Cylindrical Bending of Isotropic and Orthotropic Plate using Shear Deformation Theories under Thermal Loading

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Abstract- *Cylindrical bending of Isotropic and Orthotropic plate using shear deformation theories under thermal loading has been studied and presented in this paper. Results of deflections, displacements, bending stresses and transverse shear stresses are presented for various aspect ratios. The governing equations and boundary conditions are obtained by using principal of virtual work. Aclosed form solution is obtained by Navier solution scheme. Thus ,results obtained for different Aspect ratios and various Degree of Orthotropy, are compared with PSDT,FSDT and CPT under thermal load condition.*

Keywords- Aspect Ratio, Cylindrical Bending of Plate, Shear Deformation Theories, Thermal load.

I. INTRODUCTION

Plates are defined as plane structural elements with a small thickness compared with a small thickness compared to the planar dimensions. Plates are extensively used in many engineering applications like roof and floor of buildings, deck slab of bridges, water tanks, bulk heads, airplanes, ships, missiles, instruments, machine parts, and pavements. The needs of high rise building and aerospace industry led to the development and application of composite materials. With the increasing use of composite materials the need for advanced methods of analysis became obvious. Composite material improved stiffness, strength, weight reduction, corrosion resistance, thermal properties, fatigue life and wear resistance.

Plate theories can be developed by expanding the displacements in power series of the coordinate normal to the middle plane. The shear deformation effects are more pronounced in the plates when subjected to transverse loads. To describe the correct bending behavior of plates including shear deformation effects and the associated cross sectional warping, shear deformation theories are required. Therefore over the years researchers developed many theories. Kirchhoff,s has developed a classical plate theory (CPT) for thin plate analysis. It is not suitable for the thick plate due to neglect of shear deformation effect. Mindlin (1951) has developed first order shear deformation theory (FSDT) considering the effect of transverse shear deformation for the analysis of plate,but this theory required shear correction factor. Many of the higher order theories have been reported in the literature. Tauchert (1980) provided exact thermoelasticity solution to the plane strain deformation of orthotropic simply supported laminate using the method of displacement potentials. Reddy (1997)theory most commonly used for analysis of composite plate. This theory does not require shear correction factor also satisfies the zero stress condition at the top and bottom surface of the plate.Sayyad and Ghugal (2012) have developed an exponential shear deformation theory (ESDT) forbuckling, bending and free vibration analysis of thick isotropic plates. Ghugal and Kulkarni (2013) have used trigonometric sheardeformation theory (TSDT) for thermal analysis of composite plates. Sayyad, Chikalthankar and Nandedkar (2013) used refined plate theories for bending and free vibration analysis of isotropic plate.Sayyad and Shinde(2013) used hyperbolic shear deformation theory for thermal analysis of isotropic plates. Sayyad and Ghugal (2015) developed nth order shear deformation theory for composites laminates in cylindrical bending.

A study of literature indicates that the research work dealing mechanical and thermo-mechanical load on a plate has been done vastly. But, pure thermal loading of isotropic and orthotropic cylindrical bending of plate using refined, trigonometric, parabolic and exponential shear deformation theories is very scant.

II.MATHAMATICAL FORMULATION

Let us consider isotropic plate as shown in figure 1. Cylindrical bending is a plane strain problem. The transverse normal strain ε_z is negligible. It is assumed that the plate is of an infinite extent in the *y* direction while it is simply supported at its edges *x* = 0 and *x* = a. The plate has constant thickness '*h*'. The downward z-direction is assumed as positive, therefore, the thickness coordinate of the upper surface is +*h*/2 and lower surface is −*h*/2. A thermal load $T(x) = ZT_1$ is applied of the plate. The displacements in the *x* and *z* directions are denoted by *u* and *w* respectively.

