Equality In Between Soft Open and Soft Close In Multi Scale Environment

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Abstract- *In this paper, equality is discussed in soft open and soft-close. In mathematical morphological environment, the property of equality does not exist. But in soft mathematical morphology , the property of equality will exist. It is going to be discussed in this paper in detail.*

Keywords- Equality, Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation, open, close, soft open, soft close, Multi scale

I. INTRODUCTION

It is divided in to three parts. General introduction to Image Processing, introduction to soft morphology and introduction to multi scale mathematical morphology.

1.1. Introduction to image processing :

The human beings have the desire of recording incidents, through images. It has started from early cavemen also. Later, so many techniques, to get the images and so many techniques, to process the images are developed. After assembling of computers, image processing was expanded.

In 1964 G. Matheron was asked to investigate the relationships between the geometry of porous media and their permeability. At the same time, J. Serra was asked to quantify the petrography of iron ores, in order to predict their milling properties (1, 2). At the same time a centre was developed to study mathematical morphological techniques in Paris school of mines, France. Mathematical morphology can provide solutions to many tasks, where image processing can be applied, such as in remote sensing, optical character recognition, Radar image sequence recognition, medical image processing etc.,

The image processing algorithms or techniques can be classified in to two categories.

1) Linear methods
2) Non Linear methods.

The non linear methods will provide best results, compared to linear methods. The mathematical morphological methods / filters will come under the category of non linear methods/filters. In mathematical morphological operations, Erosion and Dilation are primitive operations (3, 4). But there exist some type of rigidity in mathematical morphological operations. That rigidity is relaxed and the morphological operations are redesigned to overcome some inconveniences, as well as to get some advantages. The primitive operations, Erosion and Dilation, now are called Soft Erosion and Soft Dilation

1.2. Introduction to soft morphology :

The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained.

Soft morphological operators are more tolerant to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. (5) In soft morphology, it preserves details, by adjusting its parameters (10). It can be designed in such a way that, it performs well in removal of salt – and – pepper noise as well as Gaussian noise, simultaneously (11).

A soft morphological filter can be designed in such a way that, it reduces periodic noise also (12). A filter designed in frequency domain, can function better for smoothening & edge enhancement, according to our requirements. The reason is that by tuning its frequency. But the design involves complex computation. But using soft morphological filters, using very simple computations we can achieve the quality of image processing, to that of filters in frequency domain, which involves complex computations (13).

Some of the soft morphological concepts are discussed in author's papers(15,16).

So, we can conclude that soft morphological filters perform excellent, compared to morphological filters.

1.3. Introduction to multi scale mathematical morphology:

Multi scale morphology has extended its applications to Image Smoothening, Edge Enhancement, Segmentation, Remote Sensing, Radar image analysis, Medical area etc.

It is having special applications, like enhancing weak Edges, Decay analysis of wood, critical analysis of (ECG) Cardio imagery (Identification of critical points), Getting results which are helpful for pilots, lunar landing etc.

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

That is why soft erosion and soft dilation are studied in multi scale environment,

II. DEFINITIONS

In some papers, researchers proposed soft morphology using two sets of structuring elements.
A) The core B) The soft boundary [7, 8, 9].

But, in some papers [5] the scientists/researchers proposed soft morphology, by counting logic. They have done the counting of ones, in the particular sub image, chosen. Then they have applied threshold value, for soft Erosion and soft dilation.

SOFT DILATION was defined as (5)

$$(I\oplus S^{(m)})[x,y] = 1 \text{ If } |I\cap S_{(x,y)}| \geq m$$
$$= 0 \quad \text{otherwise.}$$

Here "m" is threshold value where $1\leq m \leq |S|$. $|S|$ is the cardinality of S.

SOFT EROSION may be defined as

$$(I\ominus S^{(m)})[x,y] = 0 \text{ If } |\bar{I}\cap S_{(x,y)}| \geq m$$
$$= 1 \text{ otherwise.}$$

$\bar{I}$ = inversion of I;
m= threshold $\leq|S|$.

OPEN: It is a composite morphological operation. Open can be defined by the two primitive morphological operations, dilation and erosion operations.

Def: Open can be defined as, Erosion on the image, followed by Dilation. Open can be represented as, "O" symbol. B O K means image "B" is opened by the structuring element "K"
.
Operation on image I, by Structuring Element "S", may be defined as

$$\text{Open}(I, S) = ((I\ominus S)\oplus S)$$

It is further defined and explained in section 5.

CLOSE: It is also a composite morphological operation.

Def:- "Close" can be defined as, dilation on the image, followed by, erosion.

Close can be represented as ● symbol.

B ● K means, image "B" is closed by the S.E., "K".

$$B \bullet K = (B\oplus K)\ominus K.$$

Close operation, on image "I", by Structuring Element "S", may be defined as

$$\text{Close}(I,S) = ((I\oplus S)\ominus S)$$

It is further defined and explained in section 6

[The exact definition, given in "5", is slightly modified, according to the requirement, but without changing the meaning. Here the main assumption is origin is at central place of the structuring element and structuring element is assumed to be a square grid.]

