# Fuzzy Tri-Magic Labeling of Some Unicyclic Graphs

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***Abstract-*** *To prove that the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ admits fuzzy tri-magic labeling for $n \geq 6$ and $m \geq 3$.The methods involve considering G to be a finite, simple, undirected, and non-trivial graph. A fuzzy graph is said to admit tri-magic labeling if the number of magic membership values $K_i$'s and $K_j$'s($1 \leq i \leq 3$) differ by atmost 1 and $|K_i - K_j| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$, $r \geq 2$. The fuzzy graph which admits a tri-magic labeling is called a fuzzy tri-magic labeling graph. The fuzzy tri-magic labeling graphs are denoted by $\vec{T}m_0G$.This study has proved that the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ admits fuzzy tri-magic labeling for $n \geq 6$ and $m \geq 3$.This study explains the fuzzy tri-magic labeling of some unicyclic graphs.*

***Keywords***- Fuzzy labeling, Fuzzy tri-magic labeling, Magic membership value, Star graph, Unicyclic graph.

## I. INTRODUCTION

Graph labeling was first introduced in the mid-1960s. A brief explanation of the various types of graph labeling is given by Joseph A. Gallian in hisbook *A Dynamic Survey of Graph Labeling*[1].The graphs considered here are finite, simple, undirected, and nontrivial. Frank Harary, in his book *Graph Theory* [2], has standardized the terminology of graph theory and comprehensively treated the theorems based on it.Graph theory has a good development in the graph labeling and has a wide range of applications. Fuzzy is a newly emerging mathematical framework to exhibit the phenomenon of uncertainty in real-life tribulations. A fuzzy set is defined mathematically by assigning a value to each possible individual in the universe of discourse, representing its grade or membership which corresponds to the degree to which that individual is similar to or compatible with the concept represented by the fuzzy set. A complete bipartite graph with one vertex in one partition and *n* vertices in another partition is said to be a star graph, and it is denoted by $S_{1,n}$. Ameenal Bibi and Devi [3] have proved that some graphs admit fuzzy bi-magic labeling. We introduced a novel theory of the fuzzy tri-magic labeling that was inspired by the theory of fuzzy bi-magic labeling. Also, we have proved that some star-related graphs are fuzzy tri-magic [4].If a connected graph *G* contains exactly one cycle, then it is called a unicyclic graph. Sumathi, Mahalakshmi and Rathi [5] have proved various types of unicyclic graphs admitting quotient-3 labeling. Sumathi and Suresh Kumar [6] have proved various types of unicyclic graphs with pendant edges admitting fuzzy quotient-3 labeling.Likewise in this study we have proved that the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ admits fuzzy tri-magic labeling for $n \geq 6$ and $m \geq 3$.

## II. METHODOLOGY

**Definition 1 Fuzzy graph**

A fuzzy graph $G:(\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

**Definition 2 Fuzzy Labeling**

Let $G = (V,E)$ be a graph, thefuzzy graph $G:(\sigma, \mu)$ is said to have a fuzzy labeling, if $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ is bijective such that the membership value of edges and vertices is distinct and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

**Definition 3 Magic membership value (MMV)**

Let $G:(\sigma, \mu)$ be a fuzzy graph; the induced map $g:E(G) \to [0,1]$ defined by $g(uv) = \sigma(u) + \mu(uv) + \sigma(v)$ is said to be a magic membership value. It is denoted by MMV [6].

**Definition 4 Fuzzy tri-magic labeling**

A fuzzy graph is said to admit tri-magic labeling if the magic membership values $K_i$'s, $1 \leq i \leq 3$ are constants where number of $K_i$'s and $K_j$'s differ by at most 1 and $|K_i - K_j| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$, $r \geq 2$.

**Definition 5 Fuzzy tri-magic labeling graph**

A fuzzy labeling graph which admits tri-magic labeling is called a fuzzy tri-magic labeling graph. The fuzzy tri-magic labeling graphs are denoted by $\tilde{T}m_0G$.