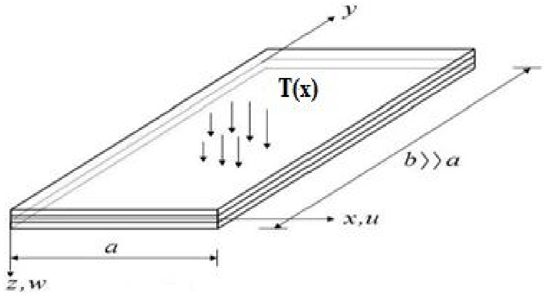

Fig. 1: Cylindrical bending of plate

A. Strain - Displacement Relations

Normal and shear strains are obtained within the framework of linear theory of elasticity using the displacement field. These relationships are given as follows:

Normal strain $\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}$

Shear strain $\gamma_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},$ (1)

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$

B. Stress – Strain Relationship

For a cylindrical bending of plate of constant thickness composed of isotropic and orthotropic material the effect of transverse normal stress σ_z is assumed to be negligible in comparison with inplane stressesσ_x.

$$\begin{Bmatrix} \sigma_x \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \gamma_{zx} \end{Bmatrix} \quad (2)$$

Where σ_x are in plane stresses and τ_{zx} are the transverse shear stresses. The ε_x are normal strain component and γ_{zx} are the shear strain components.

Using the expressions for stresses, strains, and principle of virtual work variational consistent differential equations and boundary conditions for the plate under consideration are obtained. The principle of virtual work stated below:

$$\int_0^a \int_{h/2}^{+h/2} (\sigma_x.\delta\varepsilon_x + \tau_{zx}.\delta\gamma_{zx}).dx.dz = 0 \quad (3)$$

Integrating the above equation by parts and collecting the coefficients of the following governing equations and boundary conditions are obtained in terms of displacement variables. The governing equations of parabolic shear deformation theory are as follows:

$$\partial w: Q_{11}\frac{h^3}{12}\left[\frac{\partial^4 w}{\partial x^4} - \frac{4}{5}\frac{\partial \phi}{\partial x^3} + \alpha_x \frac{\partial^2 T_1}{\partial x^2}\right] = 0 \quad (4)$$

$$\partial \phi: Q_{11}\frac{h^3}{12}\left[\frac{4}{5}\frac{\partial^3 w}{\partial x^3} - \frac{351}{560}\frac{\partial^2 \phi}{\partial x^2} + \frac{4}{5}\alpha_x \frac{\partial T_1}{\partial x}\right] + Q_{55}h\left[\frac{8}{15}\phi\right] = 0 \quad (5)$$

The governing equations of first order shear deformation theory are as follows:

$$\partial w: Q_{11}\frac{h^3}{12}\left[\frac{\partial^4 w}{\partial x^4} - \frac{\partial^3 \phi}{\partial x^3} + \alpha_x \frac{\partial^2 T_1}{\partial x^2} = 0\right] \quad (6)$$

$$\partial \phi: Q_{11}\frac{h^3}{12}\left[\frac{\partial^3 w}{\partial x^3} - \frac{\partial^2 \phi}{\partial x^2} + \alpha_x \frac{\partial T_1}{\partial x}\right] + Q_{55}hk\phi = 0 \quad (7)$$

The governing equations of classical plate theory are as follows:

$$\partial w: Q_{11}\frac{h^3}{12}\left[\frac{\partial^4 w}{\partial x^4} + \alpha_x \frac{\partial^2 T_1}{\partial x^2}\right] = 0 \quad (8)$$

III. NUMERICAL RESULT AND DISCUSSION

The results are obtained for different degree of orthotropy and a/h aspect ratios of plate. The results obtained for displacements and deflections are presented in the non dimensional parameters. The results are presented in the following non-dimensional form,

$$\bar{w} = \frac{w \times 10}{\alpha_x T_1 a}, \quad \bar{\tau}_{zx} = \frac{\tau_{zx}}{E_2 T_1 \alpha_x a^2} \times 10 \quad \bar{\sigma}_x = \frac{\sigma_x}{E_2 T_1 \alpha_x a^2} \times 10$$

$$\bar{u} = \frac{u}{T_1 \alpha_x a^2} \times 10 \quad (9)$$

Table 1 shows the value of axial displacement for isotropic plate several values of a/h. The investigation of table 1 reveals that the maximum axial displacement predicted by Reddy's parabolic shear deformation is in excellent agreement with that of FSDT and CPT consistently underestimates the value of same for a/h ratios. Through thickness variation of axial displacements are shown in figure 2. It is observed from the variation of axial displacement that the effect of nonlinearity becomes more pronounced with the increase in aspect ratios.

