

Branching fraction measurement for $B^0 \rightarrow J/\psi \pi^+ \pi^-$ using 449×10^6 BB pairs

Rameet Kaur Dahalay¹, Rajeev Kumar Sharma²

^{1,2}Department of Mathematics, Statistics & Physics

^{1,2}Punjab Agricultural University, Ludhiana.141004

Abstract- The decays of B mesons to states containing charmonium provides an excellent laboratory for the study of hadronic B decays. At the quark level, the decay process for $B^0 \rightarrow J/\psi \pi^+ \pi^-$ is described by $b \rightarrow c\bar{c}d$ transition and is a flavor non-specific state. It is both a Cabibbo and color suppressed decay and measurement of time dependent CP asymmetry may help to find CP violation parameters that are different from the values for $b \rightarrow c\bar{c}s$, if penguin or other non leading contributions are found to be significant. The data set used in present analysis contains approximately 449×10^6 BB pairs that has been collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We report the measurement of branching fraction of neutral B mesons to a charmonium (J/ψ) accompanied by a charged pion pair. The measured branching fraction $\mathcal{B}(B^0 \rightarrow J/\psi \pi^+ \pi^-) = (1.90 \pm 0.17 \pm 0.12) \times 10^{-5}$, where the first error is statistical and the second error is systematic.

Keywords- charmonium resonance, CP violation, Belle Detector, B mesons, flavor non-specific state.

I. INTRODUCTION

B decays provide a very rich ground for the test of the Standard Model (SM). The charmonium mesons from B decays play an important role in the studies of CP violation phenomena. The production mechanism for B decays also provides the test ground for low energy Quantum Chromodynamics (QCD). In the Standard Model, the decay $B^0 \rightarrow J/\psi \pi^+ \pi^-$ can give rise to CP-violating asymmetries, directly and through B^0 - \bar{B}^0 mixing [1]. Therefore, it is interesting to study the decay mode $B^0 \rightarrow J/\psi \pi^+ \pi^-$ to understand the $J/\psi \pi^+ \pi^-$ component in the final state. Since these decays are Cabibbo and color suppressed, they could be sensitive to non-Standard-Model processes which contribute through, for example, penguin amplitudes. The branching fraction may differ significantly from the Standard Model prediction of $\mathcal{B}(B^0 \rightarrow J/\psi \pi^+ \pi^-) = (4.8 \pm 0.8) \times 10^{-5}$ if the non-Standard Model effects are significantly large.[2].

The decay $B^0 \rightarrow J/\psi \pi^+ \pi^-$ [3] can also be used to measure the CP violation parameter $\sin 2\beta$. However, the measurement is not as straightforward as for $J/\psi K_S^0$ [4,5],

because it involves the decay of a pseudoscalar meson to two vector mesons, resulting in both CP-odd and CP-even final states. The decay can occur either through a color-suppressed tree diagram, or a penguin diagram, both shown in figure 1, and interference between them may result in direct CP violation [6]. Both, tree and penguin amplitudes contribute to the $b \rightarrow c\bar{c}d$ transition in the same order of $\sin \theta_c$.

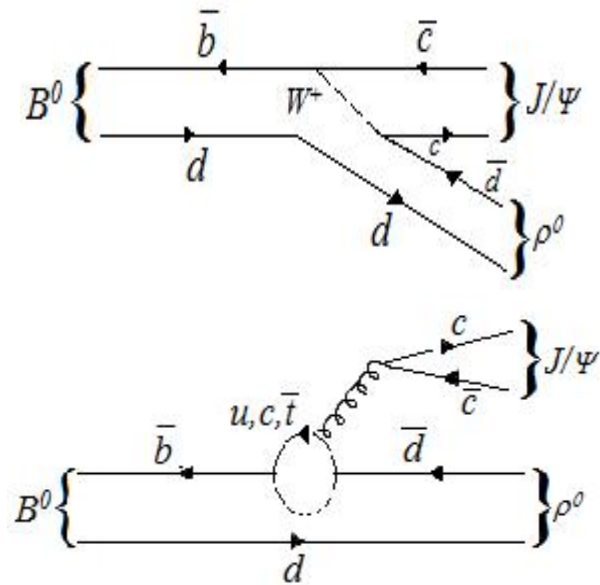


Figure 1. Tree and penguin diagrams for the process $B^0 \rightarrow J/\psi \rho^0$.

