Analysis of Laminated Composite Beam Using Higher Order Shear Deformation Theory

Keyur M. Tailor[1], Dr. Sanjay K. Kulkarni[2], Dr. Ashok S. Kasnale[3]
[1, 2, 3] Department of Civil Engineering
[1, 2, 3] Dr. D. Y. Patil School of Engineering & Technology, Pune-412105, M.S., INDIA

Abstract- *In this paper, single layer lamellae composite beam is investigated, using higher order Parabolic Shear Deformation Theory (PSDT), First Order Shear Deformation Theory (FSDT) and Classical Beam Theory (CBT). To consider transverse shear deformation effect as well as transverse normal strain effect, PSDT and FSDT is taken into account. Governing equation of higher order PSDT, FSDT and CBT is derived, using principle of virtual work. Analytical investigation includes calculation of transverse displacement, axial displacement, normal bending stresses and transverse shear stresses; varying throughout the depth of beam. Thus, results obtained for different aspect ratios and various degree of orthotropy, are compared with PSDT,FSDT and CBT under thermal load condition.*

Keywords- Classical Beam Theory, First Order Shear Deformation Theory, Normal bending & transverse shear stresses, Parabolic Shear Deformation Theory, Single Layer lamellae composite beam, sinusoidal thermal load, Transverse and axial displacement.

I. INTRODUCTION

Laminated composite beam as a structural element are increasingly finding use as primary structural component, since its excellent mechanical property of high strength to weight and stiffness to weight ratio. Structural elements are often subjected to transverse mechanical load or thermal load or combination of both loads, *i.e.*, thermo-mechanical load. To address the correct response of composite laminated beams and plates, when subjected to combine loads, is of prime interest to structural analysis. By virtue of their high strength to weight ratios and because of their mechanical properties in various directions, they can be tailored as per requirements. Further, they combine a number of unique properties, including corrosion resistance, high damping, temperature resistance and low thermal coefficient of expansion. These unique properties have resulted in the expanded use of the advanced composite materials in structures subjected to severe thermal environments. These structures are usually referred as high-temperature structures.

Examples are provided by structures used are underwater or land based structural elements, high-speed aircraft, spacecraft in aerospace industries, thin-walled members of reactor vessels, turbines, as well as the structures of future supersonic and hypersonic vehicles, such as high-speed civil transport and advanced tactical fighters etc. The high velocities of such structures give rise to aerodynamic heating, which produces intense thermal stresses that reduces the strength of the aircraft structure. Or it is often subjected to moderate to severe environment or process based thermal loading causing significant thermal stress due to thermal gradient across the thickness as well as due to widely different thermal properties of adjacent laminas.

Stress fields related to temperature variation often represent a contributing factor and, in some cases, are the main causes of failure of structures. Hence, their failure mechanisms are strongly depends on local effects at layer interface where elastic modulii differ widely. Rational design of composite and sandwich beams, plates and shell requires accurate description of in-plane stress, transverse shear stresses and warping and straining of the normal to the mid-surface, which is given by shear deformation theory.

Thermal stresses in laminated plates using classical plate theory and first-order shear deformation theory subjected to single sinusoidal thermal load and thermal stresses in a laminated plate subjected to a uniformly distributed the thermal load using classical plate theory has given by Reddy [11]. Thermal stresses in crossply laminated plates subjected to a sinusoidal thermal load through the thickness of the plate using refined shear deformation theory was studied by Ghugal and Kulkarni [5]. Thermal stresses along the bonded interfaces in a multilayer beam, caused by the different thermal expansion coefficients of the layers, were investigated by Cho et al., [2]. The presented theory treated each layer as a higher-order beam-type plate with appropriate interface and surface condition.

CPT, FSDT and the cubic Third Order Theory (TOT) for laminated beams and plates under mechanical loading, have been covered in detail by Bickford [1] and Reddy [13]. Various higher order theories (HOTs) with Taylor series type expansions in the thickness direction *z* for the displacements have been developed for composite and sandwich beams (Kant et al. [8]-[9]). CPT, FSDT, TOTs and HOTs are ESL theories, in which the functional form of the displacement expansions is independent of the material properties of the layers, with the number of primary displacement unknowns independent of the number of layers.

A new efficient higher order zigzag theory has been presented (Kapuria et al., [14]) for thermal stress analysis of laminated beams under thermal loads, with modification of the third order zigzag model by inclusion of the explicit contribution of the thermal expansion coefficient α_3 in the approximation of the transverse displacement *w*. The thermal field was approximated as piecewise linear across the thickness.

E. Correra [3], evaluated the thermal response of orthotropic laminated plates on the basis of developed classical and mixed approaches (each layer is considered as a single plate for layer wise analysis, while the unknown variables are independent of the number of the constitutive layers for the ESL cases). Linear up-to fourth-order displacement and stress field cases have been implemented to derive thermomechanical governing equations. It has been found that the thickness temperature distributions T(z) have a significant influence on the accuracy of the considered theories. Stresses and displacements for orthotropic, two-layer antisymmetric, and three-layer symmetric square cross-ply laminated plates subjected to uniformly distributed nonlinear thermo-mechanical load were obtained using Trigonometric Shear Deformation Theory (TSDT) by author Ghugal and Kulkarni [6]. Thermoelastic bending analysis of laminated composite plates for pure thermal loading (Sayyad et al., [15]) and various higher order shear deformation theories on composite and isotropic beam under mechanical loading and free vibration has been studied by Sayyad A. S. et al. [16]-[17].