**Definition 6 Unicyclic graph**

A unicyclic graph is a connected graph containing exactly one cycle [2, 4].

## III. RESULTS AND DISCUSSION

**Theorem 1:** The graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ admits fuzzy tri-magic labeling for $n \geq 6$ and $m \geq 3$.

**Proof:**

Let $G$ be a graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$. Let the vertex set and edge set of $G$ be

$V(G) = \{v_j : 1 \leq j \leq n\} \cup \{u_j : 1 \leq j \leq m\}$ and

$E(G) = \{v_j v_{j+1} : 1 \leq j \leq n-1\} \cup \{v_1 v_n\} \cup \{v_n u_j : 1 \leq j \leq m\}$

$|V(G)| = m + n$ and $|E(G)| = m + n$

Let $r \geq 2$ be any positive integer.

Define $\sigma : V \to [0,1]$ such that

$$\sigma(v_j) = (6n + 2m - 2j)\frac{1}{10^r} 1 \leq j \leq n$$

Define $\mu : V \times V \to [0,1]$ by

$$\mu(v_n u_j) = (4n - 3 + j)\frac{1}{10^r} \text{ for } 1 \leq j \leq m$$

$$\mu(v_1 v_n) = \frac{2n-1}{10^r}$$

**Case (i)** If $n \equiv 0 (mod\ 6)$

**Subcase (i)** If $m \equiv 0 (mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\frac{1}{10^r}$ for $1 \leq j \leq \frac{m}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\frac{1}{10^r}$ for $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\frac{1}{10^r}$ for $\frac{2m}{3} + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n-1$

**Subcase (ii)** If $m \equiv 1 (mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\frac{1}{10^r}$ for $1 \leq j \leq \frac{m-1}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\frac{1}{10^r}$ for $\frac{m-1}{3} + 1 \leq j \leq 2(\frac{m-1}{3})$

$\sigma(u_j) = (4n + 2m - 2 - j)\frac{1}{10^r}$ for $2(\frac{m-1}{3}) + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n-1$

**Subcase (iii)** If $m \equiv 2 (mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\frac{1}{10^r}$ for $1 \leq j \leq \frac{m-2}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\frac{1}{10^r}$ for $\frac{m-2}{3} + 1 \leq j \leq \frac{2m-1}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\frac{1}{10^r}$ for $\frac{2m-1}{3} + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n-1$

Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) are given in table 1.

**Table 1:** Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) for the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ for $n \equiv 0 (mod\ 3)$, $m \equiv 0 (mod\ 3), 1 (mod\ 3)$ and $2 (mod\ 3)$.