Therefore, if penguin or other contributions are substantial, a precision measurement of the time dependent CP asymmetry in $b \rightarrow c\bar{c}d$ may reveal the values that differ from those for $b \rightarrow c\bar{c}s$. Thus, B decays induced by the $b \rightarrow c\bar{c}d$ transition, such as $B^0 \rightarrow J/\psi \rho^0$, play an important role in probing non-tree diagram contributions. Since the ρ^0 meson has a large width, it is necessary to study all the decays of the neutral B meson that result in a $J/\psi \pi^+ \pi^-$ final state. In general, there are two possible contributions to the $B^0 \rightarrow J/\psi \pi^+ \pi^-$ mode. One is resonant, for instance, when a $\pi^+ \pi^-$ pair arises from some resonant state. The $B^0 \rightarrow J/\psi \rho^0$ decay mode has the largest contribution in this class, although it is important to search for other resonant contributions. The other contribution is non-resonant, where a neutral B meson decays directly into a J/ψ component and a $\pi^+ \pi^-$ pair. For this process, the CP eigenvalue is unknown; hence, when measuring CP violation in $B^0 \rightarrow J/\psi \rho^0$ decays the evaluation of this contribution is

important to control uncertainties. Decay channels involving J/ψ mesons are well-suited for such studies since the $J/\psi \rightarrow \mu^+ \mu^-$ decay provides a distinctive experimental signature and allows a good measurement of the secondary vertex position. The Kobayashi- Maskawa (KM) matrix [7] is incorporated in weak decays sector of the Standard Model to explain the quark mixing and CP violation. Hence, the experimental measurements of all the KM matrix elements will either complete our understanding on the weak interaction or lead to the new physics beyond the Standard Model.

II. KEKB ACCELERATOR AND THE BELLE DETECTOR

The desire to produce as many B mesons as possible forces us to operate at the $\Upsilon(4S)$ resonance, which lies right above the $B\bar{B}$ decay threshold and predominantly decays into $B\bar{B}$. The relatively high mass of the b quark ensures an abundance of possible final states and, as a result, any given decay will have a relatively low branching ratio, thus requiring a high luminosity collider. KEKB is a two-ring asymmetric energy e^+e^- collider [8] designed to produce a huge number of B and \bar{B} meson pairs. The “B” at the end of KEKB refers to its primary goal, the production of B mesons. To produce largely boosted B mesons for the time-dependent CP violation study, e^+ and e^- beam energies must be asymmetric. In that case, these beams cannot have the same orbit under common magnetic field and thus KEKB is designed to have two separate rings for the e^+ and the e^- beams. The e^+ and the e^- beams are injected directly into the main rings at Fuji area from a linear accelerator. The e^+ beam, which is called the Low Energy Ring (LER), circulates anti-clockwise with energy $E^+ = 3.5$ GeV and the e^- beam, which is called the High Energy Ring (HER), circulates clockwise with energy $E^- = 8.0$ GeV. The KEKB has two crossing points at Tsukuba and Fuji experimental hall. The beams collide at the interaction point (IP) in the Tsukuba hall, where the Belle detector is located.

The Belle detector is a large-solid-angle magnetic spectrometer. Closest to the interaction point is a three-layer silicon vertex detector (SVD), followed by a 50-layer central drift chamber (CDC), an array of aerogel threshold ~ Cerenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI (TI) crystals (ECL). These subdetectors are located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect K_L^0 mesons and to identify muons (KLM). The detector is described in detail elsewhere [9].