A review of displacement and stress based refined theories for isotropic and anisotropic laminated beams, with their merits and demerits has been presented by Ghugal Y. M. and Shimpi R. P. [4]

A study of literature indicates that the research work dealing mechanical and thermo-mechanical load has been done vastly. But, pure thermal analysis of laminated composite beams using refined trigonometric, parabolic and exponential shear deformation theories is very scant and is still in infancy.

A. Beam Under Consideration

Global coordinates of beam is as shown in figure,

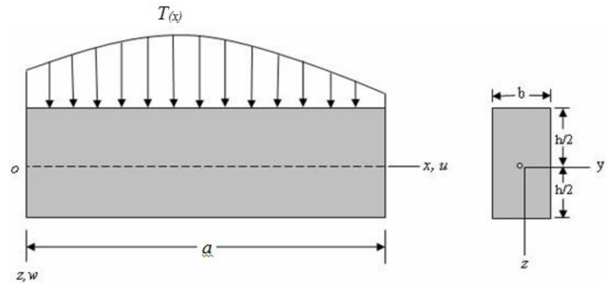

Fig. 1: Global coordinates of beam

$$0 \le x \le a;\ -\frac{b}{2} \le y \le \frac{b}{2};\ -\frac{h}{2} \le z \le \frac{h}{2} \qquad (1)$$

Where, *x*, *y*, *z* are Cartesian co-ordinates, *a*is length, *b* is width and *h* is the total depth of beam. The beam is subjected to single sine thermal load of intensity $T_{(x)}$varying across the depth of beam.

B. Assumptions Made in Theoretical Formulation

1. The in-plane displacement *u* in *x* direction consists of two parts:
   a) A displacement component analogous to displacement in elementary beam theory of bending;
   b) Displacement component due to shear deformation which is assumed to be parabolic, sinusoidal, hyperbolic and exponential in nature with respect to thickness coordinate.
2. The transverse displacement *w* in *z* direction is assumed to be a function of *x* coordinate.
3. One dimensional constitutive law is used.
4. The beam is subjected to thermal load only.

II. DISPLACEMENT FIELD

The displacement field of the present unified refined beam theory is given by,

$$U(x,z) = -z\frac{\partial w(x)}{\partial x} + f(z)\phi(x) \qquad (2)$$

$$W(x,z) = w(x) \qquad (3)$$

$$T(x) = Z.T_1(x) \qquad (4)$$

Where,
u and *w* = axial and transverse displacements of the beam in *x* and *z* directions, respectively,
ϕ = represents the rotation of the cross section of the beam at neutral axis (unknown function to be determined),
T = Thermal load
$f(z)$ = function assigned according to the shearing stress distribution through the thickness of the beam for different shear deformation theory, as stated below:

Table 1: Different shear deformation theory and its displacement field function $f(z)$

Theory	**Model**	$f(z)$
Parabolic shear deformation theory of Reddy, 1997	PSDT	$z\left[1-\frac{4}{3}\left(\frac{z}{h}\right)^2\right]$
First order shear deformation theory of Mindlin, 1951	FSDT	z
Classical Plate Theory of Kirchhoff	CPT	0

A. Normal Strain and Transverse Shear Strain for Beam:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x} \qquad (5)$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = f'(z)\frac{\partial \phi}{\partial x} \qquad (6)$$

According to one dimensional constitutive law, the axial stress normal bending stress and transverse shear stress are given by,

$$\begin{Bmatrix}\sigma_x \\ \tau_{zx}\end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x - \alpha_x T \\ \gamma_{zx}\end{Bmatrix} \quad (7)$$

Where,

Q_{11} = E = modulus of elasticity

Q_{55} = G = modulus of rigidity

α_x = coefficient of thermal expansion in x direction

T = Temperature field.

III. GOVERNING EQUATIONS

Governing equations of beam subjected to thermal loading by different shear defamation theory is determined by putting equation (7), equation (6) and equation (5) in Principle of virtual work equation which is stated below,

$$\int_0^a \int_{-h/2}^{+h/2} (\sigma_x.\delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx}).dx.dz = 0 \qquad (8)$$

By integrating preceding equation (8) by parts and collecting coefficients of δw and $\delta\phi$ terms, the governing equations in terms of displacement variables for different shear deformation theories are obtained below. Where δ is variational operator.