| Natureo f $n$ | Natureof $m$ | Edges | MMV $K_i$'s, $1 \leq i \leq 3$ | Number of $K_i$'s, $1 \leq i \leq 3$ |
|---|---|---|---|---|
| $n \equiv 0 (mod\ 3)$ | $m \equiv 0 (mod\ 3)$ | $g(v_j v_{j+1})$ if $\frac{2n}{3} + 1 \leq j \leq n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m}{3}$ | $(12n + 4m - 3)\frac{1}{10^r}$ for i = 1 | $\frac{m+n}{3}$ for i = 1 |
| | | $g(v_j v_{j+1})$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$<br>$g(v_n u_j)$ if $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$ | $(12n + 4m - 4)\frac{1}{10^r}$ for i = 2 | $\frac{m+n}{3}$ for i = 2 |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n}{3}$<br>$g(v_n u_j)$ if $\frac{2m}{3} + 1 \leq j \leq m$ | $(12n + 4m - 5)\frac{1}{10^r}$ for i = 3 | $\frac{m+n}{3}$ for i = 3 |
| | $m \equiv 1 (mod\ 3)$ | $g(v_j v_{j+1})$ if $\frac{2n}{3} + 1 \leq j \leq n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m-1}{3}$ | $(12n + 4m - 3)\frac{1}{10^r}$ for i = 1 | $\frac{m+n-1}{3}$ for i = 1 |
| | | $g(v_j v_{j+1})$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$<br>$g(v_n u_j)$ if $\frac{m-1}{3} + 1 \leq j \leq 2(\frac{m-1}{3})$ | $(12n + 4m - 4)\frac{1}{10^r}$ for i = 2 | $\frac{m+n-1}{3} + 1$ for i = 2 |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n}{3}$<br>$g(v_n u_j)$ if $2(\frac{m-1}{3}) + 1 \leq j \leq m$ | $(12n + 4m - 5)\frac{1}{10^r}$ for i = 3 | $\frac{m+n-1}{3} + 1$ for i = 3 |
| | $m \equiv 2 (mod\ 3)$ | $g(v_j v_{j+1})$ if $\frac{2n}{3} + 1 \leq j \leq n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m-2}{3}$ | $(12n + 4m - 3)\frac{1}{10^r}$ for i = 1 | $\frac{m+n-2}{3}$ for i = 1 |
| | | $g(v_j v_{j+1})$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$<br>$g(v_n u_j)$ if $\frac{m-2}{3} + 1 \leq j \leq \frac{2m-1}{3}$ | $(12n + 4m - 4)\frac{1}{10^r}$ for i = 2 | $\frac{m+n-2}{3} + 1$ for i = 2 |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n}{3}$<br>$g(v_n u_j)$ if $\frac{2m-1}{3} + 1 \leq j \leq m$ | $(12n + 4m - 5)\frac{1}{10^r}$ for i = 3 | $\frac{m+n-2}{3} + 1$ for i = 3 |

**Case (ii)** If $n \equiv 1 (mod\ 6)$

**Subcase (i)** If $m \equiv 0(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m}{3} + 1 \le j \le \frac{2m}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $\frac{2m}{3} + 1 \le j \le m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\ \frac{1}{10^r}$ for $1 \le j \le \frac{n+2}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\ \frac{1}{10^r}$ for $\frac{n+2}{3} + 1 \le j \le 2\left(\frac{n+2}{3}\right) - 1$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\ \frac{1}{10^r}$ for $2\left(\frac{n+2}{3}\right) \le j \le n - 1$

**Subcase (ii)** If $m \equiv 1(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m+2}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+2}{3} + 1 \le j \le 2(\frac{m+2}{3}) - 1$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2(\frac{m+2}{3}) \le j \le m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\ \frac{1}{10^r}$ for $1 \le j \le \frac{n+2}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\ \frac{1}{10^r}$ for $\frac{n+2}{3} + 1 \le j \le 2\left(\frac{n+2}{3}\right) - 1$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\ \frac{1}{10^r}$ for $2\left(\frac{n+2}{3}\right) \le j \le n - 1$

**Subcase (iii)** If $m \equiv 2(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m+1}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+1}{3} + 1 \le j \le 2(\frac{m+1}{3})$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2(\frac{m+1}{3}) + 1 \le j \le m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\ \frac{1}{10^r}$ for $1 \le j \le \frac{n+2}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\ \frac{1}{10^r}$ for $\frac{n+2}{3} + 1 \le j \le 2\left(\frac{n+2}{3}\right) - 1$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\ \frac{1}{10^r}$ for $2\left(\frac{n+2}{3}\right) \le j \le n - 1$

Edges with MMV $K_i$ and number of $K_i$ ($1 \le i \le 3$) are given in table 2.