III. DATA SET AND EVENT SELECTION

The results are based on 414 fb^{-1} data corresponding to 449×10^6 BB pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. The first 152×10^6 B meson pairs were collected with a 2.0 cm radius beam-pipe and a three layer SVD, and remaining 297×10^6 B meson pairs with a 1.5 cm radius beam-pipe, a four layer SVD and a small-cell inner drift chamber [10,11]. The event selection criteria are designed to select B decays with high efficiency while suppressing lepton pairs, two-photon events, and interactions with the beam pipe or residual gas in the beam line.

We require the hadronic events to satisfy the following criteria: reconstructed charged tracks must be greater than 2; a total reconstructed ECL energy in the center of mass (cms) frame in the range between $0.1 \bar{s}$ and $0.8 \bar{s}$ where \bar{s} is the total energy in cms; an average ECL cluster energy below 1 GeV; at least one ECL shower in the region $-0.7 < \cos\theta < 0.9$ in the laboratory frame where θ is the polar angle in cylindrical coordinates of the track from the beam axis (z-axis); a total visible energy E_{vis}^* , i.e. the sum of charged track momenta and total ECL energy, that must exceed $0.2 \bar{s}$ and reconstructed primary vertex that is consistent with the known interaction point. After the imposition of these requirements, the efficiency for selecting B-meson pairs that include a J/ψ meson is estimated by Monte Carlo (MC) simulation to be 99%. To suppress continuum events (qq events), we require the event shape variable R_2 to be less than 0.5, where R_2 is the ratio of the second to the zeroth Fox-Wolfram moment [12].

IV. RECONSTRUCTION OF J/ψ

J/ψ mesons are reconstructed via their decay into oppositely charged lepton pairs (e^+e^- or $\mu^+\mu^-$). Leptons are selected by starting with charged tracks satisfying $|dz| < 5$ cm, where dz is the track’s closest approach to the interaction point along the beam direction and $|dr| < 0.5$ cm. For electron identification, the ratio between the charged track’s momentum and the associated shower energy (E/p) is the most powerful discriminant. Other information including dE/dx , the distance between the ECL shower and the extrapolated track, and the shower shape are also used in this identification. Muons are identified by requiring an association between KLM hits and an extrapolated track. For the track to be identified as muon, the muon likelihood ratio must be greater than 0.1. The track is considered as electron if it satisfies the criteria of electron likelihood to be greater than 0.1. Both lepton tracks must be positively identified as such. In the e^+e^-

mode, ECL clusters that are within 50 mrad of the track's initial momentum vector are included in the calculation of the invariant mass (M_{+e^-}), in order to include photons radiated from electrons/positrons. Photon candidates are selected from clusters of up to 5×5 crystals in the ECL. Each photon candidate is required to have no associated charged track, and a cluster shape that is consistent with an electromagnetic shower. The invariant masses of e^+e^- and $\mu^+\mu^-$ combinations are required to fall in the ranges -0.06 (-0.15) $\text{GeV}/c^2 \leq (M_{l^+l^-} - M_{J/\psi}) \leq 0.036$ GeV/c^2 , respectively, where $M_{l^+l^-}$ is the invariant mass of a lepton pair and $M_{J/\psi}$ denotes the world average of the J/ψ mass [13].

Information from the ACC, TOF and CDC is combined to form π -K likelihood ratio, $\mathcal{L}_{\pi/K} = \mathcal{L}_{\pi}/(\mathcal{L}_{\pi} + \mathcal{L}_K)$, where $\mathcal{L}_{\pi}(\mathcal{L}_K)$ is the likelihood that a pion (kaon) would produce the observed detector response; we then impose a cut to reject kaons. Charged tracks with $\mathcal{R}_{\pi/K} > 0.9$ are selected as charged pions, and tracks with $\mathcal{R}_{\pi/K} \leq 0.4$ are selected as charged kaons. The efficiency for pion (kaon) identification is 75.1% (86.1%) and the probability of kaon (pion) misidentification is 4.6% (10.5%) with the above criteria. The selection criteria has been determined by optimizing the figure of merit, $S/\sqrt{S+B}$, where $S(B)$ is the number of signal (background) events in the signal region. The efficiency for pions is more than 85% while the kaon fake rate is 10%. Charged pion candidates are also rejected if they are positively identified as leptons.