A. Higher Order Parabolic Shear Deformation Theory:

$$\delta w : Q_{11}\frac{h^3}{12}\left[\frac{\partial^4 w}{\partial x^4} - \frac{4}{5}\frac{\partial^3 \phi}{\partial x^3} + \alpha_x\frac{\partial^2 T_1}{\partial x^2}\right] = 0 \qquad (9)$$

$$\delta\phi :$$

$$Q_{11}\frac{h^3}{12}\left[\frac{4}{5}\frac{\partial^3 w}{\partial x^3} - \frac{351}{560}\frac{\partial^2\phi}{\partial x^2} + \frac{4}{5}\alpha_x\frac{\partial^2 T_1}{\partial x^2}\right] + Q_{55}.h\left[\frac{8}{15}.\phi\right] = 0$$

(10)

B. First Order Shear Deformation Theory:

$$\delta w : Q_{11}\frac{h^3}{12}\left[\frac{\partial^4 w}{\partial x^4} - \frac{\partial^3 \phi}{\partial x^3} + \alpha_x\frac{\partial^2 T_1}{\partial x^2} = 0\right] \qquad (11)$$

$$\delta\phi : Q_{11}\frac{h^3}{12}\left[\frac{\partial^3 w}{\partial x^3} - \frac{\partial^2 \phi}{\partial x^2} + \alpha_x\frac{\partial T_1}{\partial x}\right] + Q_{55}hk\phi = 0$$

(12)

C. Classical Beam Theory

$$\delta w : Q_{11}\frac{h^3}{12}\left[\frac{\partial^4 w}{\partial x^4} + \alpha_x\frac{\partial^2 T_1}{\partial x^2}\right] = 0 \qquad (13)$$

D. Elastic Equilibrium Equation:

Two dimensional elastic equilibrium equation is given by

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \qquad (14)$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0 \qquad (15)$$

The transverse shear stresses are calculated using Elastic Equilibrium (EE) equations. In this paper transverse shear stresses for CBT is calculated using EE equations, which cannot be calculated by Constitutive Relationship (CR).

IV. ILLUSTRATIVE EXAMPLE

A simply supported rectangular beam having length *a in x*-direction is considered. A solution to resulting governing equations, which satisfies the associated initial conditions, is of the form,

$$w = \sum_{m=1}^{\infty} w_m \sin\left(\frac{m\pi x}{a}\right) \qquad (16)$$

$$\phi = \sum_{m=1}^{\infty} \phi_m \cos\left(\frac{m\pi x}{a}\right) \quad (17)$$

$$T = \sum_{m=1}^{\infty} T_{1m} \sin\left(\frac{m\pi x}{a}\right) \quad (18)$$

Here, w_m, ϕ_m and T_{1m} are coefficient associated with translation, rotation and temperature of beam, respectively. Sinusoidal pure thermal load is taken in to consideration.

Material Property of the beam is given below,

$$\frac{E_1}{E_2} = 25, \ \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5, \text{ and } \mu_{12} = \mu_{13} = \mu_{23} = 0.23$$

V. NUMERICAL RESULTS AND DISCUSSION

The results are obtained for different degree of orthotropy (varying from 1, 2, 10, 25) and various aspect ratio a/h (varying from 10, 25, 50, 100) of beam. The results obtained are presented in Tables 2 and 3. The results obtained for displacements are presented in the non-dimensional parameters. The results are presented in the following non dimensional form (eq. 19-22),

$$\bar{w} = \frac{w}{T_1 \alpha_x a^2} \times 10 \quad (19)$$

$$\bar{u} = \frac{u}{T_1 \alpha_x a^2} \times 10 \quad (20)$$

$$\bar{\sigma}_x = \frac{\sigma_x}{E_2 T_1 \alpha_x a^2} \times 10 \quad (21)$$

$$\bar{\tau}_{zx} = \frac{\tau_{zx}}{E_2 T_1 \alpha_x a^2} \times 10 \quad (22)$$

Table 2: Comparison of transverse displacement $\bar{w}$ at (x = a / 2, z = 0) for ESL beam subjected to single sine thermal load

a/h	Model	E_1/E_2 1	2	10	25
10	PSDT	1.0640	1.0640	1.0640	1.0640
	FSDT	1.0524	1.0524	1.0524	1.0524
	CPT	1.0132	1.0132	1.0132	1.0132
25	PSDT	1.023	1.023	1.023	1.023
	FSDT	1.020	1.020	1.020	1.020
	CPT	1.0132	1.0132	1.0132	1.0132
50	PSDT	1.0160	1.0160	1.0160	1.0160
	FSDT	1.0152	1.0152	1.0152	1.0152
	CPT	1.0132	1.0132	1.0132	1.0132
100	PSDT	1.0133	1.0132	1.0132	1.0132
	FSDT	1.0132	1.0132	1.0132	1.0132
	CPT	1.0132	1.0132	1.0132	1.0132

Table 2 shows value of the normalized transverse displacement for various degrees of orthotropy and aspect ratio. The investigation of Table 2 and Table 3 reveals that the maximum displacement predicted by present PSDT theory is in excellent agreement with that of FSDT and CBT which consistently underestimates the value of same for all, a/h, and the stiffnesses, E_1/E_2, ratios. It is seen from results that displacement does not change with change in degree of orthotropy.