Here "m,n" may be taken as 1 to 9, for $3/3$ structuring element or 1 to 25, for $5/5$ structuring element or 1 to 49, for $7/7$ structuring element.

III. DISCUSSION ON SOFT DILATION(14)

3.1 $3/3$ Structuring Elements

The number of one's (of the sub image) may be ≥ 0 & ≤ 9

If threshold value= 1 then $D_{(1)}$ may be defined as

$(I \oplus S^{(1)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| \geq 1$
$= 0 \quad \text{other wise.}$

Here, $S^{(1)}$ means, threshold value=1, in the $3/3$ sub image which is chosen, from the image.

If threshold value= 2 then $D_{(2)}$ may be defined as

$(I \oplus S^{(2)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y}| \geq 2$
$= 0 \quad \text{other wise.}$

Here, $S^{(2)}$ means, threshold value=2, in the $3/3$ sub image which is chosen, from the image.

3.2. $5/5$ Structuring Elements

For $5/5$ Structuring Element where "m" will run from 1 to 25, the soft dilations are

$D_{(1)}$ to $D_{(25).}$ They may be defined as

If threshold value = 1 then $D_{(1)}$ may be defined as

$(I \oplus S^{(1)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| \geq 1$
$= 0 \quad \text{other wise.}$

Here, $S^{(1)}$ means, threshold value=1, in the $5/5$ sub image which is chosen, in the image.

If threshold value= 11 then $D_{(11)}$ may be defined as

$(I \oplus S^{(11)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| \geq 11$
$= 0 \quad \text{other wise.}$

Here, $S^{(11)}$ means, threshold value=11 in the $5/5$ sub image, which is chosen.

In the same way $D_{(12)}$, $D_{(13)}$, $D_{(14)}$, ……. $D_{(25)}$ may be defined.

3.3. $7/7$ Structuring Elements

For $7/7$ Structuring Elements where "m" will run from 1 to 49, the soft dilations are

$D_{(1))}$ to $D_{(49).}$ They may be defined as

If threshold value= 1 then $D_{(1)}$, may be defined as

$(I \oplus S^{(1)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| \geq 1$
$= 0 \quad \text{other wise.}$

Here, $S^{(1)}$ means, threshold value=1, in the $7/7$ sub image which is chosen.

For threshold value = 27; $D_{(27)}$ may be defined as

$(I \oplus S^{(27)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| \geq 27$
$= 0 \quad \text{other wise.}$

Here, $S^{(27)}$ means, threshold value=27, in the $7/7$ sub image which is chosen, in the image.

In the same way $D_{(28)}$, $D_{(29),}$ $D_{(30),}$ $D_{(31),}$---------- $D_{(49)}$ may be defined.

3.4. $9/9$ Structuring Elements

For $9/9$ Structuring Elements where "m" will run from 1 to 81; the soft dilations are $D_{(1)}$ to $D_{(81)}$. They may be defined as

If threshold value= 1 then $D_{(1)}$ may be defined as

$(I \oplus S^{(1)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| = 1$
$= 0 \quad \text{other wise.}$

Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which having dimension $9/9$.

If threshold value= 62 then $D_{(62)}$ may be defined as

$(I \oplus S^{(62)})\,[x, y] = 1 \quad \text{If } |I \cap S_{(x,y)}| \geq 62$
$= 0 \quad \text{other wise.}$

Here, $S^{(62)}$ means, threshold value=62, in the sub image chosen, which having dimension $9/9$.

In the same way $D_{(633)}$, $D_{(64)}$,$D_{(65),}$ $D_{(66),}$--------- -------- $D_{(79)}$,$D_{(80),}$ $D_{(81)}$ may be defined.

3.5. $11/11$ Structuring Elements

For $11/11$ Structuring Elements where "m" will run from 1 to 121 , the soft dilations are $D_{(1),}$ $D_{(2)}$ ……$D_{(121)}$. They may be defined similar to above sub sections.

3.6. $13/13$ Structuring Elements

For ${}^{13}/_{13}$ Structuring Elements where "m" will run from 1 to 169, the soft dilations are $D_{(1)}, D_{(2)} \cdots\cdots D_{(169)}$. They may be defined, as defined in the above sub-sections. In the same way, the size of Structuring Elements may be extended to ${}^{15}/_{15}$, ${}^{17}/_{17}$, ${}^{19}/_{19}$ ……· to any dimension, according to our requirement.

IV. DISCUSSION ON SOFT EROSION(14)

4.1. ${}^{3}/_{3}$ Structuring Elements

In the same way, for Soft Erosion also

If threshold value= 1 then $E_{(1)}$ may be defined as

$$(I \ominus S^{(1)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 1$$
$$= 1 \text{ other wise.}$$

Here, $S^{(1)}$ means, threshold value=1, in the ${}^{3}/_{3}$ sub image which is chosen, from the image.