Table1. Comparison of axial displacement $\bar{u}$ at (x= 0, z = $\pm h/2$) for Isotropic plate in cylindrical bending subjected to single sine load.

a/h	Theory	Model	Degree of orthotropic E_1/E_2		
			10	25	40
4	Reddy	PSDT	0.4183	0.4183	0.4183
	Mindlin	FSDT	0.3977	0.3977	0.3977
	Kirchhoff	CPT	3.9260	3.9260	3.9260
10	Reddy	PSDT	0.1603	0.1603	0.1603
	Mindlin	FSDT	0.1590	0.1590	0.1590
	Kirchhoff	CPT	1.5700	1.5700	1.5700
15	Reddy	PSDT	0.1065	0.1065	0.1065
	Mindlin	FSDT	0.1061	0.1061	0.1061
	Kirchhoff	CPT	1.0470	1.0470	1.0470
20	Reddy	PSDT	0.0795	0.0795	0.0795
	Mindlin	FSDT	0.0793	0.0793	0.0793
	Kirchhoff	CPT	0.7853	0.7853	0.7853

Table 2: Comparison of axial displacement $\bar{u}$ at (x= 0, z= $\pm h/2$) for orthotropic plate in cylindrical bending subjected to single sine load.

a/h	Theory	Model	$\bar{u}$
4	Reddy	PSDT	0.4219
	Mindlin	FSDT	0.3978
	Kirchhoff	CPT	3.9260
10	Reddy	PSDT	0.1598
	Mindlin	FSDT	0.1587
	Kirchhoff	CPT	1.5700
15	Reddy	PSDT	0.1066
	Mindlin	FSDT	0.1046
	Kirchhoff	CPT	1.0470
20	Reddy	PSDT	0.0800
	Mindlin	FSDT	0.0780
	Kirchhoff	CPT	0.7853

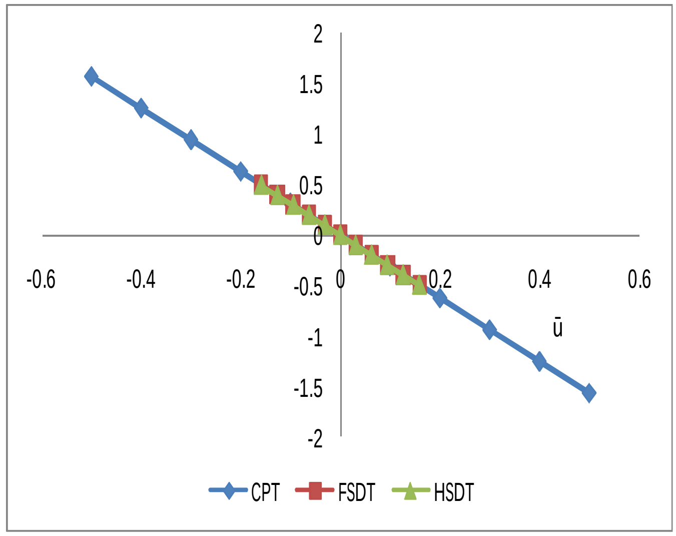

Figure 2: Through-thickness variation of displacement for isotropic plate $E_1 / E_2 = 1$ and a/h =10

Table 2 shows the value of axial displacement for orthotropic plate several values of a/h. and E_1/E_2 ratios .The investigation of table 2 reveals that the maximum axial displacement predicted by Reddy's parabolic shear deformation is in excellent agreement with that of FSDT and CPT consistently underestimates the value of same for a/h and E_1/E_2 ratios. Through thickness variation of axial displacements are shown in figure 3. It is observed from the variation of axial displacement that the effect of nonlinearity becomes more pronounced with the increase in aspect ratio.