**Table 2:** Edges with MMV $K_i$ and number of $K_i$ ($1 \le i \le 3$) for the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ for $n \equiv 1(mod\ 3)$, $m \equiv 0(mod\ 3), 1(mod\ 3)$ and $2(mod\ 3)$.

| Natureof $n$ | Natureof $m$ | Edges | MMV $K_i$'s, $1 \le i \le 3$ | Number of $K_i$'s, $1 \le i \le 3$ |
|---|---|---|---|---|
| $n \equiv 1(mod\ 3)$ | $m \equiv 0(mod\ 3)$ | $g(v_j v_{j+1})$ if $2(\frac{n+2}{3}) \le j \le n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \le j \le \frac{m}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-1}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+2}{3} + 1 \le j \le 2(\frac{n+2}{3}) - 1$<br>$g(v_n u_j)$ if $\frac{m}{3} + 1 \le j \le \frac{2m}{3}$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-1}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \le j \le \frac{n+2}{3}$<br>$g(v_n u_j)$ if $\frac{2m}{3} + 1 \le j \le m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-1}{3} + 1$ for $i = 3$ |
| | $m \equiv 1(mod\ 3)$ | $g(v_j v_{j+1})$ if $2(\frac{n+2}{3}) \le j \le n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \le j \le \frac{m+2}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-2}{3} + 1$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+2}{3} + 1 \le j \le 2\left(\frac{n+2}{3}\right) - 1$<br>$g(v_n u_j)$ if $\frac{m+2}{3} + 1 \le j \le 2\left(\frac{m+2}{3}\right) - 1$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-2}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \le j \le \frac{n+2}{3}$<br>$g(v_n u_j)$ if $2(\frac{m+2}{3}) \le j \le m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-2}{3} + 1$ for $i = 3$ |
| | $m \equiv 2(mod\ 3)$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+2}{3}\right) \le j \le n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \le j \le \frac{m+1}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+2}{3} + 1 \le j \le 2\left(\frac{n+2}{3}\right) - 1$<br>$g(v_n u_j)$ if $\frac{m+1}{3} + 1 \le j \le 2(\frac{m+1}{3})$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \le j \le \frac{n+2}{3}$<br>$g(v_n u_j)$ if $2(\frac{m+1}{3}) + 1 \le j \le m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n}{3}$ for $i = 3$ |

**Case (iii)** If $n \equiv 2(mod\ 6)$

**Subcase (i)** If $m \equiv 0(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m}{3} + 1 \le j \le \frac{2m}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $\frac{2m}{3} + 1 \le j \le m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\ \frac{1}{10^r}$ for $1 \le j \le \frac{n+1}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\ \frac{1}{10^r}$ for $\frac{n+1}{3} + 1 \le j \le 2\left(\frac{n+1}{3}\right)$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\ \frac{1}{10^r}$ for $2\left(\frac{n+1}{3}\right) + 1 \le j \le n - 1$

**Subcase (ii)** If $m \equiv 1(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m+2}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+2}{3} + 1 \le j \le 2(\frac{m+2}{3}) - 1$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2(\frac{m+2}{3}) \le j \le m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\ \frac{1}{10^r}$ for $1 \le j \le \frac{n+1}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\ \frac{1}{10^r}$ for $\frac{n+1}{3} + 1 \le j \le 2\left(\frac{n+1}{3}\right)$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\ \frac{1}{10^r}$ for $2\left(\frac{n+1}{3}\right) + 1 \le j \le n - 1$

**Subcase (iii)** If $m \equiv 2(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m+1}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+1}{3} + 1 \le j \le 2(\frac{m+1}{3})$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2(\frac{m+1}{3}) + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n+1}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n+1}{3} + 1 \leq j \leq 2\left(\frac{n+1}{3}\right)$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $2\left(\frac{n+1}{3}\right) + 1 \leq j \leq n - 1$

Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) are given in table 3.

**Table 3:** Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) for the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ for $n \equiv 2(mod\ 3)$, $m \equiv 0(mod\ 3), 1(mod\ 3)$ and $2(mod\ 3)$.