V. RECONSTRUCTION OF B

We reconstruct B candidate selection using two observables in the rest frame of the $\Upsilon(4S)$ (cms): the beam-energy constrained mass, $M_{bc} = \sqrt{(\vec{p}_{beam}^*)^2 - |\vec{p}_i^*|^2}$ and the energy difference, $E = \sum_i E_i^* - E_{beam}^*$, where $E_{beam} = s/2$ is the cms beam energy, and \vec{p}_i^* and E_i^* are the cms three-momenta and energies of the B meson decay product candidates respectively. In order to improve ΔE and M_{bc} resolutions, we applied the vertex fit to the $+\pi^-$ pairs. To reduce the $B^0 \rightarrow J/\psi K_s^0$ events and the backgrounds due to accidentally formed pion pairs, we require the distance between the reconstructed vertices of the J/ψ and the $+\pi^-$ pair to be less than 3 mm. The fraction of events that one $J/\psi\pi^+\pi^-$ combination in $-0.2 < E < 0.2$ and $5.2 \text{ GeV}/c^2 < M_{bc}$ in a event is found is 15%. The best candidate events are selected by requiring the smallest χ^2 of the vertex fit of charged pion tracks in the final state, because such multiple candidate events are totally due to the number of $+\pi^-$ combination in the event. The B meson signal region is defined as $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$ and $-0.04 \text{ GeV} <$

$E < 0.04 \text{ GeV}$. Figure 2 shows the 2D scatter plot of $M_{bc} - E$ distribution and its projection in signal region of ΔE (i.e. M_{bc} distribution) and signal region of M_{bc} (i.e. ΔE distribution).

Figure 3 shows the fit to invariant mass distributions of $\mu^+\mu^-$ and e^+e^- pairs from experimental data respectively, where the dashed lines indicate the edges of the mass window. The signal peak of J/ψ is modeled by crystal ball line shaped function and background region by third order Chebyshev polynomial for other flat backgrounds.

For the dielectrons case, mass of J/ψ comes out to be $1.37 \pm 0.23 \text{ MeV}/c^2$ more in Monte Carlo than in data (3095.87 vs. $3094.50 \text{ MeV}/c^2$). The width of the signals are in fair agreement (11.59 vs. $10.42 \text{ MeV}/c^2$). For the dimuons case, the Monte Carlo mass is greater by $1.07 \pm 0.13 \text{ MeV}/c^2$ (3097.85 vs. $3096.78 \text{ MeV}/c^2$). The width of the dimuon Monte Carlo is a bit narrower than in

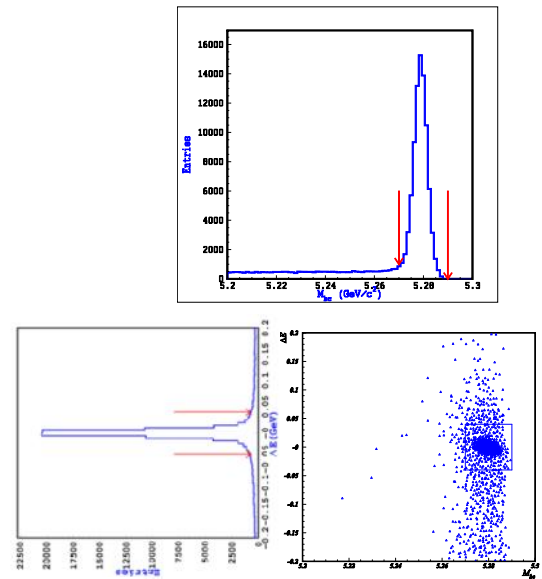
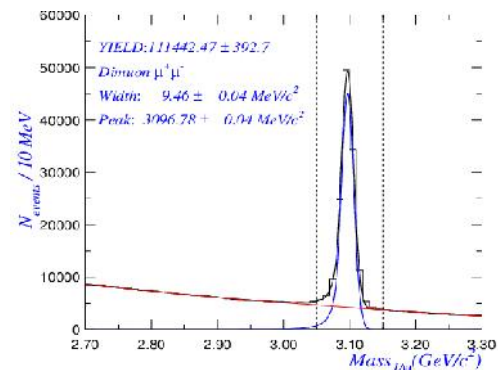