If threshold value= 2 then $E_{(2)}$ may be defined as

$$(I \ominus S^{(2)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 2$$
$$= 1 \text{ other wise.}$$

Here, $S^{(2)}$ means, threshold value=2, in the ${}^{3}/_{3}$ sub image which is chosen, from the image.

If threshold value= 9 then $E_{(9)}$ may be defined as

$$(I \ominus S^{(9)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 9$$
$$= 1 \text{ other wise.}$$

Here, $S^{(9)}$ means, threshold value=9, in the ${}^{3}/_{3}$ sub image which is chosen, from the image.

4.2. ${}^{5}/_{5}$ Structuring Elements

For ${}^{5}/_{5}$ Structuring Elements, the thresholds are 1 to 25. The soft erosions may be defined as follows.

If threshold value= 1 then $E_{(1)}$ may be defined as

$$(I \ominus S^{(1)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 1$$
$$= 1 \text{ other wise.}$$

Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which is having dimension ${}^{5}/_{5}$ from the image.

If threshold value= 2 then $E_{(2)}$ may be defined as

$$(I \ominus S^{(2)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 2$$
$$= 1 \text{ other wise.}$$

Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which is having dimension ${}^{5}/_{5}$ from the image.

If threshold value= 3 then $E_{(3)}$ may be defined as

$$(I \ominus S^{(3)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 3$$
$$= 1 \text{ other wise.}$$

Here, $S^{(3)}$ means, threshold value=3, in the sub image chosen, which is having dimension ${}^{5}/_{5}$ from the image.

In the same way $E_{(4)}$, $E_{(5)}$, $E_{(6)}$, $E_{(7)}$, $E_{(8)}$, $E_{(9)}$.

If threshold value= 10 then $E_{(10)}$ may defined as

$$(I \ominus S^{(10)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 10$$
$$= 1 \text{ other wise.}$$

Here, $S^{(10)}$ means, threshold value=10, in the sub image chosen, which is having dimension ${}^{5}/_{5}$ from the image.

If threshold value= 25 then $E_{(25)}$ may be defined as

$$(I \ominus S^{(25)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 25$$
$$= 1 \text{ other wise.}$$

Here, $S^{(25)}$ means, threshold value=25, in the sub image chosen, which is having dimension ${}^{5}/_{5}$ from the image.

4.3. ${}^{7}/_{7}$ Structuring Elements

For ${}^{7}/_{7}$ Structuring Elements, where "m" will be 1 to 49, the Soft Erosions are $E_{(1)}, E_{(2)} \ldots\ldots E_{(49)}$ they may be defined as ………

For threshold value= 1 then $E_{(1)}$ may be defined as

$$(I \ominus S^{(1)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 1$$
$$= 1 \text{ other wise.}$$

Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which is having dimension ${}^{7}/_{7}$ from the image.

For threshold value= 49 then $E_{(49)}$ may be defined as

$$(I \ominus S^{(49)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq 49$$
$$= 1 \text{ other wise.}$$

Here, $S^{(49)}$ means, threshold value=49, in the sub image chosen, which is having dimension $7/7$ from the image.

4.4. $9/9$ Structuring Elements

For $9/9$ Structuring Elements, where m=1····· 81, the Soft Erosions are $E_{(1)}$ to $E_{(81)}$. They may be defined as ········

If threshold value= 1 then $E_{(1)}$ may be defined as

$$(I \ominus S^{(1)})[x, y] = 0 \quad \text{If } |\bar{I} \cap S_{(x,y)}| \geq 1$$
$$= 1 \quad \text{other wise.}$$

Here, $S^{(1)}$ means, threshold value=1 in the sub image chosen, which is having dimension $9/9$ from the image.

If threshold value= 79 then $E_{(79)}$ may defined as

$$(I \ominus S^{(79)})[x, y] = 0 \quad \text{If } |\bar{I} \cap S_{(x,y)}| \geq 79$$
$$= 1 \quad \text{otherwise.}$$

Here, $S^{(79)}$ means, threshold value=79, in the sub image chosen, which is having dimension $9/9$ from the image.

If threshold value= 81 then $E_{(81)}$ may be defined as

$$(I \ominus S^{(81)})[x, y] = 0 \quad \text{If } |\bar{I} \cap S_{(x,y)}| \geq 81$$
$$= 1 \quad \text{otherwise.}$$

Here, $S^{(81)}$ means, threshold value=81, in the sub image chosen, which is having dimension $9/9$ from the image.

4.5. $11/11$ Structuring Elements

For $11/11$ Structuring Elements, where m=1 to 121, the Soft Erosions are $E_{(1)}$, $E_{(2)}$, $E_{(3)}$, $E_{(4)}$, $E_{(5)}$,...............$E_{(121)}$. They may be defined as

If threshold value= 1 then $E_{(1)}$ may be defined as

$$(I \ominus S^{(1)})[x, y] = 0 \quad \text{If } |\bar{I} \cap S_{(x,y)}| \geq 1$$
$$= 1 \quad \text{other wise.}$$

Here, $S^{(1)}$ means, threshold value=1 in the sub image chosen, which is having dimension $11/11$ from the image.