| Natureo f $n$ | Natureof $m$ | Edges | MMV $K_i$'s, $1 \leq i \leq 3$ | Number of $K_i$'s, $1 \leq i \leq 3$ |
|---|---|---|---|---|
| $n \equiv 2(mod\ 3)$ | $m \equiv 0(mod\ 3)$ | $g(v_j v_{j+1})$ if $2(\frac{n+1}{3}) + 1 \leq j \leq n - 1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-2}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+1}{3} + 1 \leq j \leq 2(\frac{n+1}{3})$<br>$g(v_n u_j)$ if $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-2}{3} + 1$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n+1}{3}$<br>$g(v_n u_j)$ if $\frac{2m}{3} + 1 \leq j \leq m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-2}{3} + 1$ for $i = 3$ |
| | $m \equiv 1(mod\ 3)$ | $g(v_j v_{j+1})$ if $2(\frac{n+1}{3}) + 1 \leq j \leq n - 1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m+2}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+1}{3} + 1 \leq j \leq 2\left(\frac{n+1}{3}\right)$<br>$g(v_n u_j)$ if $\frac{m+2}{3} + 1 \leq j \leq 2\left(\frac{m+2}{3}\right) - 1$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n+1}{3}$<br>$g(v_n u_j)$ if $2(\frac{m+2}{3}) \leq j \leq m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n}{3}$ for $i = 3$ |
| | $m \equiv 2(mod\ 3)$ | $g(v_j v_{j+1})$ if $2(\frac{n+1}{3}) + 1 \leq j \leq n - 1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m+1}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-1}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+1}{3} + 1 \leq j \leq 2\left(\frac{n+1}{3}\right)$<br>$g(v_n u_j)$ if $\frac{m+1}{3} + 1 \leq j \leq 2(\frac{m+1}{3})$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-1}{3} + 1$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n+1}{3}$<br>$g(v_n u_j)$ if $2(\frac{m+1}{3}) + 1 \leq j \leq m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-1}{3}$ for $i = 3$ |

**Case (iv)** If $n \equiv 3(mod\ 6)$

**Subcase (i)** If $m \equiv 0(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $\frac{2m}{3} + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n - 1$

**Subcase (ii)** If $m \equiv 1(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m-1}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m-1}{3} + 1 \leq j \leq 2(\frac{m-1}{3})$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2\left(\frac{m-1}{3}\right) + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n - 1$

**Subcase (iii)** If $m \equiv 2(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m-2}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m-2}{3} + 1 \leq j \leq \frac{2m-1}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $\frac{2m-1}{3} + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{2n}{3} + 1 \leq j \leq n - 1$

Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) are given in table 4.

**Table 4:** Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) for the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ for $n \equiv 3(mod\ 3)$, $m \equiv 0(mod\ 3), 1(mod\ 3)$ and $2(mod\ 3)$.

| Natureo f $n$ | Natureof $m$ | Edges | MMV $K_i$'s, $1 \leq i \leq 3$ | Number of $K_i$'s, $1 \leq i \leq 3$ |
|---|---|---|---|---|
| $n \equiv 2(mod\ 3)$ | $m \equiv 0(mod\ 3)$ | $g(v_j v_{j+1})$ if $\frac{2n}{3} + 1 \leq j \leq n - 1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$<br>$g(v_n u_j)$ if $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n}{3}$<br>$g(v_n u_j)$ if $\frac{2m}{3} + 1 \leq j \leq m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n}{3}$ for $i = 3$ |
| | $m \equiv 1(mod\ 3)$ | $g(v_j v_{j+1})$ if $\frac{2n}{3} + 1 \leq j \leq n - 1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m-1}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-1}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$<br>$g(v_n u_j)$ if $\frac{m-1}{3} + 1 \leq j \leq 2\left(\frac{m-1}{3}\right)$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-1}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n}{3}$<br>$g(v_n u_j)$ if $2(\frac{m-1}{3}) + 1 \leq j \leq m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-1}{3} + 1$ for $i = 3$ |
| | $m \equiv 2(mod\ 3)$ | $g(v_j v_{j+1})$ if $\frac{2n}{3} + 1 \leq j \leq n - 1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m-2}{3}$ | $(12n + 4m - 3)\ \frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-2}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n}{3} + 1 \leq j \leq \frac{2n}{3}$<br>$g(v_n u_j)$ if $\frac{m-2}{3} + 1 \leq j \leq \frac{2m-1}{3}$ | $(12n + 4m - 4)\ \frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-2}{3} + 1$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n}{3}$<br>$g(v_n u_j)$ if $\frac{2m-1}{3} + 1 \leq j \leq m$ | $(12n + 4m - 5)\ \frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-2}{3} + 1$ for $i = 3$ |