Figure 2. Top right: M_{bc} projection in $-0.04 < E < 0.04$ GeV. Bottom left: ΔE projection in $5.27 < M_{bc} < 5.29 \text{ GeV}/c^2$. Bottom right: $M_{bc} - \Delta E$ 2D distribution for $B \rightarrow J/\psi\pi^+\pi^-$.



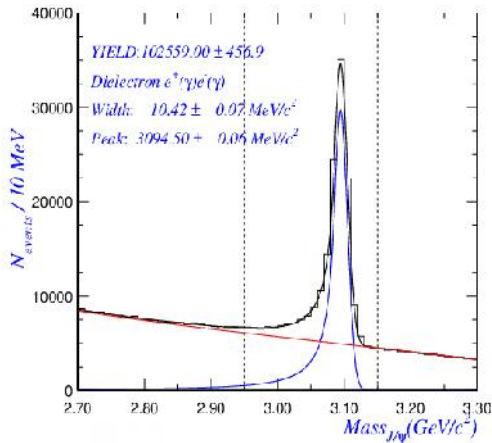


Figure 3. The fit to invariant mass distribution for $J/\psi \rightarrow \mu^+ \mu^-$ (top) and $J/\psi \rightarrow e^+ e^-$ (bottom) from experimental data. Dashed lines indicate the edges of mass window.

data (8.97 vs. 9.46 MeV/c²). The signal events for the dielectron mode are 102559 ± 457 , and, for the dimuon mode are 111443 ± 393 for $B \rightarrow J/\psi X$, where the errors are statistical.

The invariant mass and momentum distribution for charged pion pairs in centre of mass (cms) frame and laboratory (lab) frame respectively is plotted as shown in figure 4.

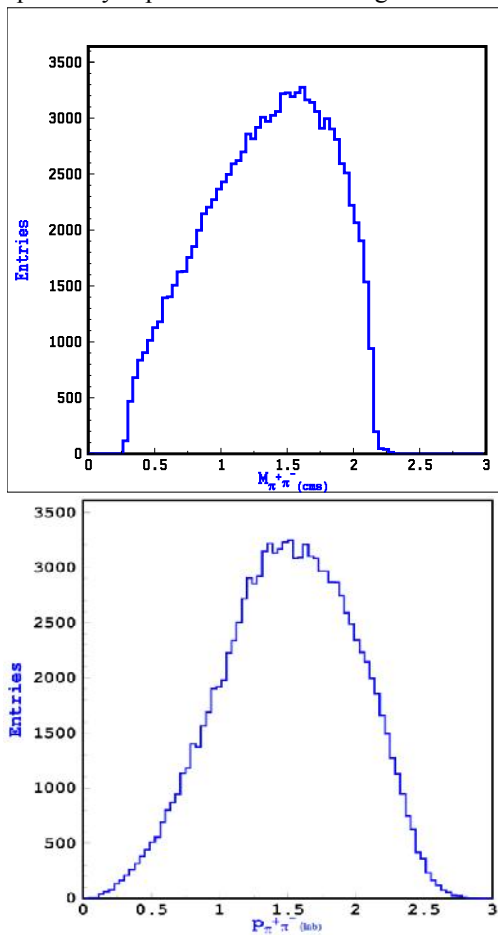


Figure 4. The invariant mass and momentum distribution for charged pion pairs in centre of mass (cms) frame (top) and laboratory (lab) frame (bottom) respectively.