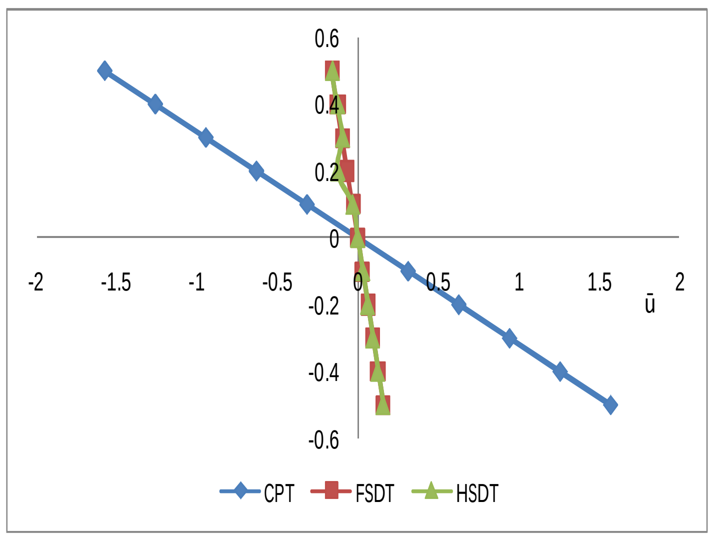

Figure 3: Through – thickness variation of displacement for orthotropic plate $E_1 / E_2 = 25$ *and a/h =10*

Table 3: Comparison of deflections $\bar{w}$ at ($x = a/2, z = 0$) for isotropic plate incylindrical bending subjected to single sine thermal load

a/h	Theory	Model	$\bar{w}$
4	Reddy	PSDT	1.366
	Mindlin	FSDT	1.365
	Kirchhoff	CPT	1.013
10	Reddy	PSDT	1.067
	Mindlin	FSDT	1.066
	Kirchhoff	CPT	1.013
15	Reddy	PSDT	1.039
	Mindlin	FSDT	1.024
	Kirchhoff	CPT	1.013
20	Reddy	PSDT	1.032
	Mindlin	FSDT	1.014
	Kirchhoff	CPT	1.013

Table 3 shows the value of deflection for isotropic plate several values of a/h. The investigation of table 3 reveals that the maximum deflection predicted by Reddy's parabolic sheared formation is in excellent agreement with that of FSDT and CPT consistently underestimates the value of same for a/h ratios. From the numerical discussion and conclusion in CPT, deflection is constant value of a/h ratios. CPT is independent of aspect ratios.

Table4: Comparison of axial displacement $\bar{w}$ at ($x = a/2, z = 0$) for orthotropic plate in cylindrical bending subjected to single sine thermal load

a/h	Theory	Model	Degree of orthotropy E_1/E_2		
			10	25	40
4	Reddy	PSDT	1.327	1.327	1.327
	Mindlin	FSDT	1.326	1.326	1.326
	Kirchhoff	CPT	1.013	1.013	1.013
10	Reddy	PSDT	1.063	1.063	1.063
	Mindlin	FSDT	1.062	1.062	1.062
	Kirchhoff	CPT	1.013	1.013	1.013
15	Reddy	PSDT	1.036	1.036	1.036
	Mindlin	FSDT	1.035	1.035	1.035
	Kirchhoff	CPT	1.013	1.013	1.013
20	Reddy	PSDT	1.024	1.024	1.024
	Mindlin	FSDT	1.023	1.023	1.023
	Kirchhoff	CPT	1.013	1.013	1.013

In Table 4 the value of deflections $\bar{w}$ is presented for several values of a/h ratios. From results maximum deflection decreases with increase in degree of orthotropy. Maximum deflection predicted by a CPT is independent of aspect ratios. The value of deflection in parabolic shear deformation theory and first order shear deformation theory is constant for different degree of orthotropy.