Table 3 shows value of the normalized axial displacement for various degrees of orthotropy and aspect ratio. The investigation of Table 3 reveals that the maximum displacement predicted by present PSDT theory is in excellent agreement with that of FSDT and CBT. Through thickness variation of axial displacements are shown in Figure 2 through 5. It is seen from results that displacement does not change with change in degree of orthotropy but it decreases as aspect ratio increases.

Axial displacement $\bar{u}$ for a/h = 10 & E_1/E_2 = 2, a/h = 25 & E_1/E_2 = 2, a/h = 50 & E_1/E_2 = 10, a/h = 100 & E_1/E_2 = 25; throughout the depth of beam is shown in fig. (2)-(5).

Table 3: Comparison of axial displacement $\bar{u} \times 10^{-3}$ at (x = 0, z = h / 2) for ESL beam subjected to single sine thermal load.

a/h	Model	E_1/E_2 1	2	10	25
10	PSDT	170.14	170.14	170.14	170.14
	FSDT	159.29	159.29	159.29	159.29
	CPT	158.74	158.74	158.74	158.74
25	PSDT	64.36	64.36	64.36	64.36
	FSDT	63.66	63.66	63.66	63.66
	CPT	63.46	63.46	63.46	63.46
50	PSDT	31.92	31.92	31.92	31.92
	FSDT	31.83	31.83	31.83	31.83
	CPT	31.73	31.73	31.73	31.73
100	PSDT	15.93	15.93	15.93	15.93
	FSDT	15.93	15.92	15.92	15.92
	CPT	15.91	15.91	15.91	15.93

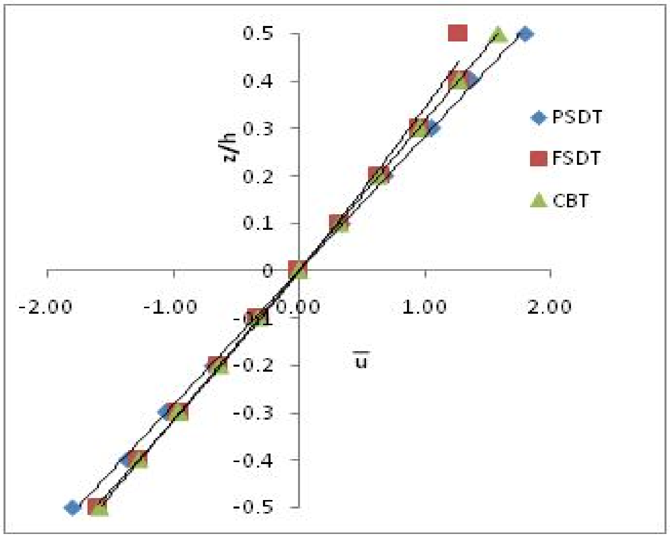

Fig. 2: Through-thickness variation of axial displacement $\overline{u}$ for E_1/E_2 = 1 and a/h = 10

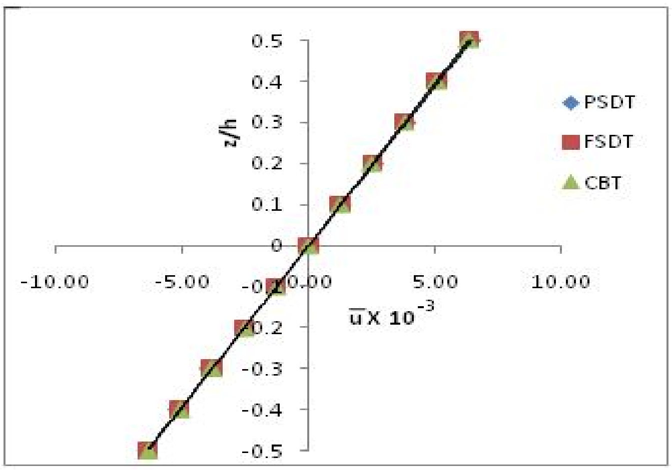

Fig. 3: Through-thickness variation of axial displacement $\overline{u}$ for E_1/E_2 = 2 and a/h = 25

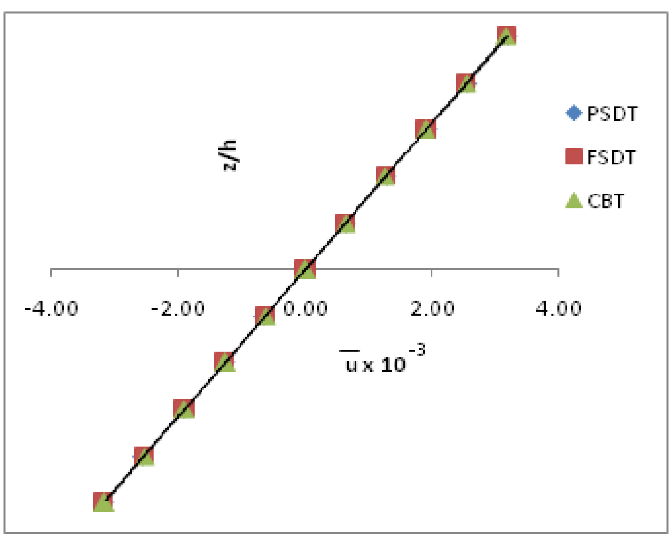

Fig. 4: Through-thickness variation of axial displacement $\overline{u}$ for E_1/E_2 = 10 and a/h = 50

