit and lower limit of p_i respectively.

VI. NUMERICAL EXAMPLE

To solve the above non linear programming using Kuhn-Tucker conditions, the following values are assumed.

i = 1, A_1 = 113, S_1 = \$100, H_1 = \$1,,B = \$1400,m_1=\$1, l_1 = 2m, b_1 = 3m, h_1 = 4m,
l = 10m, b = 12m, h = 30m and \$≤ p_1 ≤\$.

From the given values v_1 = 24 cubic m and V = 3600 cubic m.

The optimal solution is given in the following table 6.1

Table 6.1

β_1	p_1	μ_{P1} value	D_1	Q_1	M_1	V_W	Expected Total cost
1.16	6.65	0.27	22.06	93.94	46.97	62.63	193.72
1.17	5.91	0.42	24.74	100	50	66.67	196.03
1.18	5.29	0.54	27.52	105.26	52.63	70.17	198.17
1.19	4.78	0.64	30.37	110.22	55.11	73.48	200.16
1.20	4.33	0.73	33.30	114.94	57.47	76.63	201.99

VII. CONCLUSION

In this paper we have proposed a concept of the optimal solution of the inventory problem with shortages with fuzzy cost price per unit item. Fuzzy set theoretic approach of solving an inventory control problem is realistic as there is nothing like fully rigid in the world. Here, a fuzzy inventory model with shortages has been calculated with three constraints, particularly volumes of the unit items are taken in the warehouse space constraint. The result reveals the minimum expected annual total cost of the inventory model and also the optimal percentage of utilization of the volume of the warehouse. It can be increased by changing the values like volume of the warehouse space, investment cost etc.

REFERENCES

[1] L.A.Zadeh "Fuzzy sets , Inform. and Control (1965) 338-353.

[2] R.E.Bellman,L.A.Zadeh,Decision –making in a fuzzy environment Management Science 17(4)(1970) B141-B164.

[3] H.J.Zimmermann , "Discription and optimization of fuzzy systems", Internat.J. General Systems 2(4) (1976) 209-215.

[4] G.Sommer, Fuzzy inventory scheduling ,in:G.Lasker(Ed.),Applied Systems and Cybernetics, Vol.VI, Academic Press. New York,1981.

[5] K.S.Park,Fuzzy set theoretic interpretation of economic order quantity, IEEE Trans. Systems Man Cybernet. 17 (6) (1987) 1082-1084.

[6] Nirmal Kumar Mandal, Et. al., "Multi-objective fuzzy inventory model with three constraints: a geometric programming approach", Fuzzy sets and systems, (150), 87- 106, 2005.

[7] H.A.Taha,Operations Research an introduction ,Prentice-Hall of India(2005)725- 728.

[8] P.K.Gupta, Man Mohan, Problems in operations Research (Methods & Solutions), S.Chand Co., (2003)609-610.

[9] G.M.Arun Prasath, "Multi-item Multi-objective fuzzy inventory model with possible constraints" ARPN Journal of Science and Technology, Vol2, 2012.

[10] S.Ranganayaki, "Multi Objective Fuzzy inventory model with storage space and Production cost constraints via Karush Kuhn-Tucker conditions " Int.J.Contemp .Math. Sciences,Vol.7,2012,No.18,865-871.