**Case (v)** If $n \equiv 4(mod\ 6)$

**Subcase (i)** If $m \equiv 0(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $\frac{2m}{3} + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n+2}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n+2}{3} + 1 \leq j \leq 2\left(\frac{n+2}{3}\right) - 1$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $2\left(\frac{n+2}{3}\right) \leq j \leq n - 1$

**Subcase (ii)** If $m \equiv 1(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m+2}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+2}{3} + 1 \leq j \leq 2(\frac{m+2}{3}) - 1$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2\left(\frac{m+2}{3}\right) \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n+2}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n+2}{3} + 1 \leq j \leq 2\left(\frac{n+2}{3}\right) - 1$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\frac{1}{10^r}$ for $2\left(\frac{n+2}{3}\right) \leq j \leq n - 1$

**Subcase (iii)** If $m \equiv 2(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m+1}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+1}{3} + 1 \leq j \leq 2(\frac{m+1}{3})$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2(\frac{m+1}{3}) + 1 \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n+2}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n+2}{3} + 1 \leq j \leq 2\left(\frac{n+2}{3}\right) - 1$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $2\left(\frac{n+2}{3}\right) \leq j \leq n - 1$

Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) are given in table 5.

**Table 5:** Edges with MMV $K_i$ and number of $K_i$ ($1 \leq i \leq 3$) for the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ for $n \equiv 4(mod\ 3)$, $m \equiv 0(mod\ 3), 1(mod\ 3)$ and $2(mod\ 3)$.

| Natureo f $n$ | Natureof $m$ | Edges | MMV $K_i's$, $1 \leq i \leq 3$ | Number of $K_i's$, $1 \leq i \leq 3$ |
|---|---|---|---|---|
| $n \equiv 2(mod\ 3)$ | $m \equiv 0(mod\ 3)$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+2}{3}\right) \leq j \leq n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m}{3}$ | $(12n+4m-3)\frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-1}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+2}{3} + 1 \leq j \leq 2\left(\frac{n+2}{3}\right) - 1$<br>$g(v_n u_j)$ if $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$ | $(12n+4m-4)\frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-1}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n+2}{3}$<br>$g(v_n u_j)$ if $\frac{2m}{3} + 1 \leq j \leq m$ | $(12n+4m-5)\frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-1}{3} + 1$ for $i = 3$ |
| | $m \equiv 1(mod\ 3)$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+2}{3}\right) \leq j \leq n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m+2}{3}$ | $(12n+4m-3)\frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-2}{3} + 1$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+2}{3} + 1 \leq j \leq 2\left(\frac{n+2}{3}\right) - 1$<br>$g(v_n u_j)$ if $\frac{m+2}{3} + 1 \leq j \leq 2\left(\frac{m+2}{3}\right) - 1$ | $(12n+4m-4)\frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-2}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n+2}{3}$<br>$g(v_n u_j)$ if $2(\frac{m+2}{3}) \leq j \leq m$ | $(12n+4m-5)\frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-2}{3} + 1$ for $i = 3$ |
| | $m \equiv 2(mod\ 3)$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+2}{3}\right) \leq j \leq n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \leq j \leq \frac{m+1}{3}$ | $(12n+4m-3)\frac{1}{10^r}$ for $i = 1$ | $\frac{m+n}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+2}{3} + 1 \leq j \leq 2\left(\frac{n+2}{3}\right) - 1$<br>$g(v_n u_j)$ if $\frac{m+1}{3} + 1 \leq j \leq 2\left(\frac{m+1}{3}\right)$ | $(12n+4m-4)\frac{1}{10^r}$ for $i = 2$ | $\frac{m+n}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \leq j \leq \frac{n+2}{3}$ | $(12n+4m$ | $\frac{m+n}{3}$ for $i = 3$ |