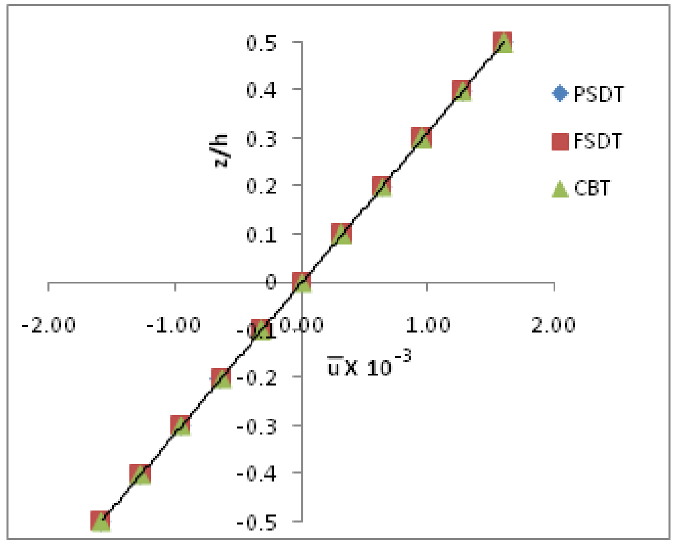

Fig. 5: Through-thickness variation of axial displacement $\overline{u}$ for E_1/E_2 = 25 and a/h = 100

Table 4: Comparison of normal bending stress $\overline{\sigma}_x$ at (x = a/2, z = ±h / 2) for ESL beam subjected to single sine thermal load

a/h	Model	E_1/E_2			
		1	2	10	25
10	PSDT	2.824×10^{-3}	4.736×10^{-3}	2.036×10^{-2}	5.0918×10^{-2}
	FSDT	4.961×10^{-3}	9.921×10^{-3}	4.961×10^{-2}	12.403×10^{-2}
	CPT	1.045×10^{-3}	2.090×10^{-3}	1.045×10^{-2}	2.615×10^{-2}
25	PSDT	6.377×10^{-4}	1.278×10^{-3}	6.377×10^{-3}	1.594×10^{-2}
	FSDT	6.295×10^{-5}	1.259×10^{-5}	6.295×10^{-4}	1.573×10^{-3}
	CPT	4.181×10^{-5}	8.362×10^{-5}	4.181×10^{-4}	1.045×10^{-3}
50	PSDT	1.353×10^{-4}	2.7072×10^{-4}	1.353×10^{-3}	3.384×10^{-3}
	FSDT	3.335×10^{-4}	7.109×10^{-4}	3.554×10^{-3}	8.887×10^{-3}
	CPT	2.090×10^{-4}	4.181×10^{-4}	2.854×10^{-3}	5.226×10^{-3}
100	PSDT	2.079×10^{-5}	4.153×10^{-5}	2.079×10^{-4}	5.198×10^{-4}
	FSDT	2.488×10^{-5}	4.976×10^{-5}	2.488×10^{-4}	6.221×10^{-4}
	CPT	2.045×10^{-5}	4.099×10^{-5}	2.045×10^{-4}	5.113×10^{-4}

Table 4 shows comparison of maximum normal bending stress for several values of degree of orthotropy and aspect ratios. Figure 6 and Figure 9 show that maximum normal bending stress predicted by present theory is in excellent agreement with that of other theories.It is seen from table 2 and table 3, displacement is independent of degree of orthotropy. But normal bending stresses changes as degree of orthotropy and aspect ratio changes. For FSDT and CBT, variation of normal bending stresses across the depth of beam is, more or less, linear. But it is non-linear for PSDT (as it can be seen from fig. (6) to fig. (9)).

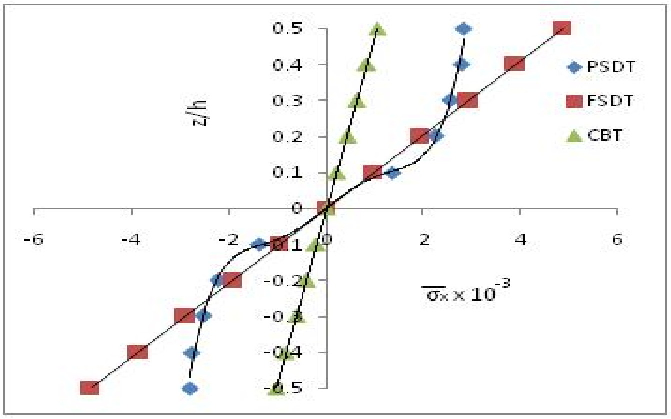

Fig. 6: Through-thickness variation of normal bending stress $\overline{\sigma}_x$ for $E_1/E_2 = 1$ and a/h = 10