If threshold value= 121 then $E_{(121)}$ may be defined as

$$(I \ominus S^{(121)})[x, y] = 0 \quad \text{If } |\bar{I} \cap S_{(x,y)}| \geq 121$$
$$= 1 \quad \text{other wise.}$$

Here, $S^{(121)}$ means, threshold value=121, in the sub image chosen, which is having dimension $11/11$ from the image.

4.6. $13/13$ Structuring Elements

For $13/13$ Structuring Elements, where m=1 to 169, the Soft Erosions are $E_{(1)}$, $E_{(2)}$, $E_{(3)}$,…………………$E_{(169)}$.

They may be defined, as described in the above sections.

In the same way, the size of Structuring Elements may be extended to $15/15, 17/17, 19/19$ ······· to any dimension, according to our requirement.

V. DISCUSSIONS ON SOFT OPEN

In soft morphological environment, soft open can be defined as, soft erosion on the image, followed by soft dilation.

So, it can be symbolically represented as, soft open (I , $S^{(m,n)}$) = $((I \ominus S^m) \oplus S^n)$.

The m, n will indicate threshold values.

According to above convention, it is under stood that, perform soft erosion on image (I), with threshold value (m) and then perform soft dilation on the soft eroded image, with threshold value "n".

Normally, the following convention will be used, for the representation of soft open.

$$O\left(\frac{m}{n}\right) \qquad \text{OR} \qquad O(m,n)$$

$O(m,n)$ means soft open, with thresholds m & n, soft erode image by threshold value "m" and then soft dilate, the resultant image by threshold value "n".

$O(1,2)$ means, soft open, with thresholds 1 & 2. Soft Erode image by threshold value "1", and then soft dilate, the resultant image by threshold value "2".

VI. DISCUSSIONS ON SOFT CLOSE

In this section, close is discussed in soft morphological environment

In soft morphological environment, soft close, can be defined as, soft dilation on the image, followed by soft erosion.

So, it can be symbolically represented as,

Soft close (I, S $^{(m, n)}$) = ((I $\oplus$ S^m) $\ominus$ S^n)

The m, n will indicate threshold values.

According to above convention, it is understood, that, perform soft dilation, on image (I), with threshold value "m", and then perform soft erosion, on the soft dilated image, with threshold value "n".

Normally (Frequently) the following convention, will be used, for the representation of soft close.

$C\left(\frac{m}{n}\right)$ OR $C(m,n)$ (where $C(m,n)$ means, soft close, with thresholds **m & n**)

Soft dilate image, by threshold value "m", and then Erode, the resultant image, by threshold value "n".

$C(1,2)$ means, soft close with thresholds 1& 2.

Soft dilate image by threshold value "1", and then soft erode, the resultant image by, threshold value "2".

VII. DISCUSSION ON EQUALITY

A relation in between soft open and soft close may be established in multi scale environment. In this section this relation is studied assuming $3/3$ structuring element, $5/5$ structuring element $7/7$ structuring element $9/9$ structuring element and so on….

To develop equality among soft open and soft close operations the concept of equality among soft erosion and soft dilation is used which is discussed in the paper 14(of the same author)

7.1. $3/3$ Structuring Element.

Here the threshold value will be 1 to 9. The definitions of soft open and soft close are already given in the previous sections and chapters.

$O(1,1) = E(1)D(1) = D(9)E(9) = C(9,9)$
$O(1,2) = E(1)D(2) = D(9)E(8) = C(9,8)$
$O(1,3) = E(1)D(3) = D(9)E(7) = C(9,7)$
$O(1,4) = E(1)D(4) = D(9)E(6) = C(9,6)$
$O(1,5) = E(1)D(5) = D(9)E(5) = C(9,5)$
$O(1,6) = E(1)D(6) = D(9)E(4) = C(9,4)$
$O(1,7) = E(1)D(7) = D(9)E(3) = C(9,3)$
$O(1,8) = E(1)D(8) = D(9)E(2) = C(9,2)$
$O(1,9) = E(1)D(9) = D(9)E(1) = C(9,1)$

$O(2,1) = E(2)D(1) = D(8)E(9) = C(8,9)$
$O(2,2) = E(2)D(2) = D(8)E(8) = C(8,8)$
$O(2,3) = E(2)D(3) = D(8)E(7) = C(8,7)$
$O(2,4) = E(2)D(4) = D(8)E(6) = C(8,6)$
$O(2,5) = E(2)D(5) = D(8)E(5) = C(8,5)$
$O(2,6) = E(2)D(6) = D(8)E(4) = C(8,4)$
$O(2,7) = E(2)D(7) = D(8)E(3) = C(8,3)$
$O(2,8) = E(2)D(8) = D(8)E(2) = C(8,2)$
$O(2,9) = E(2)D(9) = D(8)E(1) = C(8,1)$
……………
……………
……………
$O(9,1) = E(9)D(1) = D(1)E(9) = C(1,9)$
$O(9,2) = E(9)D(2) = D(1)E(8) = C(1,8)$
$O(9,3) = E(9)D(3) = D(1)E(7) = C(1,7)$
$O(9,4) = E(9)D(4) = D(1)E(6) = C(1,6)$
$O(9,5) = E(9)D(5) = D(1)E(5) = C(1,5)$
$O(9,6) = E(9)D(6) = D(1)E(4) = C(1,4)$
$O(9,7) = E(9)D(7) = D(1)E(3) = C(1,3)$
$O(9,8) = E(9)D(8) = D(1)E(2) = C(1,2)$
$O(9,9) = E(9)D(9) = D(1)E(1) = C(1,1)$