**Case (vi)** If $n \equiv 5(mod\ 6)$

**Subcase (i)** If $m \equiv 0(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m}{3} + 1 \leq j \leq \frac{2m}{3}$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $\frac{2m}{3} + 1\ u$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n+1}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n+1}{3} + 1 \leq j \leq 2\left(\frac{n+1}{3}\right)$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $2\left(\frac{n+1}{3}\right) + 1 \leq j \leq n - 1$

**Subcase (ii)** If $m \equiv 1(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{m+2}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+2}{3} + 1 \leq j \leq 2(\frac{m+2}{3}) - 1$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2\left(\frac{m+2}{3}\right) \leq j \leq m$

$\mu(v_j v_{j+1}) = 1 + 4(j - 1)\ \frac{1}{10^r}$ for $1 \leq j \leq \frac{n+1}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j - 1)\ \frac{1}{10^r}$ for $\frac{n+1}{3} + 1 \leq j \leq 2\left(\frac{n+1}{3}\right)$

$\mu(v_j v_{j+1}) = 3 + 4(j - 1)\ \frac{1}{10^r}$ for $2\left(\frac{n+1}{3}\right) + 1 \leq j \leq n - 1$

**Subcase (iii)** If $m \equiv 2(mod\ 3)$

$\sigma(u_j) = (4n + 2m - j)\ \frac{1}{10^r}$ for $1 \le j \le \frac{m+1}{3}$

$\sigma(u_j) = (4n + 2m - 1 - j)\ \frac{1}{10^r}$ for $\frac{m+1}{3} + 1 \le j \le 2(\frac{m+1}{3})$

$\sigma(u_j) = (4n + 2m - 2 - j)\ \frac{1}{10^r}$ for $2(\frac{m+1}{3}) + 1 \le j \le m$

$\mu(v_j v_{j+1}) = 1 + 4(j-1)\ \frac{1}{10^r}$ for $1 \le j \le \frac{n+1}{3}$

$\mu(v_j v_{j+1}) = 2 + 4(j-1)\ \frac{1}{10^r}$ for $\frac{n+1}{3} + 1 \le j \le 2\left(\frac{n+1}{3}\right)$

$\mu(v_j v_{j+1}) = 3 + 4(j-1)\frac{1}{10^r}$ for $2\left(\frac{n+1}{3}\right) + 1 \le j \le n-1$

Edges with MMV $K_i$ and number of $K_i$ ($1 \le i \le 3$) are given in table 6.

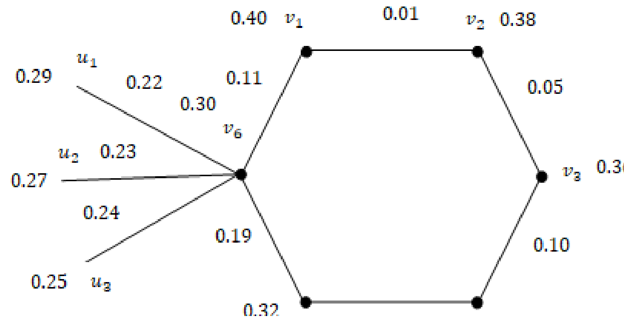
**Table 6:** Edges with MMV $K_i$ and number of $K_i$ ($1 \le i \le 3$) for the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ for $n \equiv 5(mod\ 3)$, $m \equiv 0(mod\ 3), 1(mod\ 3)$ and $2(mod\ 3)$.