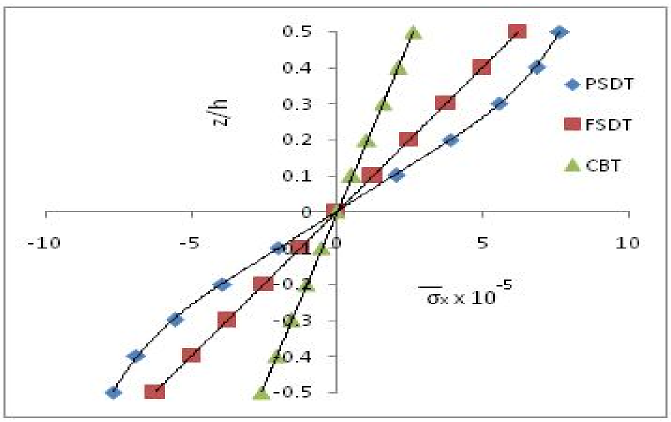

Fig. 7: Through-thickness variation of normal bending stress $\overline{\sigma}_x$ for $E_1/E_2 = 2$ and a/h = 25

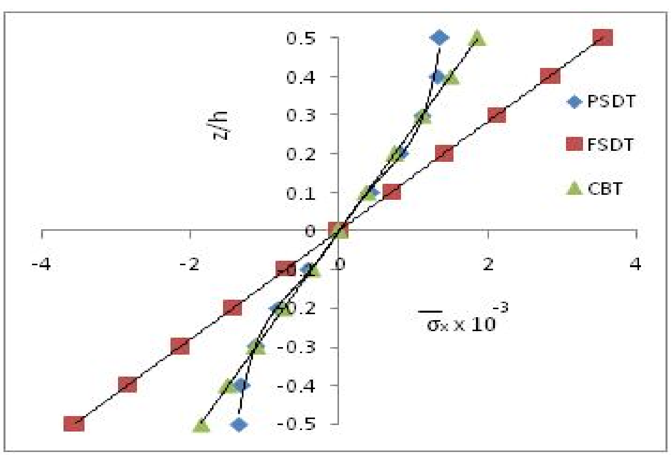

Fig. 8: Through-thickness variation of normal bending stress $\overline{\sigma}_x$ for $E_1/E_2 = 10$ and a/h = 50

Table 5: Comparison of transverse shear stress $\overline{\tau}_{zx}$ at (x = 0, z = 0) for ESL beam subjected to single sine thermal load

a/h	Model	E_1/E_2			
		1	2	10	25
10	PSDT	7.856×10^{-2}	15.69×10^{-2}	78.56×10^{-2}	196.13×10^{-2}
	FSDT	6.288×10^{-2}	12.57×10^{-2}	62.88×10^{-2}	157.21×10^{-2}
	CPT	9.275×10^{-3}	1.854×10^{-2}	9.753×10^{-2}	23.175×10^{-2}
25	PSDT	1.256×10^{-2}	2.519×10^{-2}	12.56×10^{-2}	31.401×10^{-2}
	FSDT	1.000×10^{-2}	2.000×10^{-2}	10.000×10^{-2}	25.000×10^{-2}
	CPT	0.402×10^{-3}	8.412×10^{-2}	4.181×10^{-2}	10.45×10^{-2}
50	PSDT	2.817×10^{-3}	5.631×10^{-2}	2.817×10^{-2}	7.0423×10^{-2}
	FSDT	2.5163×10^{-3}	5.032×10^{-3}	2.516×10^{-2}	6.2908×10^{-2}
	CPT	1.853×10^{-3}	3.496×10^{-3}	1.895×10^{-2}	4.632×10^{-2}
100	PSDT	7.845×10^{-4}	1.569×10^{-3}	7.845×10^{-3}	1.961×10^{-3}
	FSDT	7.488×10^{-4}	1.976×10^{-3}	7.488×10^{-3}	1.873×10^{-3}
	CPT	7.275×10^{-4}	1.499×10^{-3}	7.275×10^{-3}	1.273×10^{-3}

Table 5 shows comparison of maximum transverse shear stress for several values of degree of orthotropy and aspect ratios. Figure 10 and Figure 13 show that maximum normal bending stress predicted by present theory is in excellent agreement with that of other theories. Transverse shear stress is a function of degree of orthotropy and aspect ratio. For PSDT and FSDT, transverse shear stresses are found out using Constitutive Relation (CR); and for CBT, Elastic Equilibrium (EE) is used.

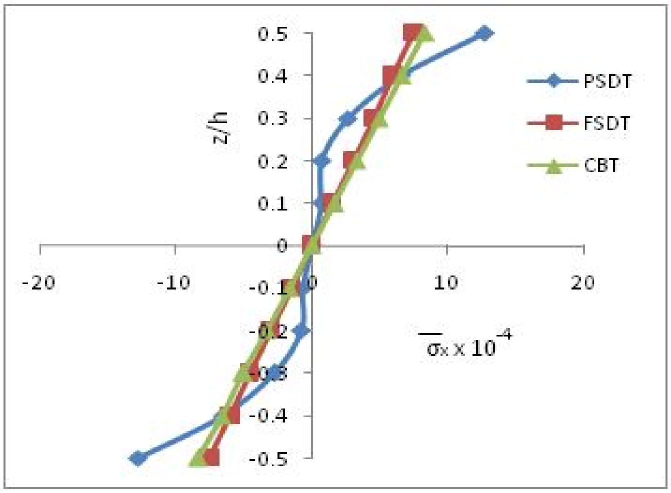

Fig. 9: Through-thickness variation of normal bending stress $\overline{\sigma}_x$ for $E_1/E_2 = 25$ and $a/h = 100$