In general $\underline{\boldsymbol{O(m,n) = C(10-m,10-n)}}$

7.2 $5/5$ Structuring Element.

$O(1,1) = E(1)D(1) = D(25)E(25) = C(25,25)$
$O(1,2) = E(1)D(2) = D(25)E(24) = C(25,24)$
$O(1,3) = E(1)D(3) = D(25)E(23) = C(25,23)$
$O(1,4) = E(1)D(4) = D(25)E(22) = C(25,22)$
$O(1,5) = E(1)D(5) = D(25)E(21) = C(25,21)$
$O(1,6) = E(1)D(6) = D(25)E(20) = C(25,20)$
$O(1,7) = E(1)D(7) = D(25)E(19) = C(25,19)$
$O(1,8) = E(1)D(8) = D(25)E(18) = C(25,18)$
$O(1,9) = E(1)D(9) = D(25)E(17) = C(25,17)$
$O(1,10) = E(1)D(10) = D(25)E(16) = C(25,16)$
$O(1,11) = E(1)D(11) = D(25)E(15) = C(25,15)$

$$O(1,12) = E(1)D(12) = D(25)E(14) = C(25,14)$$
$$O(1,13) = E(1)D(13) = D(25)E(13) = C(25,13)$$
$$O(1,14) = E(1)D(14) = D(25)E(12) = C(25,12)$$
$$O(1,15) = E(1)D(15) = D(25)E(11) = C(25,11)$$
$$O(1,16) = E(1)D(16) = D(25)E(10) = C(25,10)$$
$$O(1,17) = E(1)D(17) = D(25)E(9) = C(25,9)$$
$$O(1,18) = E(1)D(18) = D(25)E(8) = C(25,8)$$
$$O(1,19) = E(1)D(19) = D(25)E(7) = C(25,7)$$
$$O(1,20) = E(1)D(20) = D(25)E(6) = C(25,6)$$
$$O(1,21) = E(1)D(21) = D(25)E(5) = C(25,5)$$
$$O(1,22) = E(1)D(22) = D(25)E(4) = C(25,4)$$
$$O(1,23) = E(1)D(23) = D(25)E(3) = C(25,3)$$
$$O(1,24) = E(1)D(24) = D(25)E(2) = C(25,2)$$
$$O(1,25) = E(1)D(25) = D(25)E(1) = C(25,1)$$

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$$O(25,1) = E(25)D(1) = D(1)E(25) = C(1,25)$$
$$O(25,2) = E(25)D(2) = D(1)E(24) = C(1,24)$$
$$O(25,3) = E(25)D(3) = D(1)E(23) = C(1,23)$$
$$O(25,4) = E(25)D(4) = D(1)E(22) = C(1,22)$$
$$O(25,5) = E(25)D(5) = D(1)E(21) = C(1,21)$$
$$O(25,6) = E(25)D(6) = D(1)E(20) = C(1,20)$$
$$O(25,7) = E(25)D(7) = D(1)E(19) = C(1,19)$$
$$O(25,8) = E(25)D(8) = D(1)E(18) = C(1,18)$$
$$O(25,9) = E(25)D(9) = D(1)E(17) = C(1,17)$$
$$O(25,10) = E(25)D(10) = D(1)E(16) = C(1,16)$$
$$O(25,11) = E(25)D(11) = D(1)E(15) = C(1,15)$$
$$O(25,12) = E(25)D(12) = D(1)E(14) = C(1,14)$$
$$O(25,13) = E(25)D(13) = D(1)E(13) = C(1,13)$$
$$O(25,14) = E(25)D(14) = D(1)E(12) = C(1,12)$$
$$O(25,15) = E(25)D(15) = D(1)E(11) = C(1,11)$$
$$O(25,16) = E(25)D(16) = D(1)E(10) = C(1,10)$$
$$O(25,17) = E(25)D(17) = D(1)E(9) = C(1,9)$$
$$O(25,18) = E(25)D(18) = D(1)E(8) = C(1,8)$$
$$O(25,19) = E(25)D(19) = D(1)E(7) = C(1,7)$$
$$O(25,20) = E(25)D(20) = D(1)E(6) = C(1,6)$$
$$O(25,21) = E(25)D(21) = D(1)E(5) = C(1,5)$$
$$O(25,22) = E(25)D(22) = D(1)E(4) = C(1,4)$$
$$O(25,23) = E(25)D(23) = D(1)E(3) = C(1,3)$$
$$O(25,24) = E(25)D(24) = D(1)E(2) = C(1,2)$$
$$O(25,25) = E(25)D(25) = D(1)E(1) = C(1,1)$$