| Natureo f $n$ | Natureof $m$ | Edges | MMV $K_i$'s, $1 \le i \le 3$ | Number of $K_i$'s, $1 \le i \le 3$ |
|---|---|---|---|---|
| $n \equiv 2(mod\ 3)$ | $m \equiv 0$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+1}{3}\right) + 1 \le j \le n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \le j \le \frac{m}{3}$ | $(12n + 4m - 3)\frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-2}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+1}{3} + 1 \le j \le 2\left(\frac{n+1}{3}\right)$<br>$g(v_n u_j)$ if $\frac{m}{3} + 1 \le j \le \frac{2m}{3}$ | $(12n + 4m - 4)\frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-2}{3} + 1$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \le j \le \frac{n+1}{3}$<br>$g(v_n u_j)$ if $\frac{2m}{3} + 1 \le j \le m$ | $(12n + 4m - 5)\frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-2}{3} + 1$ for $i = 3$ |
| | $m \equiv 1(mod\ 3)$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+1}{3}\right) + 1 \le j \le n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \le j \le \frac{m+2}{3}$ | $(12n + 4m - 3)\frac{1}{10^r}$ for $i = 1$ | $\frac{m+n}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+1}{3} + 1 \le j \le 2\left(\frac{n+1}{3}\right)$<br>$g(v_n u_j)$ if $\frac{m+2}{3} + 1 \le j \le 2\left(\frac{m+2}{3}\right) - 1$ | $(12n + 4m - 4)\frac{1}{10^r}$ for $i = 2$ | $\frac{m+n}{3}$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \le j \le \frac{n+1}{3}$<br>$g(v_n u_j)$ if $2(\frac{m+2}{3}) \le j \le m$ | $(12n + 4m - 5)\frac{1}{10^r}$ for $i = 3$ | $\frac{m+n}{3}$ for $i = 3$ |
| | $m \equiv 2(mod\ 3)$ | $g(v_j v_{j+1})$ if $2\left(\frac{n+1}{3}\right) + 1 \le j \le n-1$<br>$g(v_1 v_n)$<br>$g(v_n u_j)$ if $1 \le j \le \frac{m+1}{3}$ | $(12n + 4m - 3)\frac{1}{10^r}$ for $i = 1$ | $\frac{m+n-1}{3}$ for $i = 1$ |
| | | $g(v_j v_{j+1})$ if $\frac{n+1}{3} + 1 \le j \le 2\left(\frac{n+1}{3}\right)$<br>$g(v_n u_j)$ if $\frac{m+1}{3} + 1 \le j \le 2\left(\frac{m+1}{3}\right)$ | $(12n + 4m - 4)\frac{1}{10^r}$ for $i = 2$ | $\frac{m+n-1}{3} + 1$ for $i = 2$ |
| | | $g(v_j v_{j+1})$ if $1 \le j \le \frac{n+1}{3}$<br>$g(v_n u_j)$ if $2\left(\frac{m+1}{3}\right) + 1 \le j \le m$ | $(12n + 4m - 5)\frac{1}{10^r}$ for $i = 3$ | $\frac{m+n-1}{3}$ for $i = 3$ |

Hence, the maximum difference between the number of $K_i$'s is 1 and

$|K_i - K_j| \le \frac{2}{10^r}$ for $1 \le i, j \le 3$. Hence, the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ admits fuzzy tri-magic labeling for $n \ge 6$ and $m \ge 3$.

**Example 1:** The cycle $C_6$ that admits fuzzy tri-magic labeling as shown in Figure 4.3.




Taking $r = 2$

$K_1 = 0.81$, $K_2 = 0.80$ and $K_3 = 0.79$

**Figure 1:** The cycle $C_6$ admits fuzzy tri-magic labeling for $n = 3$

## IV. CONCLUSION

This study explainedthe fuzzy tri-magic labeling of some unicyclic graphs. It has been proved that the graph obtained by attaching star graph $S_{1,m}$ to the end vertex of the cycle $C_n$ admits fuzzy tri-magic labeling for $n \ge 6$ and $m \ge 3$. We have given an example to prove that the cycle $C_6$ admits fuzzy tri-magic labeling for $n = 3$. We are also working on the other examples of unicyclic graphs, which will be reported in subsequent works.

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