Table 5: Comparison of bending stresses $\overline{\sigma_x}$ at ($x = a/2, z = \pm h/2$) for isotropic plate incylindrical bending subjected to single sine thermal load

a/h	Theory	Model	$\overline{\sigma_x}$
4	Reddy	PSDT	0.08013
	Mindlin	FSDT	0.00099
	Kirchhoff	CPT	0.00028
10	Reddy	PSDT	0.00495
	Mindlin	FSDT	0.00036
	Kirchhoff	CPT	0.00011
15	Reddy	PSDT	0.00145
	Mindlin	FSDT	0.00001
	Kirchhoff	CPT	0.00007
20	Reddy	PSDT	0.00067
	Mindlin	FSDT	0.000007
	Kirchhoff	CPT	0.00005

In Table 5 shows the value of bending stresses $\overline{\sigma_x}$ is presented for several values of a/h ratios. The investigation of table 5 reveals that the maximum bending stresses predicted by Reddy's parabolic shear deformation is in excellent agreement with that of FSDT and CPT consistently underestimates the value of same for a/h ratios. Fig 4 showsthrough-thickness variation of bending stresses for isotropic plate $E_1/E_2 = 1$ and a/h =10.When aspect ratio increases bending stresses decreases For FSDT and CPT, through thickness variation of normal bending stresses is, more or less, linear. But it is non-linear for PSDT.

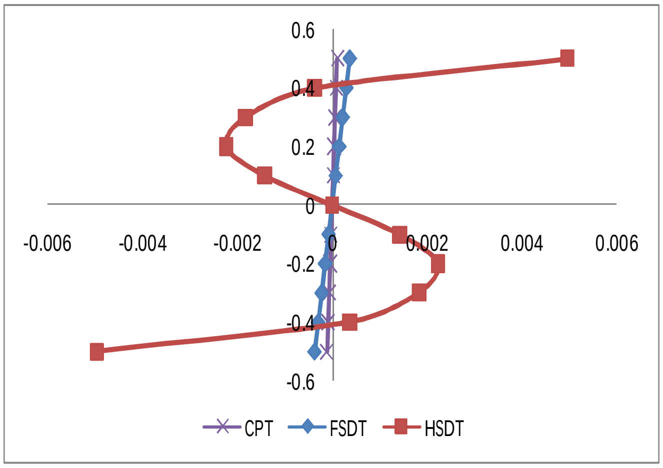

Figure 4: Through-thickness variation of bending stresses for isotropic plate $E_1 / E_2 = 1$ and a/h =10

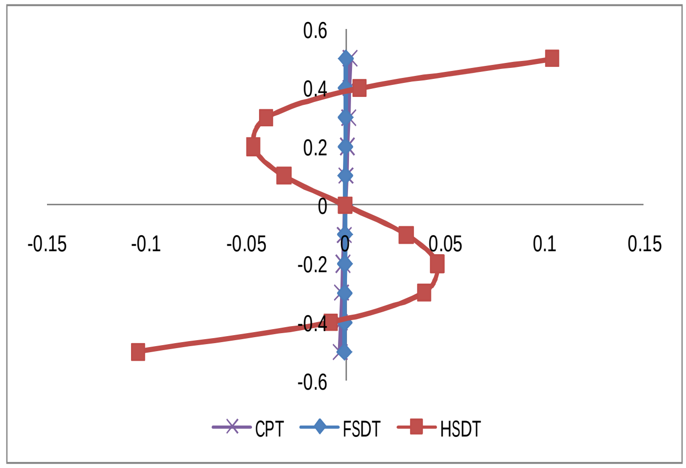

Figure 5:Through-thickness variation of transverse shear stress $E_1 / E_2 = 25$ and a/h =10