VI. RESULTS

A histogram is made of ΔE distribution for experimental data superimposed with the background expectation obtained from large sample of J/ψ Monte Carlo shown in figure 5. The fit is performed using binned maximum likelihood analysis where signal shape is modeled by two Gaussians function (shown as red in figure 5), other backgrounds from $B^0 \rightarrow J/\psi K^0$, $B^0 \rightarrow J/\psi K^0(1430)^0$, $B^0 \rightarrow J/\psi K_2^*(1430)^0$, $B^\pm \rightarrow J/\psi K^\pm$, $B^\pm \rightarrow J/\psi \pi^\pm$ are expressed by smoothed histogram and remaining combinatorial backgrounds are modeled by Chebyshev polynomial of order 1. The obtained signal yield is 435 ± 29 . The peaking background for $B \rightarrow J/\psi K_S(\rightarrow \pi^+ \pi^-)$ (100 \pm 10) events in the signal region of ΔE is taken care by subtraction from total yield obtained corresponding to ΔE distribution. Hence, the obtained signal yield for $B^0 \rightarrow J/\psi \pi^+ \pi^-$ is 334 ± 30 .

The reconstruction efficiency is determined from signal MC, as the number of reconstructed events in reconstructed space out of the total number of generated events. The number of generated events are 100,000 for present study. The signal reconstruction efficiency is determined by fitting the M_{bc} and E distributions after applying all selection criteria and background rejection cuts

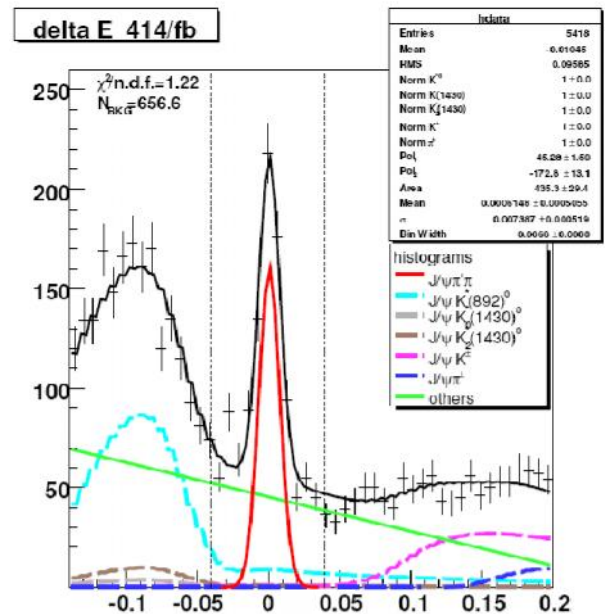


Figure 5. The fit to ΔE distribution from experimental data where signal ($B^0 \rightarrow J/\psi \pi^+ \pi^-$) is modeled by Gaussian function and combinatorial backgrounds are modeled by Chebyshev Polynomial of order 1. “Other” backgrounds are expressed by smooth histogram.

to the generated events using the EvtGen program [14] for signal MC. A GEANT4- based simulation [15] of the Belle detector is used to produce detector response to the generated events. The E distribution is modeled by two Gaussians function and a second order Chebyshev polynomial. The result of the fit to ΔE distribution is 32726.0 ± 182.1 events. We find the efficiency for reconstruction to be $(32.7 \pm 0.18) \%$ from Monte Carlo ΔE distribution, where the error is statistical.