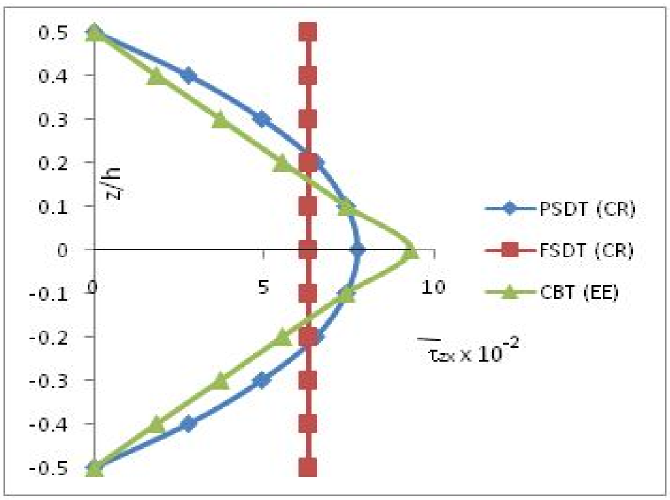

Fig. 10: Through-thickness variation of transverse shear stress $\overline{\tau}_{zx}$ for $E_1/E_2 = 1$ and $a/h = 10$

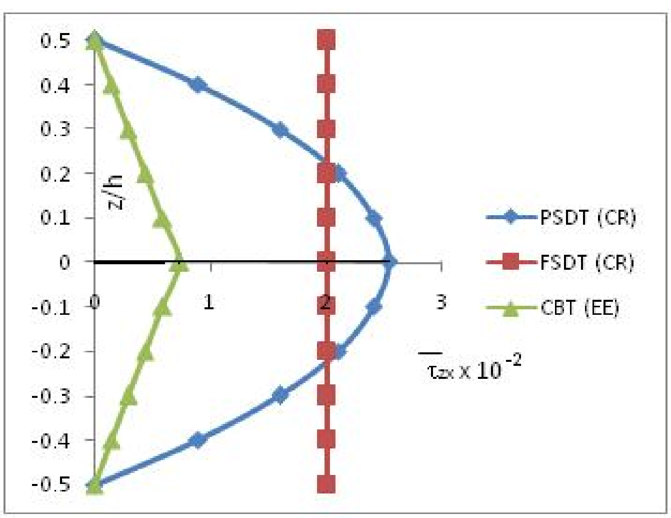

Fig. 11: Through-thickness variation of transverse shear stress $\overline{\tau}_{zx}$ for $E_1/E_2 = 2$ and $a/h = 25$

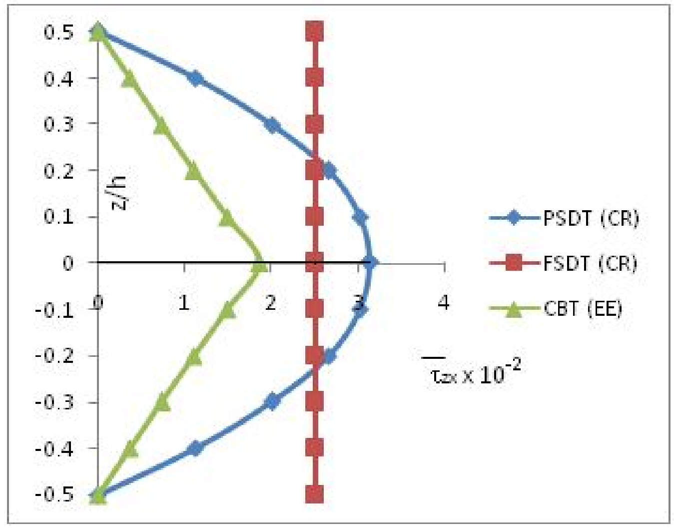

Fig. 12: Through-thickness variation of transverse shear stress $\overline{\tau}_{zx}$ for $E_1/E_2 = 10$ and $a/h = 50$

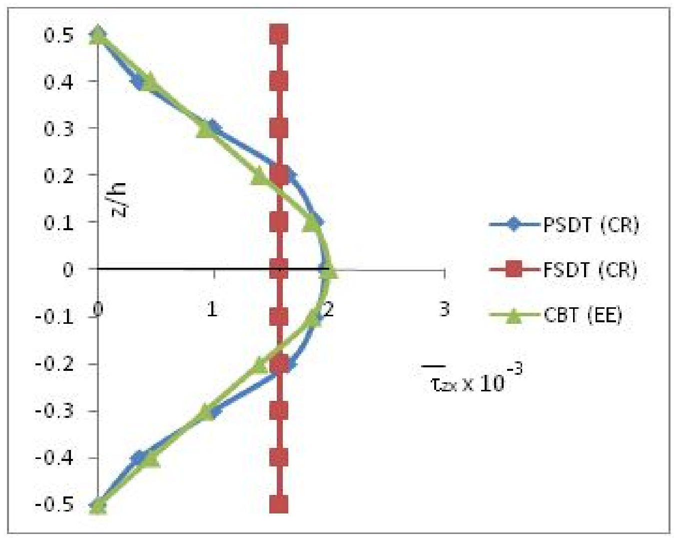

Fig. 13: Through-thickness variation of transverse shear stress $\overline{\tau}_{zx}$ for $E_1/E_2 = 25$ and $a/h = 100$