Table 6: Comparison of bending stresses $\overline{\sigma_x}$ at ($x = a/2, z = \pm h/2$) for orthotropic plate in cylindrical bending subjected to single sine thermal load

a/h	Theory	Model	Degree of orthotropy E_1/E_2		
			10	25	40
4	Reddy	PSDT	0.6518	1.6238	2.595
	Mindlin	FSDT	0.00240	0.00598	0.00956
	Kirchhoff	CPT	0.00262	0.00654	0.01046
10	Reddy	PSDT	0.04164	0.1037	0.1658
	Mindlin	FSDT	0.00013	0.00033	0.00053
	Kirchhoff	CPT	0.00105	0.00261	0.00418
15	Reddy	PSDT	0.01183	0.02947	0.0471
	Mindlin	FSDT	0.00029	0.00073	0.00117
	Kirchhoff	CPT	0.00070	0.00174	0.00279
20	Reddy	PSDT	0.006217	0.01548	0.02475
	Mindlin	FSDT	0.00110	0.00276	0.00441
	Kirchhoff	CPT	0.00052	0.00130	0.00209

In Table 6 shows the value of bending stresses $\overline{\sigma_x}$ is presented for several values of a/h ratios. The investigation of table 6 reveals that the maximum bending stresses predicted by Reddy's parabolic shear deformation is in excellent agreement with that of FSDT and CPT consistently underestimates the value of same for a/h ratios. when aspect ratio increases bending stresses decreases. Displacement is independent of degree of orthotropy. But normal bending stresses changes as degree of orthotropy and aspect ratios. For FSDT and CPT, through thickness variation of normal bending stresses is, more or less, linear. But it is non-linear for PSDT. Fig 5 shows through-thickness variation of transverse shear stress $E_1 / E_2 = 25$ and a/h =10.

Table 7: Comparison of transverse shear stresses $\overline{\tau_{xz}}$ at ($x = 0, z = 0$) for isotropic plate incylindrical bending subjected to single sine thermal load

a/h	Theory	Model	$\overline{\tau_{xz}}$
4	Reddy	PSDT	0.5299
	Mindlin	FSDT	0.3572
10	Reddy	PSDT	0.4145
	Mindlin	FSDT	0.0573
15	Reddy	PSDT	0.0692
	Mindlin	FSDT	0.0254
20	Reddy	PSDT	0.0513
	Mindlin	FSDT	0.0142

In Table 7 shows the value of transverse shear stresses $\overline{\tau_{xz}}$ is presented for several values of a/h ratios. The investigation of table 7 reveals that the maximum transverse shear stresses predicted by Reddy's parabolic shear deformation is in excellent agreement with that of FSDT consistently underestimates the value of same for a/h ratios. When aspect ratio increases transverse shear stresses decreases.Fig6 shows through-thickness variation of transverse shear stress $E_1 / E_2 = 1$ and a/h =10.

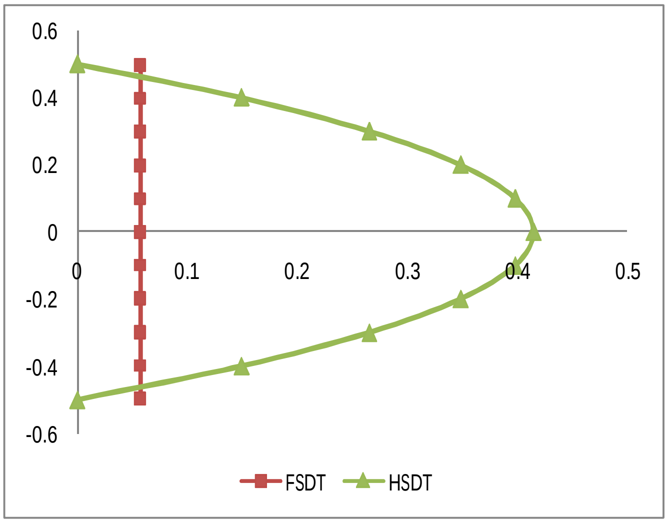

Figure 6: Through-thickness variation of transverse shear stress $E_1 / E_2 = 1$ and a/h =10