In general, $\underline{\boldsymbol{O(m,n) = C(26-m,26-n)}}$

7.3. $^{7}\!/_{7}$ Structuring Element.

$$O(1,1) = E(1)D(1) = D(49)E(49) = C(49,49)$$
$$O(1,2) = E(1)D(2) = D(49)E(48) = C(49,48)$$
$$O(1,3) = E(1)D(3) = D(49)E(47) = C(49,47)$$
$$O(1,4) = E(1)D(4) = D(49)E(46) = C(49,46)$$
$$O(1,5) = E(1)D(5) = D(49)E(45) = C(49,45)$$
$$O(1,6) = E(1)D(6) = D(49)E(44) = C(49,44)$$
$$O(1,7) = E(1)D(7) = D(49)E(43) = C(49,43)$$
$$O(1,8) = E(1)D(8) = D(49)E(42) = C(49,42)$$
$$O(1,9) = E(1)D(9) = D(49)E(41) = C(49,41)$$
$$O(1,10) = E(1)D(10) = D(49)E(40) = C(49,40)$$

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$$O(1,40) = E(1)D(40) = D(49)E(10) = C(49,10)$$
$$O(1,41) = E(1)D(41) = D(49)E(9) = C(49,9)$$
$$O(1,42) = E(1)D(42) = D(49)E(8) = C(49,8)$$
$$O(1,43) = E(1)D(43) = D(49)E(7) = C(49,7)$$
$$O(1,44) = E(1)D(44) = D(49)E(6) = C(49,6)$$
$$O(1,45) = E(1)D(45) = D(49)E(5) = C(49,5)$$
$$O(1,46) = E(1)D(46) = D(49)E(4) = C(49,4)$$
$$O(1,47) = E(1)D(47) = D(49)E(3) = C(49,3)$$
$$O(1,48) = E(1)D(48) = D(49)E(2) = C(49,2)$$
$$O(1,49) = E(1)D(49) = D(49)E(1) = C(49,1)$$

When $^{7}\!/_{7}$ structuring element is taken, the thresholds will be 1, 1 to 49, 49. Because it is not possible to mention equations for all the thresholds, in this situation limited thresholds are presented.

$$O(49,1) = E(49)D(1) = D(1)E(49) = C(1,49)$$
$$O(49,2) = E(49)D(2) = D(1)E(48) = C(1,48)$$
$$O(49,3) = E(49)D(3) = D(1)E(47) = C(1,47)$$
$$O(49,4) = E(49)D(4) = D(1)E(46) = C(1,46)$$
$$O(49,5) = E(49)D(5) = D(1)E(45) = C(1,45)$$
$$O(49,6) = E(49)D(6) = D(1)E(44) = C(1,44)$$
$$O(49,7) = E(49)D(7) = D(1)E(43) = C(1,43)$$
$$O(49,8) = E(49)D(8) = D(1)E(42) = C(1,42)$$
$$O(49,9) = E(49)D(9) = D(1)E(41) = C(1,41)$$
$$O(49,10) = E(49)D(10) = D(1)E(40) = C(1,40)$$

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$$O(49,40) = E(49)D(40) = D(1)E(10) = C(1,10)$$
$$O(49,41) = E(49)D(41) = D(1)E(9) = C(1,9)$$
$$O(49,42) = E(49)D(42) = D(1)E(8) = C(1,8)$$
$$O(49,43) = E(49)D(43) = D(1)E(7) = C(1,7)$$
$$O(49,44) = E(49)D(44) = D(1)E(6) = C(1,6)$$
$$O(49,45) = E(49)D(45) = D(1)E(5) = C(1,5)$$
$$O(49,46) = E(49)D(46) = D(1)E(4) = C(1,4)$$
$$O(49,47) = E(49)D(47) = D(1)E(3) = C(1,3)$$
$$O(49,48) = E(49)D(48) = D(1)E(2) = C(1,2)$$
$$O(49,49) = E(49)D(49) = D(1)E(1) = C(1,1)$$

In general, $\underline{\boldsymbol{O(m,n) = C(49-m,49-n)}}$

7.4. $\frac{9}{9}$ Structuring Element.