Experimentally, the branching ratio of a decay to a certain final state is determined by measuring the fraction of events into this specific final state compared to all events. The branching fraction of total $B^0 \rightarrow J/\psi \pi^+ \pi^-$ is calculated using the formula:

$$\mathcal{B}(B^0 \rightarrow J/\psi \pi^+ \pi^-) = \frac{N_{J/\psi \pi^+ \pi^-}}{N_{B\bar{B}} \times \epsilon \times \mathcal{B}(J/\psi \rightarrow l^+ l^-)} \quad (1)$$

where $N_{J/\psi \pi^+ \pi^-}$ is the total signal yield obtained from the fit to ΔE distribution. $N_{B\bar{B}}$ are the total number of B mesons in the data sample and ϵ is the reconstruction efficiency for $B^0 \rightarrow J/\psi \pi^+ \pi^-$ decays and $\mathcal{B}(J/\psi \rightarrow l^+ l^-)$ is the daughter branching fraction.

The systematical uncertainties are summarized in Table 1. Significant sources of systematic uncertainty are in the tracking efficiency (1.0% per track), lepton identification (2.0% per track), charged pion pair identification (1.7%), Background PDF normalization (2.8%) and for daughter branching fraction $\mathcal{B}(J/\psi \rightarrow l^+ l^-)$ (0.7%). The total systematic error is the sum of all these uncertainties in quadrature.

Source	Uncertainty ($B^0 \rightarrow J/\psi \pi^+ \pi^-$)
Track Reconstruction	$\pm 4.0\%$
Lepton- ID	$\pm 4.0\%$
π^\pm - ID	$\pm 1.7\%$
Background PDF normalization	$\pm 2.8\%$
MC- statistics	$\pm 1.0\%$
$\mathcal{B}(J/\psi \rightarrow l^+ l^-)$	$\pm 0.7\%$
Total	6.6%

Table 1. Summary of systematic errors on branching fraction

The total yield obtained from the fit to ΔE distribution for experimental data is $N_{J/\psi \pi^+ \pi^-} = 334 \pm 30$. The data sample used for this analysis consists of $N_{B\bar{B}} = 449 \times 10^6$ $B\bar{B}$ pairs. Particle Data Group (PDG) values have been used for daughter branching fraction $\mathcal{B}(J/\psi \rightarrow l^+ l^-) = (11.93$

$\pm 0.06)\%$. The obtained branching fraction $\mathcal{B}(B^0 \rightarrow J/\psi \pi^+ \pi^-) = (1.90 \pm 0.17 \pm 0.12) \times 10^{-5}$, where the first error is statistical and the second error is systematic.

The branching fraction for $B^0 \rightarrow J/\psi \pi^+ \pi^-$ has previously been measured at BABAR to be $(4.6 \pm 0.7 \pm 0.6) \times 10^{-5}$, including a $J/\psi \rho^0$ component with a branching fraction of $(1.6 \pm 0.6 \pm 0.4) \times 10^{-5}$ [16]. This measurement used a data sample containing approximately 56 million $B\bar{B}$ pairs. The final state composition of this channel was also investigated using a 1.0 fb^{-1} sample of data produced in 7 TeV pp collisions at the LHC and collected by the LHCb experiment [17]. An improved measurement of the $B^0 \rightarrow J/\psi \pi^+ \pi^-$ branching fraction of $(3.97 \pm 0.09 \pm 0.11 \pm 0.16) \times 10^{-5}$ was reported where the first uncertainty is statistical, the second is systematic and the third is due to the uncertainty of the branching fraction of the decay $B^- \rightarrow J/\psi K^-$ used as a normalization channel. Also, CLEO collaboration reported the branching fraction measurements of $B^0 \rightarrow J/\psi \rho^0$ decays [18] with a data sample of 5.1 million $B\bar{B}$ pairs. The calculated branching fraction is in well agreement with the standard model prediction value.

VII. SUMMARY

The branching fraction for decay of neutral B mesons to charmonium and charged pion pair is reported in this paper. We reconstruct J/ψ from lepton pairs (e^+e^- and $\mu^+\mu^-$). The reconstructed signal efficiency from ΔE distribution is $(32.7 \pm 0.18) \%$. The measured branching fraction $\mathcal{B}(B^0 \rightarrow J/\psi \pi^+ \pi^-) = (1.90 \pm 0.17 \pm 0.12) \times 10^{-5}$, where the first error is statistical and the second error is systematic.