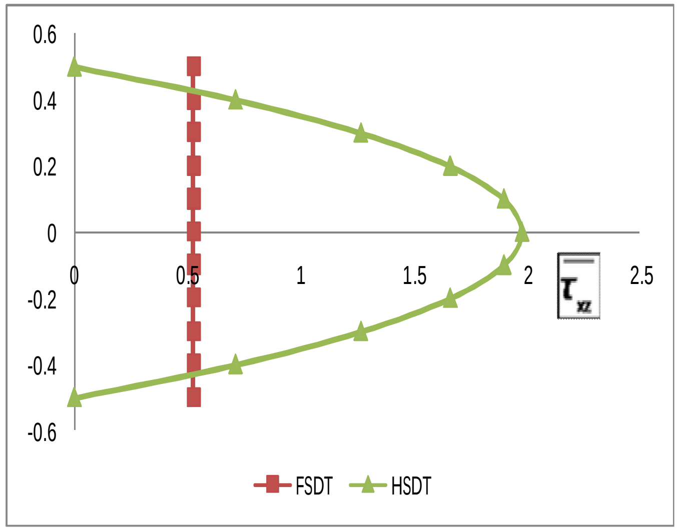

Figure 7:Through-thickness variation of transverse shear stress $E_1 / E_2 = 25$ and a/h =10

Table 8: Comparison of transverse shear streses $\overline{\tau_{xz}}$ at ($x = 0, z = 0$) for orthotropic plate in cylindrical bending subjected to single sine thermal load

a/h	Theory	Model	Degree of orthotropy E_1/E_2		
			10	25	40
4	Reddy	PSDT	4.954	12.385	19.816
	Mindlin	FSDT	3.2923	8.2308	13.169
10	Reddy	PSDT	0.79	1.975	3.16
	Mindlin	FSDT	0.5263	1.3158	2.1053
15	Reddy	PSDT	0.3512	0.878	1.4048
	Mindlin	FSDT	0.2343	0.5858	0.9373
20	Reddy	PSDT	0.1972	0.493	0.7888
	Mindlin	FSDT	0.1313	0.3283	0.5253

In Table 8 shows the value of transverse shear stresses $\overline{\tau_{xz}}$ is presented for several values of a/h ratios. The investigation of table 8 reveals that the maximum transverse shear stresses predicted by Reddy's parabolic shear deformation is in excellent agreement with that of FSDT consistently underestimates the value of same for a/h ratios· when aspect ratio increases transverse shear stresses decreases. Displacement is independent of degree of orthotropy. But transverse shear stresses changes as degree of orthotropy and aspect ratios. For FSDT andthrough thickness variation of transverse shear stresses is constant. fig 7 shows the through thickness variation transverse shear stress $E_1 / E_2 = 25$ and a/h =10.

IV. CONCLUSION

From numerical results and discussion it is observed that

1. In case of Isotropic and Orthotropic plate the displacements ,bending stresses, transverse shear stresses and deflections obtained by the parabolic shear deformation theory are in excellent agreement with those of other FSDT and CPT theories.
2. As plate is subjected to pure thermal load, it has no effect on degree of orthotropy but it affects on aspect ratio.
3. Reddy's PSDT takes in to account shear deformation effect which is not taken by other conventional theory.
4. The CPT shows same numerical values of transverse displacement for different aspect ratios and degree of orthotropy, due to neglect of shear deformation effect.
5. The classical plate theory which is not suitable thick plate due to neglect of shear deformation effect.
6. PSDT obviates need of shear correction factor as it satisfies zero transverse shear stress at top and bottom of plate. FSDT gives constant transverse shear stress throughout thickness of plate.

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