VI. CONCLUSIONS

1. The beam is subjected to pure thermal load and not subjected to transverse mechanical load, therefore the transverse and axial displacements predicted by all the theories for various Degree of Orthotropy, are more or less same.*i.e.*, it is independent of Degree of Orthotropy.
2. PSDT obviates need of shear correction factor as itsatisfies zero transverse shear stress at top and bottom ofbeam. FSDT gives constant transverse shear stressthroughout depth of beam.Transverse shear stresses throughout depth of beam, are calculated by CBT from elastic equilibrium equations in terms of stress field of CBT.

3. The CBT shows same numerical values of transverse displacement for different aspect ratios and Degree of Orthotropy, due to neglect of shear deformation effect.
4. Normal bending stresses and transverse shear stresses are the function of aspect ratio and Degree of Orthotropy in case of beam which is subjected to pure thermal load.
5. Normal bending stress profile, throughout the depth of the beam, calculated by PSDT is non-linear; whereas, it is linear for CBT and FSDT.
6. The parabolic shear deformation theory takes into account shear deformationwhich is more predominant in the precious analysis of composite beams. Other conventional theory neglects the effect of shear deformation, thus the results in lagging if accuracy in analysis.

REFERENCES

[1] Bickford W.B., 1982, A Consistent higher-order beam theory. Dev. Th. Appl. Mech, 11, 137–142.

[2] Cho K. N., Striz A. G., and Bert W. B. 1989, ―Thermal Stress Analysis of Laminate Using Higher - Order Theory In Each Layer, J. of Thermal Stresses, 12, 321-332. 1989.

[3] Erasmo Carrera, 2000, ―An assessment of mixed and classical theories for the thermal stress analysis of orthotropic multilayered plates, J. of Thermal Stresses, 23,797-831.

[4] Ghugal Y. M. and Shimpi R. P., 2001, ―A Review of Refined Shear Deforamation Theory for Isotropic and Anisotropic Laminated Beams, J. of Reinforced Plastic and Composites, 20, 255-272

[5] Ghugal Y. M. and Kulkarni S. K., 2011, ―Thermal stress analysis of cross-ply laminated plates using refined shear deformation theory, J. of Experimental and Applied Mechanics, 2, 19-27

[6] Ghugal Y. M. and Kulkarni S. K., 2013, ―Flexural Analysis of cross-ply laminated plates subjected to non-linear thermal and mechanical loadings, Acta Mech, 224, 675–690

[7] Heyliger P. R., Reddy J. N., 1988, ―A higher-order beam finite element for bending and vibration problems, J. of Sound Vibration, 126, 309–326.

[8] Kant T., Manjunath B.S., 1989, ―Refined theories for composite and sandwich beams with C_0 finite elements, Computer and Structure J., 33, 755–764.

[9] Kant T., Manjunath B.S., 1990, ―Higher-order theories for symmetric and unsymmetrical fibre reinforced composite beams with C_0 finite elements, Finite Element Analysis & Design, 6, 303- 320.

[10] Marur S.R., Kant T., 1997, ―On the performance of higher order theories for transient dynamic analysis of sandwich and composite beams, Computer and Structure J, 65, 741–759

[11] Mindlin R. D., 1951, ―Influence of rotatory inertia and shear on flexural vibrations of isotropicelastic plates, ASME J. of Applied Mechanics, 18, 31-38.

[12] Reddy J. N., 1997, ―Mechnics of Laminated Composite Plates, CRC Press, Boca Raton.

[13] Reddy J. N., 1984, ― A simple higher-order theory for laminated composite plates, J. of Applied Mechanics, 51, 745–752.

[14] S. Kapuria , P.C. Dumir, 2013, ―An efficient higher order zigzag theory for composite and sandwich beams subjected to thermal loading, International J. of Solids and Structures, 40, 6613–6631,

[15] Sayyad A. S., Shinde B. M. and Ghugal Y. M., 2014, ―Thermoelastic bending analysis of laminated composite plates according to various shear deformation theories, De Gruyter Open, 5, 18-30.

[16] Sayyad A. S., 2011, ―Comparison of various refined beam theories for the bending and free vibration analysis of thick beams, Applied and Computational Mechanics, 5, 217–230.

[17] Sayyad A. S., 2011, ―Comparison of various shear deformation theories for the free vibration of thick isotropic beams using refined shear deformation Theory, International J. of Civil and Structural Engineering, 2, 81-97.