$$O(1,1) = E(1)D(1) = D(81)E(81) = C(81,81)$$
$$O(1,2) = E(1)D(2) = D(81)E(80) = C(81,80)$$
$$O(1,3) = E(1)D(3) = D(81)E(79) = C(81,79)$$
$$O(1,4) = E(1)D(4) = D(81)E(78) = C(81,78)$$
$$O(1,5) = E(1)D(5) = D(81)E(77) = C(81,77)$$
$$O(1,6) = E(1)D(6) = D(81)E(76) = C(81,76)$$
$$O(1,7) = E(1)D(7) = D(81)E(75) = C(81,75)$$
$$O(1,8) = E(1)D(8) = D(81)E(74) = C(81,74)$$
$$O(1,9) = E(1)D(9) = D(81)E(73) = C(81,73)$$
$$O(1,10) = E(1)D(10) = D(81)E(72) = C(81,72)$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$O(1,72) = E(1)D(72) = D(81)E(10) = C(81,10)$$
$$O(1,73) = E(1)D(73) = D(81)E(9) = C(81,9)$$
$$O(1,74) = E(1)D(74) = D(81)E(8) = C(81,8)$$
$$O(1,75) = E(1)D(75) = D(81)E(7) = C(81,7)$$
$$O(1,76) = E(1)D(76) = D(81)E(6) = C(81,6)$$
$$O(1,77) = E(1)D(77) = D(81)E(5) = C(81,5)$$
$$O(1,78) = E(1)D(78) = D(81)E(4) = C(81,4)$$
$$O(1,79) = E(1)D(79) = D(81)E(3) = C(81,3)$$
$$O(1,80) = E(1)D(80) = D(81)E(2) = C(81,2)$$
$$O(1,81) = E(1)D(81) = D(81)E(1) = C(81,1)$$

When $\frac{9}{9}$ structuring element is taken, the thresholds will be 1, 1 to 81, 81. Because it is not possible to mention equations for all the thresholds, in this situation limited thresholds are presented.

$$O(81,1) = E(81)D(1) = D(1)E(81) = C(1,81)$$
$$O(81,2) = E(81)D(2) = D(1)E(80) = C(1,80)$$
$$O(81,3) = E(81)D(3) = D(1)E(79) = C(1,79)$$
$$O(81,4) = E(81)D(4) = D(1)E(78) = C(1,78)$$
$$O(81,5) = E(81)D(5) = D(1)E(77) = C(1,77)$$
$$O(81,6) = E(81)D(6) = D(1)E(76) = C(1,76)$$
$$O(81,7) = E(81)D(7) = D(1)E(75) = C(1,75)$$
$$O(81,8) = E(81)D(8) = D(1)E(74) = C(1,74)$$
$$O(81,9) = E(81)D(9) = D(1)E(73) = C(1,73)$$
$$O(81,10) = E(81)D(10) = D(1)E(72) = C(1,72)$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$O(81,72) = E(81)D(72) = D(1)E(10) = C(1,10)$$
$$O(81,73) = E(81)D(73) = D(1)E(9) = C(1,9)$$
$$O(81,74) = E(81)D(74) = D(1)E(8) = C(1,8)$$
$$O(81,75) = E(81)D(75) = D(1)E(7) = C(1,7)$$
$$O(81,76) = E(81)D(76) = D(1)E(6) = C(1,6)$$
$$O(81,77) = E(81)D(77) = D(1)E(5) = C(1,5)$$
$$O(81,78) = E(81)D(78) = D(1)E(4) = C(1,4)$$
$$O(81,79) = E(81)D(79) = D(1)E(3) = C(1,3)$$
$$O(81,80) = E(81)D(80) = D(1)E(2) = C(1,2)$$
$$O(81,81) = E(81)D(81) = D(1)E(1) = C(1,1)$$

In general $\underline{\boldsymbol{O(m,n) = C(81-m,81-n)}}$

7.5. General case:

For $\frac{w}{w}$ structuring element size

- $\boldsymbol{O(m,n) = C(w^2+1-m, w^2+1-n)}$

7.6 Equality in between Soft close and Soft open

In the same way, when $\frac{3}{3}$ structuring element is taken equality in between soft close and soft open can be described as the following equation.

$\underline{\boldsymbol{C(m,n) = O(10-m,10-n)}}$

For $\frac{5}{5}$ structuring element this relation can be derived as

$\underline{\boldsymbol{C(m,n) = O(26-m,26-n)}}$

For $\frac{7}{7}$ structuring element this relation can be derived as

$\underline{\boldsymbol{C(m,n) = O(50-m,50-n)}}$

For $\frac{9}{9}$ structuring element this relation can be derived as

$\underline{\boldsymbol{C(m,n) = O(82-m,82-n)}}$

For $\frac{11}{11}$ structuring element this relation can be derived as

$\underline{\boldsymbol{C(m,n) = O(122-m,122-n)}}$

In the same way the equivalency formulae for soft close and soft open for $\frac{13}{13}$, $\frac{15}{15}$, $\frac{17}{17}$, $\frac{19}{19}$ ……. structuring elements can be derived.