The authors are highly grateful to the KEKB group for excellent operation of the accelerator, the KEK cryogenics group for efficient solenoid operations, and the KEK computer group and the NII for valuable computing and SINET3 network support. Author are further grateful to the Belle collaboration and Indian Belle group for their feedback. The financial support of the DST (SERB), Govt. of India is hereby acknowledged under grant number SR/S2/HEP-05/2011.

REFERENCES

- [1] I. Dunietz, H. R. Quinn, A. Snyder, W. Toki and H. J. Lipkin, Angular distributions and lifetime differences in $B_s \rightarrow J/\psi \phi$ decays. Phys. Rev. D, 1991, 43, 2193 .
- [2] K. Hagiwara et al (Particle Data Group Collaboration), Review of Particle properties. Phys. Rev. D., 2002, 66, 010001.

- [3] Charge conjugation is implied throughout this paper unless stated otherwise.
- [4] B. Aubert et al (BaBar Collaboration), Improved measurement of CP Asymmetries in $B^0 \rightarrow (c\bar{c}) K^{0(\prime)}$ decays. Phys. Rev. Lett., 2005, 94, 161803.
- [5] K. Abe et al (Belle Collaboration), Improved measurement of CP- violation parameters $\sin 2\phi_1$ and $|\lambda|$, B meson lifetimes, and B^0 - B^0 mixing parameter Δm_d . Phys. Rev. D., 2005,71, 072003.
- [6] I. Dunietz, CP violation in $b \rightarrow d^+cc$ without tagging. Phys. Lett. B., 1993, 316,561.
- [7] M. Kobayashi and T. Maskawa, CP Violation in the Renormalizable Theory of Weak Interaction. Prog. Theor. Phys., 1973, 49,pp. 652-57.
- [8] S. Kurokawa and E. Kikutani, Overview of the KEKB Accelerator. Nucl. Instrum. Meth. A., 2003, 499, 1.
- [9] A. Abashian et al (Belle Collaboration), The Belle Detector. Nucl. Instr. and Meth. A., 2002, 479,117.
- [10] Y. Ushiroda, A. Mohapatra, H. Sakamoto, Y. Sakai, M. Nakao, Q. An, Y. F. Wang, Development of the central trigger system for the BELLE detector at the KEK B-factory. Nucl. Instrum. Meth. A., 1999, 438,pp. 460-71.
- [11] Z. Natkaniec et al (Belle SVD2 Group), Status of the Belle Silicon Vertex Detector. Nucl. Instr. Meth. A., 2006, 560,1.
- [12] G. C. Fox and S. Wolfram, Observables for the analysis of Event Shapes in e^+e^- Annihilation and Other Processes. Phys. Rev. Lett., 1978, 41,1581.
- [13] K. A. Olive et al, Review of Particle Physics. Chin. Phys. C., 2014, 38,090001.
- [14] D. J. Lange, The EvtGen particle decay simulation package. Nucl. Instrum. Meth. A., 2001, 462,152.
- [15] S. Agostinelli et al, GEANT4 - a simulation toolkit. Nucl. Instrum. Meth. A., 2003, 506,250.
- [16] B. Aubert et al (BaBar Collaboration), Branching fraction and charge asymmetry measurements in B to J/psi pi pi decays. Phys Rev D., 2007, 76, 031101.
- [17] R. Aaij et al (LHCb Collaboration), Analysis of the resonant components. in $B^0 \rightarrow J/\psi^+ \pi^-$. Phys. Rev. D., 2013, 87, 052001.
- [18] M. Bishai et al (CLEO Collaboration), Study of $B \rightarrow \psi \rho$. Phys. Rev. Lett B., 1996, 369, pp. 186-192.