In general for $\frac{w}{w}$ structuring element size

- $\boldsymbol{C(m,n) = O(w^2+1-m, w^2+1-n)}$

VIII. RESULTS AND ANALYSIS:

In this section the results of experiments are presented. Actually two diagrams are taken, a Semi circle shape and a dumbbell shape. On these images various morphological and soft morphological operations are applied. The output is got in the form of tables, diagrams and graphs, around 1000 pages. But here some important as well as samples outputs are presented

Images: Semi circle and dumbbell

Semi-circle

dumbbell

In the following tables soft open and soft close values are given in 3/3, 5/5 window environments. But in my database these tables are available for 7/7, 9/9 …….window environments in tables as well as graphical representations.

They give the proof practically about equality of soft open and soft close.

Tables relating soft open and soft close in 3/3 window environment:-

Window 3/3		
Threshold values of soft close	Threshold values of soft open	Area
8,1	2,9	3602
8,2	2,8	3723
8,3	2,7	3792
8,4	2,6	4043
8,5	2,5	4132
8,6	2,4	4211
8,7	2,3	4472
8,8	2,2	4581
8,9	2,1	4722

Window 3/3		
Threshold values of soft close	Threshold values of soft open	Area
2,1	8,9	4426
2,2	8,8	4586
2,3	8,7	4799
2,4	8,6	4935
2,5	8,5	5052
2,6	8,4	5111
2,7	8,3	5415
2,8	8,2	5526
2,9	8,1	5706

Window 3/3		
Threshold values of soft close	Threshold values of soft open	Area
1,1	9,9	4582
1,2	9,8	4730
1,3	9,7	4799
1,4	9,6	5104
1,5	9,5	5211
1,6	9,4	5273
1,7	9,3	5601
1,8	9,2	5712
1,9	9,1	5870

Window 5/5		
Threshold values of soft close	Threshold values of soft open	Area
2,1	24,25	4446
2,2	24,24	4602
2,3	24,23	4665
2,4	24,22	4756
2,5	24,21	4809
2,6	24,20	5001
2,7	24,19	5120
2,8	24,18	5185
2,9	24,17	5233
2,10	24,16	5285
2,11	24,15	5497
2,12	24,14	5605
2,13	24,13	5682
2,14	24,12	5727
2,15	24,11	5781
2,16	24,10	6006
2,17	24,09	6132
2,18	24,08	6202
2,19	24,07	6249
2,20	24,06	6359
2,21	24,05	6603
2,22	24,04	6702
2,23	24,03	6867
2,24	24,02	6978
2,25	24,01	7158

Window 5/5		
Threshold values of soft close	Threshold values of soft open	Area
2,1	24,25	4446
2,2	24,24	4602
2,3	24,23	4665
2,4	24,22	4756
2,5	24,21	4809
2,6	24,20	5001
2,7	24,19	5120
2,8	24,18	5185
2,9	24,17	5233
2,10	24,16	5285
2,11	24,15	5497
2,12	24,14	5605
2,13	24,13	5682
2,14	24,12	5727
2,15	24,11	5781
2,16	24,10	6006
2,17	24,09	6132
2,18	24,08	6202
2,19	24,07	6249
2,20	24,06	6359
2,21	24,05	6603
2,22	24,04	6702
2,23	24,03	6867
2,24	24,02	6978
2,25	24,01	7158

Window 5/5		
Threshold values of soft close	Threshold values of soft open	Area
25,1	1,25	2588
25,2	1,24	2672
25,3	1,23	2729
25,4	1,22	2789
25,5	1,21	2833
25,6	1,20	2992
25,7	1,19	3051
25,8	1,18	3093
25,9	1,17	3122
25,10	1,16	3175
25,11	1,15	3340
25,12	1,14	3391
25,13	1,13	3429
25,14	1,12	3467
25,15	1,11	3521
25,16	1,10	3700
25,17	1,09	3247
25,18	1,08	3812
25,19	1,07	3839
25,20	1,06	3924
25,21	1,05	4115
25,22	1,04	4191
25,23	1,03	4281
25,24	1,02	4375
25,25	1,01	4549

Window 5/5		
Threshold values of soft close	Threshold values of soft open	Area
24,1	2,25	2674
24,2	2,24	2759
24,3	2,23	2831
24,4	2,22	2903
24,5	2,21	2941
24,6	2,20	3094
24,7	2,19	3146
24,8	2,18	3212
24,9	2,17	3241
24,10	2,16	3297
24,11	2,15	3455
24,12	2,14	3502
24,13	2,13	3564
24,14	2,12	3603
24,15	2,11	3660
24,16	2,10	3825
24,17	2,09	3883
24,18	2,08	3957
24,19	2,07	4002
24,20	2,06	4106
24,21	2,05	4262
24,22	2,04	4335
24,23	2,03	4450
24,24	2,02	4570
24,25	2,01	4703

IX. CONCLUSION

In this paper the equality property is established in between soft open and soft close in multi scale environment.

The relevalent formulae are For structuring element size: w/w

- $O(m,n) = C(w^2+1-m, w^2+1-n)$
- $C(m,n) = O(w^2+1-m, w^2+1-n)$

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[16] KOMPELLA VENKATA RAMANA------- DUALITY IN SOFT EROSION IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT IJARCSSE FEB 2016

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